Coalition Analysis on Two Manufactures and Two Retailers Supply Chain via Cooperative Game Theory

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Abstract. In this study, we consider a coalition analysis on the pricing problem for a decentralized supply chain model in which two manufacturers and two retailers with price competitions. In the pricing game, we analyze the equilibrium solutions with perfect competition, grand coalition and partial cooperation between manufacturers and retailers. The results show the externality between coalitions for supply chain members. Therefore, the pricing game is represented as a partition function game. The stable profit allocation in each alliance structure is obtained based on cooperative game theory for the partition function game. We derive the new finding that if there are multiple partial alliances within the same alliance structure, the profit within the partial alliance is smaller than the profit when there is only one partial alliance. Then, it is shown that the pessimistic and optimistic Shapley values of the manufacturers are lower than the optimistic personal alliance value of the manufacturer when the product substitutability is lower and the store substitutability is higher.

Keywords: Supply chain management · Cooperative game theory · Shapley value · Partial coalition

1 Introduction

In recent years, coalition analysis for supply chain management has increased to realize an efficient optimization and collaboration of the entire supply chain. Game theoretical models for supply chain management have been used to analyze cooperation and competition between supply chain members. However, there are some cases that companies partially cooperation in the real world. In addition to the conventional examination of two manufacturers and one retailer, it is important to analyze coalitions for a supply chain consisting of multiple manufacturers and multiple retailers in consideration of competitions between retailers. In this study, we study a supply chain model in which two manufacturers and two retailers which has the minimum number of companies that can consider both competitions, rep- resented by product substitutability and represented by store substitutability. In the pricing game, we analyze both equilibrium solutions with perfect competition and grand coalition and partial cooperation between companies. The stable profit allocation in each alliance structure is obtained based on the cooperative game theory.

Many game-theoretic approaches to supply chain management studies analyze only perfect competition and grand coalition. Conventional supply chain models are limited two manufacturers and one retailer. Choi (1991) studied a supply chain pricing problem for two manufacturers and one retailer, and the case of two producers and one seller become leaders, respectively [1]. Trivedi (1998) studied two manufacturers and two retailers, and the case of two manufacturers and two retailers become leaders, respectively [2]. Feng and Lu (2012) compared the results of the Stackelberg game with the results of the negotiation set for the case of changing the contract form in the case of two manufacturers and two retailers [3]. Chung and Lee (2016) discussed the strategic choices for two manufacturers and one retailer when changing the asymmetric leader-follower relationship structure between manufacturers and retailer [4]. Sakurai (2016) studied the changes in the profit due to changes in the leader-follower relationship structure in the case of two manufacturers and two retailers [5]. Hasegawa (2019) examines the changes in the profits of each company due to changes in the alliance structure in the case of two manufacturers and one retailer in the case of considering partial alliances between companies [6]. Granot and Sosic (2005) examined the changes in the profits of each company due to changes in the alliance structure in the case of three retailers and considering partial alliances between companies [7]. Since there are the supply chain models of two manufacturers and two retailers that considers partial alliances between companies, it is possible that there are a partial alliance between them in the supply chains consisting of multiple manufacturers and retailers in the real world. From the above past studies, it is considered that the profit and stable allocation between companies may change when changing the alliance structure in consideration of partial cooperation between companies. In this study, we extend the supply chain model to four companies from the work conducted by Hasegawa, which consist of two manufacturers and two retailers [6]. The total profit of partial cooperation between companies is examined due to changes in the alliance structure. Manufacturers sells differentiated product to both retailers. The model features both manufacturer competition represented by product substitution. The retail competition is represented by store substitution.

Supply chains with similarly structured two level competitions are commonly observed in practice. For example, both Calvin Klein and Ralph Lauren sell their products to Macy's and Lord and Taylor, two of the largest department stores. The analysis target is the supply chain model that was studied by Trivedi (1998), which also consists of four companies consisting of two manufacturers and two retailers with partial cooperation between companies. In this study, we analyze all possible alliance structures in this supply chain model to study the total profit between companies due to changes in the alliance structure. Also, from the perspective of the total profit of the entire supply chain, it is obvious that integrated decision making through a grand coalition is the most ideal relationship structure. However, when distributing the profits from the whole entity to each individual company, each company has to understand the distribution of the total profit. Therefore, in addition to the analysis of the equilibrium solution, we analyze the stable profit allocation for each company when forming a grand coalition by applying the idea of cooperative game theory. Game theory is explained in more detail in reference [8]. The objective of this study is to consider the stable allocation that can be finally obtained by using the core and Shapley values, which are typical solution concepts of the grand coalition.

2 Model

The supply chain model proposed by Trivedi (1998), which is the model target of this study, is introduced. We consider a pricing game in which a product produced by two competing manufacturers is sold at two competing retailers. See the reference of [9] for more information on the most basic models of the pricing games. In the following, two manufacturers are M1 and M2, and two retailers are R1 and R2. In addition, they may be referred to as M without distinction as manufacturers and R without distinction as retailers. The numbers for this company are for distinguishing between the two companies , and there is no difference in conditions between the two companies. The figure of the two manufacturers and two retailers' model is shown in Fig. 1, and the arrows indicate the flow to the product.

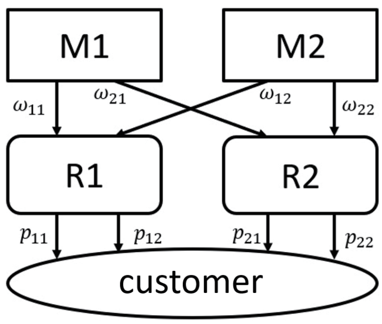


Fig. 1. Two manufacturers and two retailers' model

In this model, two manufacturers produce a different product. Each manufacturer can set a wholesale price to sell the product to two retailers. Two retailers can also set a selling price for each product purchased from a manufacturer to sell it to consumers. The demand of consumer is determined by the demand function described later. This model does not consider the cost of producing the product. The cost of transportation from the manufacturer to the retailer is negligible.

Also, since the demand is decisive, the cost which are the product inventory, raw material inventory, and inventory are stored is not taken into consideration. The products are produced according to the demand determined by the demand function, and all the produced products are supplied to the consumer. The decision variables are the following two types, w_{ij}: wholesale price determined by the manufacturer ($i = 1, 2; j = 1, 2$), p_{ij}: The selling price determined by the retailer ($i = 1, 2; j = 1, 2$). Also, the parameters are product substitutability is given ($0 \leq a < 1$), store substitutability is given as ($0 \leq x < 1$). Given the demand function and the profit function below that compose the model.

Demand

$$q_{ij} = 1 - p_{ij} + (1-a)xp_{kj} + a(1-x)p_{il} + axp_{kl} \qquad (1)$$
$$(\ i = 1, 2; j = 1, 2; k = 3 - i; l = 3 - j)$$

Demand q_{ij}, that is the demand for a product produced by manufacturer j and sold by retailer i, can be seen to decrease as the selling price of the product p_{ij} increases. Also, since the parameters $aandx$ take values from 0 to 1, $a, x, (1 - a), and$ $(1 - x)$ each take a value of 0 or more. Therefore, it can be seen that the higher the selling prices of three types of products p_{il}, p_{kj}, p_{kl} which are sold from the same store and two types of products sold at different stores, the larger the demand q_{ij}.

Profit Function

$$Manufacturer : \Pi_{M_j} = \sum_i w_{ij}q_{ij} \qquad (i, j = 1, 2) \qquad (2)$$

$$Retailer : \Pi_{R_i} = \sum_j (p_{ij} - w_{ij})q_{ij} \qquad (i, j = 1, 2) \qquad (3)$$

3 Analysis

In this study, we analyze the equilibrium solutions in all the alliance structures that can be considered in the two manufacturers and two retailers model.

3.1 Analysis of equilibrium solution

By using the Nash equilibrium solution, the optimum coefficient and the equilibrium solution can be obtained for each company in the coalition. The procedure for analyzing the equilibrium solution is explained below.

STEP 1. If a partial cooperation exists, the sum of the objective functions of the companies participating in the cooperation is the objective function of the coalition group.

STEP 2. Find the Nash equilibrium solution by partially differentiating with the determinants of each company or group of companies and solving simultaneous equations.

The notation of the profit function for each coalition structure is shown in

Table 1. Definition of the profit function for each coalition structure

coalition structure	Profit function
MM-R-R	$\Pi_{MM}^{MM-R-R} + \Pi_{R}^{MM-R-R} + \Pi_{R}^{MM-R-R}$
M-M-RR	$\Pi_{MM}^{M-M-RR} + \Pi_{R}^{M-M-RR} + \Pi_{R}^{M-M-RR}$
MM-RR	$\Pi_{MM}^{MM-RR} + \Pi_{RR}^{MM-RR}$
MR-M-R	$\Pi_{MR}^{MR-M-R} + \Pi_{M}^{MR-M-R} + \Pi_{R}^{MR-M-R}$
MR-MR	$\Pi_{MR}^{MR-MR} + \Pi_{MR}^{MR-MR}$
MMR-R	$\Pi_{MMR}^{MMR-R} + \Pi_{R}^{MMR-R}$
M-MRR	$\Pi_{MRR}^{M-MRR} + \Pi_{M}^{M-MRR}$
MMRR	Π_{MMRR}^{MMRR}

Table 1. The coalition structure MM-R-R means that manufacturer cooperation (MM) with two decentralized retailers R-R. In this case, the profit function of the two manufacturer MM in the sense of coalition MM-R-R is written by Π_{MM}^{MM-R-R} and the profit function for decentralized retailer is written by Π_{R}^{MM-R-R}. The total profit for the coalition structure is $\Pi_{MM}^{MM-R-R} + \Pi_{R}^{MM-R-R} + \Pi_{R}^{MM-R-R}$. The superscript of Π is the coalition structure of the game and the subscript of Π is the cooperating group companies or a single company.

3.2 Analytical results

The following propositions can be mentioned as new findings obtained in this study.

[Proposition]

The following relationship holds for the equilibrium solution of the profits of companies that have the same alliance.

$$\hat{\Pi}_{MM}^{MM-R-R} \geq \hat{\Pi}_{MM}^{MM-RR} \quad (4)$$

$$\hat{\Pi}_{RR}^{M-M-RR} \geq \hat{\Pi}_{RR}^{MM-RR} \quad (5)$$

$$\hat{\Pi}_{MR}^{MR-M-R} \geq \hat{\Pi}_{MR}^{MR-MR} \quad (6)$$

where $\hat{\Pi}$ is the equilibrium solution.

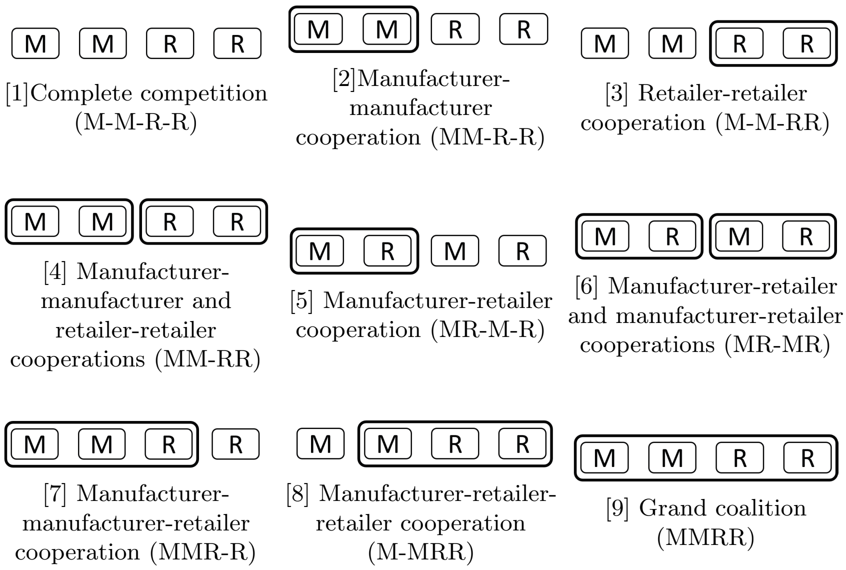


Fig. 2. All coalition structures

[Proof]

$$\hat{\Pi}_{MM}^{MM-R-R} - \hat{\Pi}_{MM}^{MM-RR} = \frac{4x(6ax - 6a - x + 6)}{9(ax - a - x + 1)(3ax - 3a - x + 3)^2} \geq 0 \quad (7)$$

(equality holds when $x = 0$)

$$\hat{\Pi}_{RR}^{M-M-RR} - \hat{\Pi}_{RR}^{MM-RR} = \frac{4a(6ax - a - 6x + 6)}{9(ax - a - x + 1)(3ax - a - 3x + 3)^2} \geq 0 \quad (8)$$

(equality holds when $a = 0$)

The calculated values of $\hat{\Pi}_{MR}^{MR-M-R} - \hat{\Pi}_{MR}^{MR-MR}$ are shown in Fig. 3. It can be seen that $\hat{\Pi}_{MR}^{MR-M-R} - \hat{\Pi}_{MR}^{MR-MR} \geq 0$ from Fig. 3. Accordingly,

$\hat{\Pi}_{MR}^{MR-M-R} - \hat{\Pi}_{MR}^{MR-MR} = \ldots \geq 0$
(equality holds when $a = 0, x = 0, \ldots$ is omitted due to the huge number of expressions)

From the proposition, we obtained the new findings that could not be obtained in the two manufacturers and one retailer model that the profit within a partial cooperation is greater in a alliance structure that includes only one partial cooperation than in a alliance structure that includes multiple partial cooperations. From this, it is considered that the existence of multiple partial cooperations is disadvantageous for the companies within the partial cooperation.

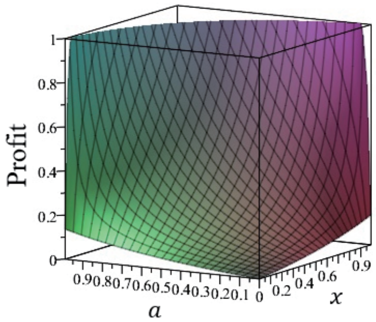


Fig. 3. $\hat{\Pi}_{MR}^{MR-M-R} - \hat{\Pi}_{MR}^{MR-MR}$

4 Allocation analysis

Based on the equilibrium solution obtained in the analysis in the previous section, the idea of cooperative game theory is utilized to analyze the stable allocation in each alliance structure. We examine the stability of the allocation using the concept of the Shapley value, which is the allocation obtained from the contribution of the player and the core which is a collection of allocations that are not controlled by any allocation.

The core, Shapley value, and bargaining set are defined on the game of characteristic function and should be applied to the partition function game. When $v_{\min}^{P}(S) = \min\{v^P(S), P \ni S\}$, $v_{\max}^{P}(S) = \max\{v^P(S), P \ni S\}$, the core under pessimistic conjecture C^{pes} and the core under optimistic conjecture C^{opt} in the partition function game are given by the following equations, respectively.

$$C^{pes} = \{x| \sum_{i \in S} x_i \geq v_{\min}^{P}(S) \forall S \subset N, \sum_{i \in N} x_i = v^{P^N}(N)\} \tag{9}$$

$$C^{opt} = \{x| \sum_{i \in S} x_i \geq v_{\max}^{P}(S) \forall S \subset N, \sum_{i \in N} x_i = v^{P^N}(N)\} \tag{10}$$

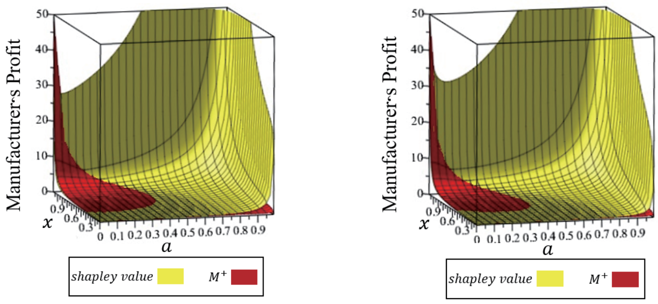
The Shapley value in the game of characteristic function (N, v) is given by the following equation.

$$\phi_i = \sum_{S \subset N} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S - \{i\})] \tag{11}$$

The following results were obtained as new findings in this study. The Shapley values of pessimistic and optimistic manufacturers are shown in Fig.4, and the analysis results of pessimistic core and Shapley values are shown in Fig.5. The analysis results of optimistic core and Shapley value are shown in Fig.6.

The Shapley value of pessimistic and optimistic manufacturers is shown in yellow, and the individual partnership value of optimistic manufacturers is shown in red.

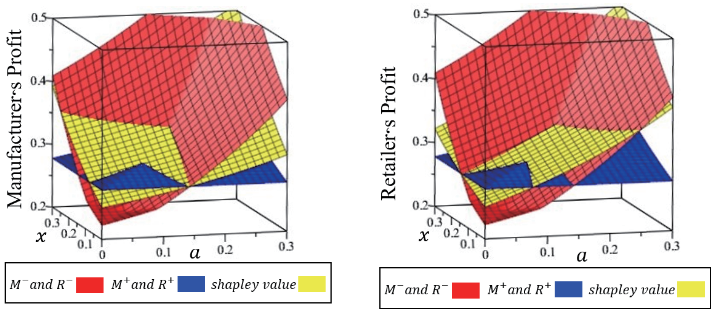
Fig. 4 shows that when a is small and x is large, the Shapley value of the pessimistic and optimistic manufacturer is lower than the optimistic personal alliance value. This means that the manufacturers have not made a sufficient contribution to the grand coalition. This is because there may be competition between retailers due to the fact that there are two retailers. Therefore, if the sellers are competing and the manufacturers are not competing (x is large, a is small), manufacturers can set higher wholesale prices, so their profits increase. Therefore, it is considered that the optimistic personal alliance value M^+ exceeds the Shapley value when a is small and x is large.



[1] Pessmistic [2] Optimistic

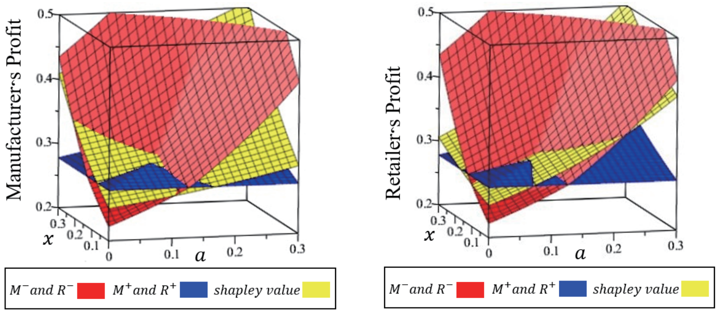
Fig. 4. Shapley value of manufacturers

In the graph below, M^- and R^- (red) is the lower limit of the core, M^+ and R^+ (blue) is the upper limit of the core, and yellow is the Shapley value.



[1] Manufacturer [2] Retailer

Fig. 5. Pessimistic core and Shapley value



[1] Manufacturer [2] Retailer

Fig. 6. Optimistic core and Shapley value

The analytical results of the pessimistic core and Shapley value of the manufacturer and the retailer are shown in Fig. 5. It can be confirmed that the core is not empty, when a and x are close to 0 in both the manufacturer and the retailer. In other words, it shows that when a grand coalition is formed, there is an allocation that allows all companies to make more profits than if they formed an alliance of two or a three companies. In other ranges, the core is empty because the magnitude relations of M^- and R^- and M^+ and R^+ are swapped. This means that there are some companies that can make more profits by forming an alliance of two or three companies than the allocation when forming a grand coalition. It can be seen that the Shapley value may exist outside the core in the region where both a and x are close to 0. In other words, profits based on the degree of contribution to the alliance cannot be a stable distribution in the core sense.

Then, the analysis results of the optimistic core and Shapley values of the manufacturer and the retailer are shown in Fig.6. It has he same tendency as Fig. 5. In other words, it shows that when a grand coalition is formed, there is an allocation that allows all companies to make more profits than if they formed an alliance of two or a three companies alliance, and the profits based on the degree of contribution to the alliance cannot be a stable distribution in the core sense.

5 Conclusion

In this study, we analyzed the equilibrium solution of a supply chain consisting of two manufacturers and two retailers using a cooperative game theory. For all alliance structures, the changes of the total profit between companies due to changes in the alliance structure were examined. As a result, we have obtained a new finding that if there are multiple partial alliances within the same alliance structure, the profit within the partial alliance will be smaller than if there is only one partial alliance. Beside, the stable distribution that can be finally obtained was examined by using the concept of core and Shapley value, which is a typical solution concept of the grand coalition. As a result, we have a new finding that the Shapley value of pessimistic and optimistic manufacturers fall below the optimistic personal alliance valueM^+ when product substitutability is low and store substitutability is high in the two manufacturers and two retailers model. The result of the Shapley value obtained this time is pseudo by applying it to the characteristic function game based on the idea of the core under pessimistic prediction and the core under optimistic prediction in the partition function game. Therefore, it is necessary to apply the Shapley value to the partition function game in order to examine the strict stability. In addition, it is a future task to analyze the stable distribution in the partial alliance using the concept of the negotiation to consider the leader-follower relationship structure in addition to the alliance structure as the relationship structure.

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