Studies in Fuzziness and Soft Computing

Volume 371

Series editor

Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

The series "Studies in Fuzziness and Soft Computing" contains publications on various topics in the area of soft computing, which include fuzzy sets, rough sets, neural networks, evolutionary computation, probabilistic and evidential reasoning, multi-valued logic, and related fields. The publications within "Studies in Fuzziness and Soft Computing" are primarily monographs and edited volumes. They cover significant recent developments in the field, both of a foundational and applicable character. An important feature of the series is its short publication time and world-wide distribution. This permits a rapid and broad dissemination of research results.

More information about this series at http://www.springer.com/series/2941

Muhammad Akram

m–Polar Fuzzy Graphs

Theory, Methods & Applications

Springer

Muhammad Akram
Department of Mathematics
University of the Punjab
Lahore, Pakistan

ISSN 1434-9922 ISSN 1860-0808 (electronic)
Studies in Fuzziness and Soft Computing
ISBN 978-3-030-03750-5 ISBN 978-3-030-03751-2 (eBook)
https://doi.org/10.1007/978-3-030-03751-2

Library of Congress Control Number: 2018960747

© Springer Nature Switzerland AG 2019
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

I dedicate this book to my worthy teachers who taught me at any stage of learning!

Foreword

Graph theory is one of the branches of mathematics that has experienced an impressive development in recent years. The origin of graph theory started with the Königsberg bridge problem. This problem led to the concept of the Eulerian graph. Euler studied the Königsberg bridge problem and constructed a structure that solves the problem that is referred to as Eulerian graph. The first book on graph theory was by König in 1935 followed by a book by Frank Harary in 1969.

Encouraged by Lotfi A. Zadeh, C.-L. Chang wrote a paper on fuzzy topological spaces. Upon seeing this paper, Azriel Rosenfeld wrote a paper on fuzzy group theory and then followed this paper with a paper on fuzzy graphs. Both of Rosenfeld's papers opened the door for the development of the entire field of fuzzy abstract algebra and fuzzy graph theory, respectively.

Fuzzy graph theory has also experienced an impressive growth in recent years. Fuzzy graph theory has been important in technological development. Fuzzy graph theory has paved the way for engineers to build many rule-based expert systems. Applications can also be found in computer science, artificial intelligence, decision analysis, information science, systems science, control engineering, expert systems, pattern recognition, management science, operations research, robotics, and social situations. The theory and application of fuzzy graphs is currently a hot topic. One can thank the seminal work by Rosenfeld and Yeh and Bang for this important development.

Recently, Enric Trillas and Itziar Garcia-Honrado have analyzed dialectic synthesis using the methodology of fuzzy logic and limiting the antithesis to either negation or opposite. Dialectic synthesis is concerned with a method of reasoning by means of the triplet Thesis–Antithesis–Synthesis triad. Their analysis uses very general operations. For example, for conjunction and disjunction, it is not assumed that the commutative laws, associative laws, distributive laws, idempotent laws nor those of duality hold. This generality agrees with a recent new approach in fuzzy graph theory to relax the basic definitions in fuzzy graph theory by replacing the operations minimum and maximum with t-norms and t-conorms and even aggregation operators. It has recently been shown that the dialectic synthesis method can be used to construct certain types of fuzzy graphs. The journal New Mathematics

and Natural Computation and the publisher Springer International Publishing are currently supporting this effort.

In many real-world problems, data sometimes comes from two or more agents and so multi-polar information exists. This information cannot be well represented by means of fuzzy graphs or bipolar fuzzy graphs. In these situations, *m*–polar fuzzy set theory is applied to graphs to describe the relationships among several agents.

This book introduces readers to fundamental theories such as *m*–polar sets and graphs, metrics in *m*–polar graphs, labeling in *m*–polar graphs, and *m*–polar fuzzy matroids. The interesting concept of domination in *m*–polar fuzzy graphs is also presented. It also discusses in detail the important applications in decision-making problems and imaging processing. One of the objectives of the book is to present the development of a mathematical approach to multi-index, multi-polar, and multi-attribute data. The author is a well-known researcher in fuzzy graphs. He has made another important contribution to fuzzy graph theory with the publication of this book.

Omaha, Nebraska, USA

John N. Mordeson
Creighton University

Preface

Fuzzy set theory owes its origin to the work of Zadeh. After the significant introduction of fuzzy set theory, this remarkable approach has been applied to various domains. In 1975, Rosenfeld first discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann in 1973. The invaluable contribution of Profs. Mordeson and Nair on "Fuzzy Graphs and Fuzzy Hypergraphs" inspired me to work in this domain.

In most of the real problems, information consistently comes from more than one agents or sources. We must acknowledge that multi-agent, multi-attribute, multi-object, multi-index, and multipolar information exist, and these inputs cannot be correctly expressed by the fuzzy modeling. These concerns motivated the birth of many wider or alternative models. Particularly, Chen et al. introduced the notion of m–polar fuzzy sets that inspire the concept to which this monograph is devoted, namely, m–polar fuzzy graphs. The work presented here intends to overcome the lack of a mathematical approach toward multi-index, multi-polar, and multi-attribute data. This monograph deals with certain m–polar fuzzy graphs and their applications. It is based on a number of papers by the author, which have been published in various scientific journals. The book may be useful for researchers in mathematics, computer scientists, and social scientists alike. I believe that this book will be helpful to students and researchers wishing to learn and apply m–polar fuzzy graph theory.

In Chap. 1, I present a concise review of m–polar fuzzy sets. Then I consider m–polar fuzzy graphs, and define isomorphism of m–polar fuzzy graphs. I present a characterization of m-polar fuzzy graphs in terms of their level graphs. I also consider energy of m-polar fuzzy graphs.

In Chap. 2, I introduce some relevant types of m–polar fuzzy graphs, including regular m–polar fuzzy graphs, irregular m–polar fuzzy graphs, k-neighborly irregular m–polar fuzzy graphs, edge m–polar fuzzy graphs, edge irregular m–polar fuzzy graphs. I also explore their applications. In Chap. 3, I present certain metrics in m–polar fuzzy graphs. I also describe applications of m–polar fuzzy graphs in decision-making problems.

In Chap. 4, I present m–polar fuzzy labeling graphs, distances in m–polar fuzzy labeling graphs, labeling tree based on m–polar fuzzy sets, and bipartite m–polar fuzzy labeling graphs. I also describe an application of m–polar fuzzy labeling graphs in image processing.

In Chap. 5, I present domination in m–polar fuzzy graphs. I consider the concepts of strong domination, weak domination, edge domination, equitable domination, k-domination, and restrained domination in m–polar fuzzy graphs. I also present their applications in Chap. 5.

In Chap. 6, I present m–polar fuzzy concept lattice and its applications. Chapter 7 is devoted to m–polar fuzzy graph structures.

Chapter 8 deals with various concepts in m–polar fuzzy hypergraphs. Certain algorithms are developed in order to construct dual m–polar fuzzy hypergraphs and m–polar fuzzy line graphs. In addition, m–polar fuzzy transversals and locally minimal m–polar fuzzy transversals of m–polar fuzzy hypergraphs with their applications are studied. Chapter 9 deals with m–polar fuzzy matroids and their applications.

Lahore, Pakistan Muhammad Akram

Acknowledgements

I am grateful to the administration of University of the Punjab who provided the facilities which were required for successful completion of this monograph. I pay my gratitude to Prof. Dr. Mujahid Kamran (Ex-Vice Chancellor, University of the Punjab) for his encouraging attitude and for supplying the necessary research facilities during his tenure. I would like to acknowledge the assistance of my students Musavarah Sarwar, Arooj Adeel, and Neha Waseem. At the last but not the least, I thank my wife Safoora and my four children: Zainab, Muhammad Abdullah, Humaira Fatima and Muhammad Abdul Rehman, who sacrificed their leisure time for my work. Without their cooperation, it was hard for me to concentrate on my scientific research and successfully accomplish my task.

Contents

1 *m*–Polar Fuzzy Sets and *m*–Polar Fuzzy Graphs 1
1.1 Fuzzy Notions 2
1.2 *m*–Polar Fuzzy Sets 3
1.3 *m*–Polar Fuzzy Graphs 7
1.4 Energy of *m*–Polar Fuzzy Graphs 34

2 Certain Types of *m*–Polar Fuzzy Graphs 51
2.1 Regular *m*–Polar Fuzzy Graphs 51
2.2 $(2,(p_1,p_2,\ldots,p_m))$-Regular *m*–Polar Fuzzy Graphs 55
2.3 d_2-Degrees of Certain Products of *m*–Polar Fuzzy Graphs 64
2.4 Irregular *m*–Polar Fuzzy Graphs 68
2.5 *k*-Neighbourly Irregular *m*–Polar Fuzzy Graphs 76
2.6 Edge *m*–Polar Fuzzy Graphs 83
2.7 Edge Irregularity in *m*–Polar Fuzzy Graphs 94
2.8 Applications 106

3 Certain Metrics in *m*–Polar Fuzzy Graphs 113
3.1 Metric in *m*–Polar Fuzzy Graphs 113
3.2 Antipodal *m*–Polar Fuzzy Graphs 121
3.3 Applications 130

4 *m*–Polar Fuzzy Labeling Graphs 135
4.1 *m*–Polar Fuzzy Labeling Graphs 135
4.2 Distances in *m*–Polar Fuzzy Labeling Graphs 140
4.3 Labeling Tree Based on *m*–Polar Fuzzy Sets 145
4.4 Bipartite *m*–Polar Fuzzy Labeling Graph 148
4.5 Application 150

5 Certain Dominations in *m*–Polar Fuzzy Graphs 153
5.1 Domination in *m*–Polar Fuzzy Graphs 153
5.2 Strong and Weak Domination in *m*–Polar Fuzzy Graphs 160
5.3 Edge Domination in *m*–Polar Fuzzy Graphs 161
5.4 Equitable Domination in *m*–Polar Fuzzy Graphs 167
5.5 *k*–Domination and Restrained Domination 174
5.6 Applications 180

6 *m*–Polar Fuzzy Concept Lattice 185
6.1 *m*–Polar Fuzzy Concept Lattice 185
6.2 Applications 199

7 *m*–Polar Fuzzy Graph Structures 209
7.1 *m*–Polar Fuzzy Graph Structures 209

8 *m*–Polar Fuzzy Hypergraphs 235
8.1 *m*–Polar Fuzzy Hypergraphs 236
8.2 Dual *m*–Polar Fuzzy Hypergraph 249
8.3 Applications 251
8.4 Transversals of *m*–Polar Fuzzy Hypergraphs 254
8.5 Applications 266

9 *m*–Polar Fuzzy Matroids 271
9.1 *m*–Polar Fuzzy Matroids 271
9.2 Applications 282

Bibliography 285

Index 293

About the Author

Dr. Muhammad Akram has received M.Sc. degree in Mathematics and Computer Science, M.Phil. in Computational Mathematics, and Ph.D. in Fuzzy Mathematics. He is currently a Professor in the Department of Mathematics at the University of the Punjab, Lahore, Pakistan. Dr. Akram's research interests include numerical solutions of parabolic PDEs, fuzzy graphs, fuzzy algebras, and fuzzy decision support systems. He has published 5 monographs over 270 research articles in international peer-reviewed journals. He has been an Editorial Member of 10 international academic journals and reviewer/referee for 114 international journals, including Mathematical Reviews (USA) and Zentralblatt MATH (Germany). Seven students have successfully completed their Ph.D. under his supervision and he is currently supervising five more Ph.D. students.

List of Figures

Fig. 1.1 Fuzzy graph G 4
Fig. 1.2 Order relation when $m = 2$ 5
Fig. 1.3 Order relation when $m = 4$ 5
Fig. 1.4 3–polar fuzzy graph 8
Fig. 1.5 Underlying graph 10
Fig. 1.6 Strong 4–polar fuzzy graph. 10
Fig. 1.7 Complement of a 3–polar fuzzy graph 11
Fig. 1.8 Homomorphic 4–polar fuzzy graphs 11
Fig. 1.9 Isomorphic 3–polar fuzzy graphs 12
Fig. 1.10 Weak isomorphic 3–polar fuzzy graphs 13
Fig. 1.11 Co-weak isomorphic 3–polar fuzzy graphs 13
Fig. 1.12 $\widetilde{P_5}$ 15
Fig. 1.13 Complete 3–polar fuzzy graph 16
Fig. 1.14 Bipartite 3–polar fuzzy graph 16
Fig. 1.15 Complete bipartite 3–polar fuzzy graph 17
Fig. 1.16 3–polar fuzzy graph 17
Fig. 1.17 Cartesian product $\widetilde{P}_4 \square \widetilde{P}_4$ 18
Fig. 1.18 Direct product $\widetilde{P}_3 \times \widetilde{P}_3$ 21
Fig. 1.19 Direct product $\widetilde{P}_3 \times \widetilde{C}_3$ 23
Fig. 1.20 Strong product $\widetilde{P}_3 \boxtimes \widetilde{P}_3$ 24
Fig. 1.21 Lexicographic product $\widetilde{P}_3 \bullet \widetilde{P}_3$ 25
Fig. 1.22 3–polar fuzzy graph $G = (C, D)$ 26
Fig. 1.23 4–polar fuzzy graph 40
Fig. 1.24 5–polar fuzzy graph 43
Fig. 1.25 4–polar fuzzy graph 48
Fig. 2.1 Regular and totally regular 3–polar fuzzy graph 52
Fig. 2.2 Totally regular 3–polar fuzzy graph 53
Fig. 2.3 4–polar fuzzy graph 53

Fig. 2.4 Totally regular 3–polar fuzzy graph 54
Fig. 2.5 3–polar fuzzy graph 55
Fig. 2.6 d_2-degree of an 3–polar fuzzy graph. 56
Fig. 2.7 d_2-degree of 3–polar fuzzy graph 57
Fig. 2.8 $(2,(0.2,0.1,0.4))$-regular 3–polar fuzzy graph 58
Fig. 2.9 Totally $(2,(0.8,1.2,1.1))$-regular 3–polar fuzzy graph 58
Fig. 2.10 3–polar fuzzy graph 59
Fig. 2.11 $(2,(0.2,0.3,0.2))$-regular 3–polar fuzzy graph 60
Fig. 2.12 $(2,(0.4,0.3,0.5))$-regular 3–polar fuzzy graph 61
Fig. 2.13 Barbell graph 62
Fig. 2.14 3–polar fuzzy graph of length 6 62
Fig. 2.15 3–polar fuzzy graph G 63
Fig. 2.16 $G_1 \times_\alpha G_2$ (alpha product). 65
Fig. 2.17 $G_1 \times_\beta G_2$ (beta product) 66
Fig. 2.18 $G_1 \times_\gamma G_2$ (gamma product) 67
Fig. 2.19 3–polar fuzzy graph 68
Fig. 2.20 4–polar fuzzy graph 69
Fig. 2.21 3–polar fuzzy graph 69
Fig. 2.22 4–polar fuzzy graph 70
Fig. 2.23 3–polar fuzzy graph 70
Fig. 2.24 3–polar fuzzy graph 71
Fig. 2.25 3–polar fuzzy graph 71
Fig. 2.26 4–polar fuzzy graph 71
Fig. 2.27 3–polar fuzzy graph 72
Fig. 2.28 4–polar fuzzy graph 73
Fig. 2.29 3–polar fuzzy graph 73
Fig. 2.30 3–polar fuzzy graph and 3–polar fuzzy subgraph 75
Fig. 2.31 5–polar fuzzy graph and 5–polar fuzzy subgraph 75
Fig. 2.32 3–polar fuzzy graph 76
Fig. 2.33 2–neighbourly irregular 3–polar fuzzy graph 77
Fig. 2.34 3–neighbourly totally irregular 3–polar fuzzy graph 78
Fig. 2.35 2–neighbourly totally irregular 4–polar fuzzy graph 79
Fig. 2.36 Edge regular 5–polar fuzzy graph. 84
Fig. 2.37 Totally edge regular 4–polar fuzzy graph 85
Fig. 2.38 Complete 4–polar fuzzy graph 86
Fig. 2.39 4–polar fuzzy graph 87
Fig. 2.40 3–polar fuzzy graph 90
Fig. 2.41 Full edge regular 4–polar fuzzy graph 93
Fig. 2.42 Strongly edge irregular 5–polar fuzzy graph. 94
Fig. 2.43 Strongly edge totally irregular 5–polar fuzzy graph 96
Fig. 2.44 3–polar fuzzy graph 96
Fig. 2.45 3–polar fuzzy graph 97

Fig. 2.46 4–polar fuzzy graph 98
Fig. 2.47 Neighborly edge irregular 5–polar fuzzy graph. 99
Fig. 2.48 4–polar fuzzy graph 100
Fig. 2.49 Highly irregular 3–polar fuzzy graph 101
Fig. 2.50 Path on $2n$ vertices 101
Fig. 2.51 Star $K_{1,s}$ 103
Fig. 2.52 Barbell graph $B_{n,n}$ 104
Fig. 2.53 5–polar fuzzy graph 106
Fig. 2.54 4–polar fuzzy graph 107
Fig. 2.55 3–polar fuzzy graph of an airline network 109
Fig. 2.56 5–polar fuzzy graph 110
Fig. 2.57 5–polar fuzzy digraph. 110
Fig. 3.1 3–polar fuzzy graph 115
Fig. 3.2 2–polar fuzzy graph 118
Fig. 3.3 Center of a 2–polar fuzzy graph 118
Fig. 3.5 Antipodal 4–polar fuzzy graph 122
Fig. 3.6 3–polar fuzzy graphs G and its antipodal $A(G)$. 125
Fig. 3.7 3–polar fuzzy graphs $\bar{G}$ and its antipodal $A(\bar{G})$ 126
Fig. 3.8 Complement of $A(G)$ 126
Fig. 3.9 4–polar fuzzy graphs G and its antipodal $A(G)$. 126
Fig. 3.10 4–polar fuzzy graphs 127
Fig. 3.11 Complement of 4–polar fuzzy graph $A(G)$ 127
Fig. 3.12 Self median 4–polar fuzzy graph 128
Fig. 3.13 4–polar fuzzy model of product manufacturing. 130
Fig. 3.14 3–polar fuzzy model of shortest path problem 132
Fig. 3.15 3–polar fuzzy digraph of a road network 133
Fig. 4.1 3–polar fuzzy graph 136
Fig. 4.2 3–polar fuzzy path 136
Fig. 4.3 4–polar fuzzy labeling graph 137
Fig. 4.4 3–polar fuzzy star. 138
Fig. 4.5 2–polar fuzzy labeling graph 139
Fig. 4.6 3–polar fuzzy labeling with four vertices and five edges 140
Fig. 4.7 4–polar fuzzy labeling graph 141
Fig. 4.8 3–polar fuzzy labeling with four vertices 143
Fig. 4.9 3–polar fuzzy labeling with five vertices 144
Fig. 4.10 3–polar fuzzy labeling tree 146
Fig. 4.11 3–polar fuzzy labeling graph 146
Fig. 4.12 4–polar fuzzy labeling tree 147
Fig. 4.13 Bipartite 3–polar fuzzy labeling graph 148
Fig. 4.14 3–polar fuzzy labeling tree when G^* is complete 149
Fig. 4.15 Connected graph of image blocks. 150

Fig. 4.16 4–polar fuzzy labeling graph of image processing 151
Fig. 5.1 Effective edges of 5–polar fuzzy graph G. 154
Fig. 5.2 Dominating set of a 5–polar fuzzy graph 155
Fig. 5.3 Minimal dominating set of G 155
Fig. 5.4 Isolated vertex of a 5–polar fuzzy graph. 156
Fig. 5.5 Independent set of a 4–polar fuzzy graph 157
Fig. 5.6 5–polar fuzzy graph 158
Fig. 5.7 Total dominating set of a 5–polar fuzzy graph 159
Fig. 5.8 Strong dominating set of G. 160
Fig. 5.9 Edge dominating set of a 6–polar fuzzy graph 162
Fig. 5.10 Minimal edge dominating set of G 162
Fig. 5.11 Isolated edge of a 5–polar fuzzy graph. 163
Fig. 5.12 Edge independent set of G 164
Fig. 5.13 4–polar fuzzy graph 165
Fig. 5.14 Total edge dominating set of G 166
Fig. 5.15 Equitable neighborhood of vertices of 5–polar fuzzy graph 167
Fig. 5.16 Equitable isolated vertex of G 168
Fig. 5.17 Equitable dominating set of a 5–polar fuzzy graph. 168
Fig. 5.18 Degree equitable 6–polar fuzzy graph. 169
Fig. 5.19 Equitable independent set of a 4–polar fuzzy graph 169
Fig. 5.20 Strong equitable dominating set of G 170
Fig. 5.21 Total equitable dominating set of G 172
Fig. 5.22 Bi-regular 4–polar fuzzy graph 174
Fig. 5.23 4-dominating set of a 5–polar fuzzy graph 175
Fig. 5.24 Total 3-dominating set of a 4–polar fuzzy graph 175
Fig. 5.25 Global restrained dominating set of G 176
Fig. 5.26 3–polar fuzzy graphs 177
Fig. 5.27 Direct product of G_1 and G_2. 178
Fig. 5.28 Cartesian product of G_1 and G_2 178
Fig. 5.29 4–polar fuzzy graph of representatives 181
Fig. 5.30 3–polar fuzzy graph of a wireless network 182
Fig. 5.31 4–polar fuzzy graph of towns 183
Fig. 6.1 3–polar fuzzy concept lattice $\mathcal{L}(Y, Z, \xi)$ 189
Fig. 6.2 3–polar fuzzy concept lattice $\mathcal{L}(Y, Z, \xi)$ 191
Fig. 6.3 3–polar fuzzy concept lattice $\mathcal{L}(Y, Z_d, \xi_d)$ 194
Fig. 6.4 3–polar fuzzy concept lattice $\mathcal{L}(Y, Z_{d_1}, \xi_{d_1})$ 195
Fig. 6.5 3–polar fuzzy concept lattice 203
Fig. 6.6 Actions of the suspect. 204
Fig. 6.7 3–polar fuzzy concept lattice 207
Fig. 7.1 4–polar fuzzy graph structure 211
Fig. 7.2 4–polar fuzzy subgraph structures. 211
Fig. 7.3 Strong 4–polar fuzzy graph structure 212

Fig. 7.4 4–polar fuzzy D_2-cycle 214
Fig. 7.5 4–polar fuzzy $D_{1,1}$-tree 215
Fig. 7.6 Two isomorphic 4–polar fuzzy graph structures 216
Fig. 7.7 Identical 4–polar fuzzy graph structures 217
Fig. 7.8 Strong self-complementary 4–polar fuzzy graph structure. 219
Fig. 7.9 Totally strong self-complementary 4–polar fuzzy graph structure 220
Fig. 7.10 4–polar fuzzy graph structure 220
Fig. 7.11 Cartesian product of two 4–polar fuzzy graph structures 221
Fig. 7.12 Cross product of two 4–polar fuzzy graph structures 222
Fig. 7.13 Lexicographic product of tTwo 4–polar fuzzy graph structures 223
Fig. 7.14 Strong product of two 4–polar fuzzy graph structures 225
Fig. 7.15 Composition of two 4–polar fuzzy graph structures 227
Fig. 7.16 Union of two m–polar fuzzy graph structures. 229
Fig. 7.17 Join of two m–polar fuzzy graph structures 231
Fig. 8.1 3–polar fuzzy hypergraph 237
Fig. 8.2 3–polar fuzzy hypergraph 237
Fig. 8.3 3–polar fuzzy hypergraph H 239
Fig. 8.4 3–polar fuzzy hypergraph $H^{'}$ 239
Fig. 8.5 3–polar fuzzy hypergraph H_1 240
Fig. 8.6 3–polar fuzzy hypergraph H_2 241
Fig. 8.7 3–polar fuzzy hypergraph H_1 241
Fig. 8.8 3–polar fuzzy hypergraph H_2 242
Fig. 8.9 3–polar fuzzy hypergraph H_1 243
Fig. 8.10 3–polar fuzzy hypergraph H_2 243
Fig. 8.11 2–polar fuzzy hypergraph 244
Fig. 8.12 3–polar fuzzy line graph $L(H)$ 247
Fig. 8.13 3–polar fuzzy hypergraph from 3–polar fuzzy graph 248
Fig. 8.14 2–section of a 3–polar fuzzy hypergraph 249
Fig. 8.15 Mapping $f : Y \rightarrow E$ between vertices and hyperedges 250
Fig. 8.16 Dual 2–polar fuzzy hypergraph H^D 250
Fig. 8.17 Planet surface communication model 252
Fig. 8.18 4–polar fuzzy social hypergraph 253
Fig. 8.19 Grouping of incompatible chemicals. 254
Fig. 8.20 2–polar fuzzy hypergraph H 255
Fig. 8.21 $(0.2, 0.4)$-level hypergraph $H_{(0.2,0.4)}$ 255
Fig. 8.22 $(0.3, 0.4)$-level hypergraph $H_{(0.3,0.4)}$ 256
Fig. 8.23 3–polar fuzzy hypergraph H 256
Fig. 8.24 H_{r_1} 256
Fig. 8.25 H_{r_2} 257
Fig. 8.26 H_{r_3} 257

Fig. 8.27 H_{r_4} 257
Fig. 8.28 3–polar fuzzy hypergraph H 258
Fig. 8.29 3–polar fuzzy hypergraph $\widehat{H}$ 258
Fig. 8.30 Elementary 3–polar fuzzy hypergraph. 258
Fig. 8.31 Non-elementary 4–polar fuzzy graph 259
Fig. 8.32 A 3–polar fuzzy hypergraph model for investment management 267
Fig. 8.33 Multiple channel mesh network 268
Fig. 9.1 3–polar fuzzy multigraph 276
Fig. 9.2 Wireless communication 284
Fig. 9.3 Communication network with minimum connections 284

List of Tables

Table 1.1 Fuzzy sets on network servers 2
Table 1.2 Fuzzy sets of species 2
Table 1.3 Fuzzy set μ 3
Table 1.4 Fuzzy relation λ. 3
Table 1.5 3–polar fuzzy set C 6
Table 1.6 3–polar fuzzy set on Y. 8
Table 1.7 3–polar fuzzy relation in Y 8
Table 1.8 3–polar fuzzy set C 15
Table 1.9 3–polar fuzzy relation D 15
Table 1.10 4–polar fuzzy set C 40
Table 1.11 4–polar fuzzy relation D 40
Table 1.12 5–polar fuzzy set C 41
Table 1.13 5–polar fuzzy relation D 41
Table 1.14 4–polar fuzzy set C 47
Table 1.15 4–polar fuzzy relation D 48
Table 2.1 4–polar fuzzy set on Y. 53
Table 2.2 4–polar fuzzy relation in Y 53
Table 3.1 Distances in m–polar fuzzy graphs. 129
Table 4.1 Labeling of 4–polar fuzzy set C_p^ω 136
Table 4.2 Labeling of 4–polar fuzzy relation D_p^ω 137
Table 4.3 4–polar fuzzy set of image block B_p^ω 150
Table 4.4 4–polar fuzzy relation R_p^ω on image blocks 151
Table 4.5 4–polar fuzzy relation R_p^ω on image blocks 151
Table 6.1 3–polar fuzzy formal context 186
Table 6.2 m–polar fuzzy formal concepts 187
Table 6.3 Fuzzy context 1 190
Table 6.4 Fuzzy context 2 190
Table 6.5 Fuzzy context 3 190
Table 6.6 3–polar fuzzy formal context 191

Table 6.7 3–polar fuzzy formal concepts 191
Table 6.8 3–polar fuzzy formal concepts 194
Table 6.9 3–polar fuzzy formal concepts 194
Table 6.10 3–polar fuzzy formal context 197
Table 6.11 Discernibility matrix. 197
Table 6.12 Discernibility matrix. 198
Table 6.13 Women and child trafficking indicators 201
Table 6.14 Description of degree of membership. 201
Table 6.15 3–polar fuzzy formal context 201
Table 6.16 3–polar fuzzy formal concepts 202
Table 6.17 Movements of suspect in different time dimensions. 203
Table 6.18 Degree of existence of trafficking indications of the suspect. 204
Table 6.19 Degree of being suspicious corresponding to trafficking indications 204
Table 6.20 Child abduction indicators 205
Table 6.21 Description of degree of membership of child abduction indications 205
Table 6.22 3–polar fuzzy formal context 206
Table 6.23 3–polar fuzzy formal concepts 206
Table 7.1 4–polar fuzzy sets on Y, E_1 and E_2. 210
Table 7.2 4–polar fuzzy set on Y. 212
Table 7.3 4–polar fuzzy sets on E_1 and E_2 212
Table 7.4 Connectedness between vertices. 213
Table 8.1 3–polar fuzzy subsets on Y 236
Table 8.2 3–polar fuzzy subsets ξ_1 and ξ_2. 237
Table 8.3 4–polar fuzzy set on Y. 238
Table 8.4 4–polar fuzzy set on Y' 239
Table 8.5 Range of fuzzy values for the amount of taxes 266
Table 8.6 Strength of level of risk and profit on investment assets 267