

# **Studies in Systems, Decision and Control**

Volume 265

## **Series Editor**

Janusz Kacprzyk, Systems Research Institute, Polish Academy of Sciences,  
Warsaw, Poland

The series “Studies in Systems, Decision and Control” (SSDC) covers both new developments and advances, as well as the state of the art, in the various areas of broadly perceived systems, decision making and control—quickly, up to date and with a high quality. The intent is to cover the theory, applications, and perspectives on the state of the art and future developments relevant to systems, decision making, control, complex processes and related areas, as embedded in the fields of engineering, computer science, physics, economics, social and life sciences, as well as the paradigms and methodologies behind them. The series contains monographs, textbooks, lecture notes and edited volumes in systems, decision making and control spanning the areas of Cyber-Physical Systems, Autonomous Systems, Sensor Networks, Control Systems, Energy Systems, Automotive Systems, Biological Systems, Vehicular Networking and Connected Vehicles, Aerospace Systems, Automation, Manufacturing, Smart Grids, Nonlinear Systems, Power Systems, Robotics, Social Systems, Economic Systems and other. Of particular value to both the contributors and the readership are the short publication timeframe and the world-wide distribution and exposure which enable both a wide and rapid dissemination of research output.

\*\* Indexing: The books of this series are submitted to ISI, SCOPUS, DBLP, Ulrichs, MathSciNet, Current Mathematical Publications, Mathematical Reviews, Zentralblatt Math: MetaPress and Springerlink.

More information about this series at <http://www.springer.com/series/13304>

Yinyan Zhang · Shuai Li · Xuefeng Zhou

# Deep Reinforcement Learning with Guaranteed Performance

A Lyapunov-Based Approach

Yinyan Zhang  
College of Cyber Security  
Jinan University  
Guangzhou, China

Shuai Li  
School of Information Science and  
Engineering  
Lanzhou University  
Lanzhou, China

Xuefeng Zhou  
Guangdong Institute of Intelligent  
Manufacturing  
Guangdong Academy of Science  
Guangzhou, China

ISSN 2198-4182                      ISSN 2198-4190 (electronic)  
Studies in Systems, Decision and Control  
ISBN 978-3-030-33383-6              ISBN 978-3-030-33384-3 (eBook)  
<https://doi.org/10.1007/978-3-030-33384-3>

© Springer Nature Switzerland AG 2020

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG  
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*To our ancestors and parents, as always*

# Preface

In the past decades, both optimal control and adaptive control have been widely investigated to solve control problems of nonlinear systems arising from engineering applications. Optimal control aims at finding a control law to drive a control system to a desired state while optimizing certain performance index with or without constraints. Adaptive control is a tool to handle parameter uncertainty or structure uncertainty of control systems. In most works, the two types of control methods are separated. Reinforcement learning and, in particular, deep reinforcement learning have attracted more and more research interest in recent years. Such type of learning methods could be a powerful tool for the control of nonlinear systems.

In this book, we present our systematic investigations on a near-optimal adaptive control method based on the Taylor expansion, neural networks, estimator design approaches, and the idea of sliding mode control. We mainly handle the tracking control problem of nonlinear systems under different scenarios, for which the output of a controlled nonlinear system is expected to track a desired reference output trajectory with respect to time  $t$ . The performance of the presented approach is theoretically guaranteed. Specifically, the performance index is asymptotically convergent to the optimal and the tracking error is asymptotically convergent to 0. Issues like actuator saturations are also considered in this book. By combining the design method of the near-optimal adaptive control method and the novel ideas in the developments of methods for the redundancy resolution of redundant manipulators, two new redundancy resolution methods are also presented.

To make the contents clear and easy to follow, in this book, each part (and even each chapter) is written in a relatively self-contained manner.

This book is divided into the following seven chapters.

Chapter 1—In this chapter, a brief review is provided about recent advancements in the field of optimal control.

Chapter 2—In this chapter, a time-scale expansion-based scheme is presented for approximately solving the optimal control problem of continuous-time underactuated nonlinear systems subject to input constraints and system dynamics. By time-scale Taylor approximation of the original performance index, the optimal control problem is relaxed into an approximated optimal control problem. Based on

the system dynamics, the problem is further reformulated as a quadratic program, which is solved by a projection neural network. Theoretical analysis on the closed-loop system synthesized by the controlled system and the projection neural network is conducted, which reveals that, under certain conditions, the closed-loop system possesses exponential stability and the original performance index converges to zero as time tends to infinity.

**Chapter 3**—In this chapter, a unified online adaptive near-optimal control framework is presented for linear and nonlinear systems with parameter uncertainty. Under this framework, auxiliary systems converging to the unknown dynamics are constructed to approximate and compensate the parameter uncertainty. With the aid of the auxiliary system, future outputs of the controlled system are predicted recursively. By utilizing a predictive time-scale approximation technique, the nonlinear dynamic programming problem for optimal control is significantly simplified and decoupled from the parameter learning dynamics: the finite-horizon integral-type objective function is simplified into a quadratic one relative to the control action and there is no need to solve time-consuming Hamilton equations. Theoretical analysis shows that closed-loop systems are asymptotically stable. It is also proved that the presented adaptive near-optimal control law asymptotically converges to the optimal. The efficacy of the presented framework and the theoretical results are validated by an application to underactuated surface vessels.

**Chapter 4**—In this chapter, an adaptive near-optimal control law, which is inherently real time, is designed to tackle the contradictory between solution accuracy and solution speed for the optimal control of a general class of nonlinear systems with fully unknown parameters. The key technique in the presented adaptive near-optimal control is to design an auxiliary system with the aid of the sliding mode control concept to reconstruct the dynamics of the controlled nonlinear system. Based on the sliding-mode auxiliary system and approximation of the performance index, the presented control law guarantees asymptotic stability of the closed-system and asymptotic optimality of the performance index with time. Two illustrative examples and an application of the presented method to a van der Pol oscillator are presented to validate the efficacy of the presented adaptive near-optimal control. In addition, physical experiment results based on a DC motor are also presented to show the realizability, performance, and superiority of the presented method.

**Chapter 5**—In this chapter, the receding-horizon near-optimal tracking control problem about a class of continuous-time nonlinear systems with fully unknown dynamics is considered. A novel model-free adaptive near-optimal control method is presented to solve this problem via utilizing the Taylor expansion-based problem relaxation, the universal approximation property of sigmoid neural networks, and the concept of sliding mode control. By making approximation for the performance index, it is first relaxed to a quadratic program, and then a linear algebraic equation with unknown terms. An auxiliary system is designed to reconstruct the input-to-output property of the control systems with unknown dynamics so as to tackle the difficulty caused by the unknown terms. Then, by considering the property of the sliding mode surface, an explicit adaptive near-optimal control law is derived from the linear algebraic equation. Theoretical analysis shows that the

auxiliary system is convergent, the resultant closed-loop system is asymptotically stable, and the performance index asymptotically converges to optimal. An illustrative example and experimental results are presented, which substantiate the efficacy of the presented method and verify the theoretical results.

Chapter 6—In this chapter, an adaptive projection neural network (PNN) with online learning is presented for the redundancy resolution of manipulators with unknown physical parameters, which tackles the dilemmas in existing methods. The presented method is capable of simultaneously optimizing performance indices subject to physical constraints and handling parameter uncertainty. Theoretical results are presented to guarantee the performance of the presented neural network. Besides, simulations based on a PUMA 560 manipulator with unknown physical parameters together with the comparison of an existing PNN substantiate the efficacy and superiority of the presented neural network and verify the theoretical results.

Chapter 7—In this chapter, a novel recurrent neural network is presented to simultaneously address the periodic input disturbance, joint angle constraint, and joint velocity constraint, and optimize a general quadratic performance index. The presented recurrent neural network applies to both regulation and tracking tasks. Theoretical analysis shows that, with the presented neural network, the end-effector tracking and regulation errors asymptotically converge to zero in the presence of both input disturbance and the two constraints. Simulation examples and comparisons with an existing controller are also presented to validate the effectiveness and superiority of the presented controller.

In summary, this book mainly presents methods and algorithms for the near-optimal adaptive control of nonlinear systems, together with the corresponding theoretical analysis and simulative examples. Based on these methods, two new methods for the redundancy resolution of redundant manipulators with consideration of parameter uncertainty and periodic disturbances are also presented. This book is written for graduate students and academic and industrial researchers in the field of adaptive/optimal control, robotics, and dynamic neural networks. We hope that this book will benefit the readers and could give them some inspirations in related fields. In particular, we hope that the results presented in this book could help develop deep reinforcement learning approaches for the control of nonlinear systems with performance guarantee through Lyapunov-based approaches, for which we have the current title of this book.

Any comments or suggestions are welcome, and the authors can be contacted via e-mails: [zhangyinyan12@163.com](mailto:zhangyinyan12@163.com) (Yinyan Zhang), [shuaili@ieee.org](mailto:shuaili@ieee.org) (Shuai Li), and [xuefengzhou@vip.qq.com](mailto:xuefengzhou@vip.qq.com) (Xuefeng Zhou).

Guangzhou, China  
Lanzhou, China  
Guangzhou, China  
August 2019

Yinyan Zhang  
Shuai Li  
Xuefeng Zhou



# Acknowledgements

During the work on this book, we have had the pleasure of discussing its various aspects and results with many cooperators and students. We highly appreciate their contributions, which help improve the quality and presentation of this book.

Guangzhou, China

Lanzhou, China

Guangzhou, China

August 2019

Yinyan Zhang

Shuai Li

Xuefeng Zhou

# Contents

<b>1</b>	<b>A Survey of Near-Optimal Control of Nonlinear Systems</b>	<b>1</b>
1.1	Introduction	1
1.2	Dynamic Programming	3
1.2.1	Example for Continuous-Time Nonlinear System	3
1.2.2	Example for Discrete-Time Nonlinear System	4
1.2.3	Adaptive Dynamic Programming for Approximately Solving HJB Equations	5
1.2.4	Numerical Methods Without NNA for Approximately Solving HJB Equations	7
1.3	Near-Optimal Control via Nonlinear Programming	8
1.3.1	Methods via Control Parameterization	8
1.3.2	Model Predictive Control	9
1.3.3	Method via Taylor Expansion	10
1.4	Summary	13
	References	14
<b>2</b>	<b>Near-Optimal Control with Input Saturation</b>	<b>21</b>
2.1	Introduction	21
2.2	Preliminary and Notations	23
2.3	Problem Formulations	25
2.3.1	Original Formulation	25
2.3.2	Reformulation	26
2.4	Projection Neural Network Design	27
2.5	Theoretical Analysis	29
2.6	Numerical Investigation	34
2.6.1	Performance Comparison for Manipulator Control	35
2.6.2	Application to Underactuated Ships	37
2.7	Questions and Answers	41
2.8	Summary	44
	References	44

<b>3</b>	<b>Adaptive Near-Optimal Control with Full-State Feedback</b>	49
3.1	Introduction	49
3.2	Preliminary	51
3.3	General Linear Systems	52
3.3.1	Problem Formulation	52
3.3.2	Nominal Design	53
3.3.3	Adaptive Design	55
3.4	Extension to Nonlinear Systems	56
3.4.1	Problem Formulation	56
3.4.2	Nominal Design	58
3.4.3	Adaptive Design	59
3.4.4	Computational Complexity Analysis	63
3.5	Theoretical Results	64
3.5.1	Convergence of Auxiliary Systems	64
3.5.2	Stability of Closed-Loop Systems	66
3.5.3	Asymptotic Optimality	69
3.6	Application to Uncertain Underactuated Surface Vessel	72
3.6.1	Without Measurement Noises	72
3.6.2	With Measurement Noises	74
3.6.3	Capability for Real-Time Control	75
3.7	Questions and Answers	79
3.8	Summary	92
	References	92
<b>4</b>	<b>Adaptive Near-Optimal Control Using Sliding Mode</b>	97
4.1	Introduction	97
4.2	Problem Formulation and Preliminary	99
4.3	Nominal Near-Optimal Design	100
4.4	Adaptive Near-Optimal Design	101
4.5	Illustrative Examples	107
4.5.1	Example 1	108
4.5.2	Example 2	111
4.6	Application to van der Pol Oscillator	112
4.7	Experimental Validation	113
4.8	Questions and Answers	119
4.9	Summary	125
	References	125
<b>5</b>	<b>Model-Free Adaptive Near-Optimal Tracking Control</b>	129
5.1	Introduction	129
5.2	Problem Description and Preliminary	131
5.2.1	Problem Description	131
5.2.2	Sigmoid Neural Network	133
5.2.3	Problem Reformulation	134

5.3	Control Design . . . . .	134
5.3.1	Problem Relaxation . . . . .	135
5.3.2	Reconstruction of Input-to-Output Dynamics . . . . .	137
5.3.3	Adaptive Near-Optimal Control Law . . . . .	139
5.4	Theoretical Analysis . . . . .	142
5.4.1	Confirmation of No Singularity Problems . . . . .	142
5.4.2	Convergence of the Auxiliary System . . . . .	144
5.4.3	Stability of the Closed-Loop System . . . . .	147
5.4.4	Asymptotic Optimality of Performance Index . . . . .	149
5.5	Illustrative Example . . . . .	151
5.6	Experimental Validation . . . . .	152
5.7	Questions and Answers . . . . .	157
5.8	Summary . . . . .	160
	References . . . . .	161
<b>6</b>	<b>Adaptive Kinematic Control of Redundant Manipulators . . . . .</b>	<b>167</b>
6.1	Introduction . . . . .	167
6.2	Preliminary and Problem Formulation . . . . .	169
6.2.1	Forward Kinematics . . . . .	169
6.2.2	QP-Type Problem Formulation . . . . .	170
6.3	Nominal Design . . . . .	171
6.4	Adaptive Design . . . . .	172
6.4.1	Adaptive Projection Neural Network . . . . .	172
6.4.2	Theoretical Analysis . . . . .	175
6.5	Simulative Verifications and Comparisons . . . . .	179
6.5.1	PUMA 560 Description . . . . .	179
6.5.2	Minimum-Velocity-Norm Redundancy Resolution . . . . .	180
6.5.3	Repetitive-Motion Redundancy Resolution . . . . .	185
6.6	Experimental Verification . . . . .	186
6.7	Questions and Answers . . . . .	188
6.8	Summary . . . . .	192
	References . . . . .	195
<b>7</b>	<b>Redundancy Resolution with Periodic Input Disturbance . . . . .</b>	<b>199</b>
7.1	Introduction . . . . .	199
7.2	Preliminary and Problem Description . . . . .	201
7.2.1	Manipulator Kinematics Model . . . . .	201
7.2.2	Problem Description . . . . .	201
7.2.3	Problem Reformulation as Quadratic Program . . . . .	203
7.3	Neural Network Design . . . . .	204
7.3.1	Step 1: Nominal Design Without Disturbance . . . . .	204
7.3.2	Step 2: Modified Controller with Disturbance Rejection . . . . .	206

7.4	Simulation Examples and Comparisons . . . . .	209
7.4.1	Simulation Setup . . . . .	210
7.4.2	End-Effector Regulation . . . . .	210
7.4.3	End-Effector Tracking . . . . .	215
7.5	Questions and Answers . . . . .	221
7.6	Summary . . . . .	222
	References . . . . .	222

# Abbreviations

ADP	Adaptive dynamic programming
APNN	Adaptive projection neural network
D-H	Denavit–Hartenberg
DOF	Degrees of freedom
FPGA	Field-programmable gate array
HJB	Hamilton–Jacobi–Bellman
MPC	Model predictive control
NNA	Neural network approximation
PE	Persistent excitation
PID	Proportional–integral–derivative
PNN	Projection neural network
PSO	Particle swarm optimization
QP	Quadratic program