A Dictionary-Passing Translation of Featherweight Go

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Abstract. The Go programming language is an increasingly popular language but some of its features lack a formal investigation. This article explains Go's resolution mechanism for overloaded methods and its support for structural subtyping by means of translation from Featherweight Go to a simple target language. The translation employs a form of dictionary passing known from type classes in Haskell and preserves the dynamic behavior of Featherweight Go programs.

1 Introduction

The Go programming language [22], introduced by Google in 2009, is syntactically close to C and incorporates features that are well-established in other programming languages. For example, a garbage collector as found in Java [6], built-in support for concurrency and channels in the style of Concurrent ML [17], higher-order and anonymous functions known from functional languages such as Haskell [12]. Go also supports method overloading for structures where related methods can be grouped together using interfaces. Unlike Java, where subtyping is nominal, Go supports structural subtyping among interfaces.

Earlier work by Griesmer and co-authors [7] introduces Featherweight Go (FG), a minimal core calculus that includes the essential features of Go. Their work specifies static typing rules and a run-time method lookup semantics for FG. However, the actual Go implementation appears to employ a different dynamic semantics. Quoting Griesmer and co-workers:

Go is designed to enable efficient implementation. Structures are laid out in memory as a sequence of fields, while an interface is a pair of a pointer to an underlying structure and a pointer to a dictionary of methods.

To our knowledge, nobody has so far formalized such a dictionary-passing translation for FG and established its semantic equivalence with the FG run-time method lookup dynamic semantics. Hence, we make the following contributions:

- Section 5 specifies the translation of source FG programs to an untyped lambda calculus with pattern matching. We employ a dictionary-passing translation scheme à la type classes [8] to statically resolve overloaded FG method calls. The translation is guided by the typing of the FG program.

— Section 6 establishes the semantic correctness of the dictionary-passing translation. The proof for this result is far from trivial. We require step-indexed logical relations [1] as there can be cyclic dependencies between interfaces and method declarations.

Section 3 specifies Featherweight Go (FG) and Section 4 specifies our target language. Section 7 covers related works and concludes. The upcoming section gives an overview.

2 Overview

We introduce Featherweight Go [7] (FG) by an example and then present the ideas of our dictionary-passing translation for FG.

2.1 FG by Example

FG is a syntactic subset of the full Go language, supporting structures, methods and interfaces. The upper part in Figure 1, lines 1-22, shows an example slightly adopted from [7]. The original example covers equality in FG. We extend the example and include an ordering relation (less or equal than) as well.

FG programs consist of a sequence of declarations defining structures, methods, interfaces and a main function. Method bodies in FG only consist of a return statement. For clarity, we sometimes identify subexpressions via variable bindings introduced with var. In such a declaration, the name of a variable precedes its type, the notation var $_{-}$ (line 21) indicates that we do not care about the variable name given to the main expression. The example uses primitive types int and bool and several operations on values of these types (==, &&, ...). These are not part of FG.

Structures in FG are similar to structures known from C/C++. A syntactic difference is the FG convention that field names precede the types. In FG, structures and methods are always declared separately, whereas C++ groups methods together in a class declaration. Methods in FG can be overloaded on the *receiver*. The receiver is the value on which the method operates on.

Interfaces in FG consist of a set of method declarations that share the same receiver. For example, interface Eq introduces method eq and interface Ord introduces methods eq and 1t (line 3 and 4). The (leading) receiver argument is left implicit and method names in interfaces must always be distinct. Interfaces are types and can be used in type declarations for structures and methods. For example, structure Pair defines two fields left and right, each of type Eq. Declarations of structures must be non-recursive whereas an interface may appear in the method declaration of the interface itself. For example, see interface Eq.

FG uses the keyword func to introduce methods and functions. Methods can be distinguished from ordinary functions as the receiver argument always precedes the method name. In FG, the only function is the main function, all other declarations introduced by func are methods.

```
type Int   struct { val int }
type Pair struct { left Eq; right Eq }
                 interface { eq(that Eq) bool }
    type Eq
    type Ord interface { eq(that Eq) bool; lt(that Ord) bool }
    func (this Int) eq(that Eq) bool {
7
           return this.val == (that.(Int)).val
    func (this Pair) eq(that Eq) bool {
           return this.left.eq(that.(Pair).left) &&
10
                     this.right.eq(that.(Pair).right)
11
   func (this Int) lt(that Ord) bool {
14
           return this.eq(that) ||
15
                     (this.val < (that.(Int)).val)</pre>
16
17
    func main() {
      var i Int = Int{1}
18
      var j Int = Int{2}
19
      var p Pair = Pair{i, j}
21
      var _bool = p.eq(p)
22
    -- Field access assuming constructors K_{\text{Int}} and K_{\text{Pair}}.
23
   val (K_{Int} y) = y
25
   left (K_{Pair}(x, _)) = x
   right (K_{Pair}(_,x)) = x
26
27
    -- Interface-value construction assuming constructors K_{Eq}, K_{Ord}.
28
29
   toEq_{Int} y = K_{Eq} (y, eq_{Int})
   toEq_{Pair} y = K_{Eq} (y, eq_{Pair})
30
    toEq_{Ord} (K_{Ord} (x,eq,_)) = K_{Eq} (x,eq)
    -- Interface - value destruction.
33
    fromEq_{Int} (K_{Eq} (K_{Int} y, _)) = K_{Int} y
34
    fromEq_{Pair} (K_{Eq} (K_{Pair} (x,y) , _)) = K_{Pair} (x,y)
35
    fromOrd<sub>Int</sub> (K_{Ord} (K_{Int} y, _{-}, _{-})) = K_{Int} y
37
    -- Method definitions.
38
   eq_{Eq} (K_{Eq} (x, eq)) = eq x
   eq<sub>Int</sub> this that = val this == val (fromEq<sub>Int</sub> that)
    eq_{Pair} \ this \ that = eq_{Eq} \ (left \ this) \ (left \ (from Eq_{Pair} \ that))
41
                          && eq_{Eq} (right this) (right (fromEq_{Pair} that))
42
43
    lt_{Int} this that = eq_{Int} this (toEq<sub>Ord</sub> that)
44
                          || (val this < val (fromOrd<sub>Int</sub> that))
    main =
45
    let i = K_{Int} 1
47
          j = K_{Int} 2
48
          p = K_{Pair} (toEq_{Int} i, toEq_{Int} j)
     in eq<sub>Pair</sub> p (toEq<sub>Pair</sub> p)
```

Fig. 1. Equality and ordering in FG and its translation

Consider the method implementation of eq for receiver this of type Int starting at line 6. This definition takes care of equality among Int values by making use of primitive equality == among int. We would expect argument that to be of type Int. However, to be able to use an Int value everywhere an Eq value is expected (to be discussed shortly), the signature of eq for Int must match exactly the signature declared by interface Eq. Hence, that has declared type Eq, and we resort to a type assertion, written that. (Int), to convert it to Int. Type assertions involve a run-time check that may fail. The same observation applies to the implementation of Eq for receiver Pair (line 9).

FG supports structural subtyping among structures and interfaces. A structure is a subtype of an interface if the structure implements the methods as declared by the interface. For example, Int and Pair both implement interface Eq. This implies the structural subtype relations (1) Int <: Eq and (2) Pair <: Eq. Relation (1) ensures that the construction of the pair at line 20 type checks: variables i and j have type Int but can also be viewed as type Eq thanks to structural subtyping. Relation (2) resolves the method call p.eq(p) at line 21 as the Pair variable p also has the type Eq. The method definition starting at line 9 will be chosen.

An interface I is a structural subtype of another interface J if I contains all of J's method declarations. For example, the set of methods of interface Ord is a superset of the method set of Eq. This implies (3) Ord <: Eq, which is used in the method implementation of 1t for receiver type Int. See line 14 where (3) yields that variable that with declared type Ord also has type Eq. Thus, the method call this.eq(that) is resolved via the method definition from line 6.

2.2 Dictionary-Passing Translation

We translate FG programs by applying a form of dictionary-passing translation known from type classes [8]. As our target language we consider an untyped functional language with pattern matching where we use Haskell style syntax for expressions and patterns. Each FG interface translates to a pair consisting of a structure value and a dictionary. The dictionary holds the set of methods available as specified by the interface whereas the structure implements these methods. We refer to such pairs as *interface-values*. The translation is type-directed as we need type information to resolve method calls and construct the appropriate dictionaries and interface values.

Lines 23-49 show the result of applying our dictionary-passing³ translation scheme to the FG program (lines 1-22). We use a tagged representation to encode FG structures in the target language. Hence, for each structure S, we assume a data constructor K_S , where we use pattern matching to represent field access (lines 23-26). For example, structure Pair implies the data constructor $\mathsf{K}_{\mathsf{Pair}}$. For convenience, we assume tuples and make use of don't care patterns $_$.

A method call on an interface type translates to a lookup of the method in the dictionary of the corresponding interface-value. Like structures, interface-

³ Technically, we are passing around interface-values wrapping dictionaries of methods.

values are tagged in the target language. For example, line 39 introduces the helper function eq_{Eq} to perform method lookup for method eq of interface Eq. The constructor for an Eq interface-value is K_{Eq} . Hence, we pattern match on K_{Eq} and extract the underlying structure value and method definition. A method call such as this.left.eq(...) in the source program (line 10) with receiver this.left of type Eq then translates to eq_{Eq} (left this)... (line 41).

A method call on a structure translates to the method definition for this receiver type. For example, we write eq_{Int} to refer to the translation of the method definition of eq for receiver type Int. A method call such as this.eq(...) in the source program (line 14) with receiver this of type Int then translates to eq_{Int} this... (line 43).

The construction of interface-values is based on structural subtype relations. Recall the three structural subtype relations we have seen earlier: (1) Int <: Eq and (2) Pair <: Eq and (3) Ord <: Eq. Relation (1) implies the interface-value constructor toEq_{Int} (line 29), which builds an Eq interface-value via the given structure value y and a dictionary consisting only of the method eq_{Int}. Relation (2) implies a similar interface-value constructor toEq_{Pair} (line 30). Relation (3) gives raise to the interface-value constructor toEq_{Ord} (line 31), which transforms some Ord into an Eq interface-value. We assume that in case a dictionary consists of several methods, methods are kept in fixed order.

Type assertions imply interface-value destructors. For example, the source expression that. (Int) (line 7) performs a run-time check, asserting that that has type Int. In terms of the dictionary-passing translation, function from Eq Int (line 34) performs this check. Via the pattern K_{Eq} (K_{Int} y,) _), we assert that the underlying target structure must result from Int. If the interface-value contains a value not tagged with K_{Int} , the pattern matching fails at run-time, just as the type assertion in FG. Interface-value destructors from Eq $_{Pair}$ and from Ord $_{Int}$ (lines 35, 36) result from similar uses of type assertions.

To summarize, each use of structural subtyping implies a interface-value constructor being inserted in the target program. For example, typing the source expression p.eq(p) in line 21 relies on structural subtyping Pair <: Eq because argument p has type Pair but method eq requires a parameter of type Eq. Thus, the translation of this expression is eq_{Pair} p (toEq_{Pair} p) in line 49.

Similarly, type assertions imply interface-value destructors. For example, the source expression that.(Pair).left in line 10 use a type assertion on that, which has type Eq. Thus, it translates to the target expression left (fromEq_{Pair} that) in line 41.

We continue by introducing FG and our target language followed by the full details of the dictionary-passing translation.

3 Featherweight Go

Featherweight Go (FG) [7] is a tiny fragment of Go containing only structures, methods and interfaces. Figure 2 gives the syntax of FG. With the exception of variable bindings in function bodies, the primitive type int with operations ==

```
Field name
                                                                  f
                   Method name
                                                                                                   Expression
                                                                                                                                            d, e ::=
                                                                 m
                    Variable name
                                                                                                         Variable
                                                                 x, y
                                                                                                        Method call
                   Structure type name t_S, u_S
                                                                                                                                                 e.m(\overline{e})
                                                                                                        Structure literal
                   Interface type name t_I, u_I
                                                                                                                                                t_S\{\overline{e}\}
                   Type name
                                                                 t, u ::= t_S \mid t_I
                                                                                                        Select
                                                                                                                                                 e.f
                    Method signature M := (\overline{x_i \ t_i}) \ t
                                                                                                        Type assertion
                                                                                                                                                 e.(t)
                   Method specification R, S := mM
         Type literal L ::=
                                                                               Declaration D ::=
                                        struct \{\overline{f}\ t\}
                                                                                    Type
                                                                                                              \mathbf{type}\ t\ L
               Structure
                                                                                    Method
                                                                                                              func (x \ t_S) \ mM \ \{ return \ e \}
               Interface
                                         interface \{\overline{S}\}
                                                   Program P := \overline{D} func main(){\bot = e}
 |\overline{D} \vdash_{\mathsf{FG}} t <: u|
                                                                                                                                                                 Subtyping
                            METHODS-STRUCT
                            \mathsf{methods}(\overline{D}, t_S) = \{ mM \mid \mathsf{func}\ (x\ t_S)\ mM\ \{\mathsf{return}\ e\} \in \overline{D} \}
                                                                                                                \frac{\text{SUB-IFACE}}{\text{methods}(\overline{D},t)\supseteq \text{methods}(\overline{D},u_I)}
METHODS-IFACE
                                                                 {\tt SUB\text{-}STRUCT\text{-}REFL}
type t_I interface \{\overline{S}\} \in \overline{D}
    \mathsf{methods}(\overline{D},t_I) = \{\overline{S}\}
                                                                                                                                    \overline{D} \vdash_{\mathsf{FG}} t <: u_I
  \overline{D} \vdash_{\mathsf{FG}} d \longrightarrow e
                                                                                                                                                               Reductions
                                                        \begin{array}{ll} v & ::= t_S\{\overline{v}\} \\ \mathcal{E} & ::= \left[ \mid t_S\{\overline{v}, \mathcal{E}, \overline{e}\} \mid \mathcal{E}.f \mid \mathcal{E}.(t) \mid \mathcal{E}.m(\overline{e}) \mid v.m(\overline{v}, \mathcal{E}, \overline{e}) \end{array} \right]
     Evaluation context
     Substitution (FG values) \Phi_{\mathsf{v}} ::= \langle \overline{x_i \mapsto v_i} \rangle
                                                                             \begin{array}{c} \text{FG-FIELD} \\ \textbf{type} \ t_S \ \textbf{struct} \ \{\overline{f} \ t\} \in \overline{D} \\ \hline \overline{D} \vdash_{\mathsf{FG}} \ t_S\{\overline{v}\}.f_i \longrightarrow v_i \end{array} 
                             \begin{array}{c} \text{FG-CONTEXT} \\ \overline{D} \vdash_{\mathsf{FG}} d \longrightarrow e \\ \overline{\overline{D}} \vdash_{\mathsf{FG}} \mathcal{E}[d] \longrightarrow \mathcal{E}[e] \end{array} 
FG-CALL
                                                                                                                                  \frac{t_S\{\overline{v}\}}{\overline{D}} \vdash_{\mathsf{FG}} t_S <: t
                                                                                                                       v = t_S\{\overline{v}\}
                          func (x t_S) m(\overline{x t}) t \{ return e \} \in \overline{D}
 v = t_S\{\overline{v}\}
               \overline{D} \vdash_{\mathsf{FG}} v.m(\overline{v}) \longrightarrow \langle x \mapsto v, \overline{x_i \mapsto v_i} \rangle e^{-\overline{v}}
```

Fig. 2. Featherweight Go (FG)

and \lt , and the primitive type bool with operations && and | |, we can represent the example from Figure 2 in FG. Compared to the original presentation of FG [7] we use symbol L instead of T (for type literals), and omit the **package** keyword at the start of a FG program. Overbar notation $\overline{\xi}^n$ denotes the sequence $\xi_1 \dots \xi_n$ for some syntactic construct ξ , where in some places commas separate the sequence items. If irrelevant, we omit the n and simply write $\overline{\xi}$. Using the index variable i under an overbar marks the parts that vary from sequence item to sequence item; for example, $\overline{\xi'} \, \overline{\xi_i}^n$ abbreviates $\xi' \, \xi_1 \dots \xi' \, \xi_n$ and $\overline{\xi_j}^q$ abbreviates $\xi_{j1} \dots \xi_{jq}$.

FG is a statically typed language. For brevity, we omit a detailed description of the FG typing rules as they will show up in the type-directed translation. The following conditions must be satisfied.

FG1: Structures must be non-recursive.

FG2: For each struct, field names must be distinct.

FG3: For each interface, method names must be distinct.

FG4: Each method declaration is uniquely identified by the receiver type and method name.

FG supports structural subtyping, written $\overline{D} \vdash_{\mathsf{FG}} t <: u$. A struct t_S is a subtype of an interface t_I if t_S implements all the methods specified by the interface t_I . An interface t_I is a subtype of another interface u_I if the methods specified by t_I are a superset of the methods specified by u_I . The structural subtyping relations are specified in the middle part of Figure 2.

Next, we consider the dynamic semantics of FG. The bottom part of Figure 2 specifies the reduction of FG programs by making use of structural operational semantics rules of the form $\overline{D} \vdash_{\mathsf{FG}} d \longrightarrow e$ to reduce expression d to expression e under the sequence \overline{D} of declarations.

Rule FG-CONTEXT makes use of evaluation contexts with holes to apply a reduction inside an expression. Rule FG-FIELD deals with field access. Condition FG2 guarantees that field name lookup is unambiguous. Rule FG-CALL reduces method calls. Condition FG4 guarantees that method lookup is unambiguous. The method call is reduced to the method body e where we map the receiver argument to a concrete value v and method arguments x_i to concrete values v_i . This is achieved by applying the substitution $\langle x \mapsto v, \overline{x_i \mapsto v_i} \rangle$ on e, written $\langle x \mapsto v, \overline{x_i \mapsto v_i} \rangle e$.

Rule FG-ASSERT covers type assertions. We need to check that the type t_S of value v is consistent with the type t asserted in the program text. If t is an interface, then t_S must implement all the methods as specified by this interface. If t is a struct type, then t must be equal to t_S . Both checks can be carried out by checking that t_S and t are in a structural subtype relation.

We write $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^* v$ to denote that under the declarations \overline{D} , expression e reduces to the value v in a finite number of steps. We write $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^k v$ to denote that under the declarations \overline{D} , expression e reduces to the value v within at most k steps. This means we might need fewer than k steps but k are clearly sufficient to reduce the expression to some value. If there is no such v for any number of steps, we say that e is irreducible w.r.t. \overline{D} , written $irred(\overline{D}, e)$.

4 Target Language

```
Expression
                                                                                                                                                               E ::=
                                                                                                                                                                                 X \mid Y
                                                                                                                                                                                                                                                                                                                                                                                      Pattern clause Cls ::= Pat \rightarrow E
                                          Variable
                                         Constructor
                                                                                                                                                                                 K
                                                                                                                                                                                                                                                                                                                                                                                      Pattern
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Pat ::= K \overline{X}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Prog ::= let \overline{Y_i = \lambda X_i.E_i}
                                                                                                                                                                                  E E
                                         Application
                                                                                                                                                                                                                                                                                                                                                                                    Program
                                          Abstraction
                                                                                                                                                                               \lambda X.E
                                                                                                                                                                             case E of \overline{Cls}
                                         Pattern case
                                                                                                                                                                                                                                                                 V ::= X \mid K \overline{V}
           TL values
                                                                                                                                                                                                                                                               \mathcal{R} ::= [] \mid K \ \overline{V} \mathcal{R} \overline{E} \mid \mathbf{case} \ \mathcal{R} \ \mathbf{of} \ [\overline{Pat \to E}] \mid \mathcal{R} \ E \mid V \ \mathcal{R}
         TL evaluation context
         Substitution (TL values) \Phi_{\mathsf{V}} ::= \langle \overline{X} \mapsto \overline{V} \rangle
           Substitution (TL methods) \Phi_{\mathsf{m}} ::= \langle \overline{Y \mapsto \lambda X.E} \rangle
    \begin{array}{c|c} \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E' \\ \\ \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E' \\ \hline \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \mathcal{R}[E] \longrightarrow \mathcal{R}[E'] \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   TL expression reductions
                                                                                                                                                                                                                                                                                                                                                                       TL-LAMBDA \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X.E) V \longrightarrow \langle X \mapsto V \rangle E
TL-CASE
 \frac{K \ \overline{X_i}^n \to E' \in [\overline{Pat} \to E]}{\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \mathsf{case} \ K \ \overline{V_i}^n \ \mathsf{of} \ [\overline{Pat} \to E] \longrightarrow \langle \overline{X_i} \mapsto \overline{V_i}^n \rangle E'} \qquad \qquad ^{\mathsf{TL-METHOD}} \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} Y \ E \longrightarrow \Phi_{\mathsf{m}}(Y) \ 
           \vdash_{\mathsf{TL}} Prog \longrightarrow Prog'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    TL reductions
                                                                                                                                                TL-PROG
                                                                                                                                                  \frac{\langle \overline{Y_i} \mapsto \lambda X_i.\overline{E_i} \rangle \vdash_{\mathsf{TL}} E \longrightarrow E'}{\vdash_{\mathsf{TL}} \mathsf{let} \ \overline{Y_i} = \lambda X_i.\overline{E_i} \mathsf{ in } E \longrightarrow \mathsf{let} \ \overline{Y_i} = \lambda X_i.\overline{E_i} \mathsf{ in } E'}
```

Fig. 3. Target Language (TL)

Figure 3 specifies the syntax and dynamic semantics of our target language (TL). We use capital letters for constructs of the target language. Target expressions E include variables X,Y, data constructors K, function application, lambda abstraction and case expressions to pattern match against constructors. In a case expression with only one pattern clause, we often omit the brackets and just write **case** E **of** $Pat \rightarrow E$. A program consists of a sequence of function definitions and a (main) expression. The function definitions are the result of translating FG method definitions.

We assume data constructors for tuples up to some fixed but arbitrary size. The syntax (\overline{E}^n) constructs an n-tuple when used as an expression, and deconstructs it when used in a pattern context. At some places, we use nested patterns as an abbreviation for nested case expressions. The notation $\lambda Pat.E$ stands for $\lambda X.\mathbf{case}\ X$ of $[Pat \to E]$, where X is fresh.

Representing the example from Figure 2 in the target language requires some more straightforward extensions: integers with operations == and <, booleans

with operations && and ||, let-bindings inside expressions, and top-level bindings. The target language can encode the last two features via lambda-abstractions and top-level let-bindings.

The structural operational semantics employs two types of substitutions. Substitution $\Phi_{\rm V}$ records the bindings resulting from pattern matching and function applications. Substitution $\Phi_{\rm m}$ records the bindings for translated method definitions (i.e. for top-level let-bindings). Target values consist of constructors and variables. A variable may be a value if it refers to a yet to be evaluated method binding.

Reduction of programs is mapped to reduction of expressions under a method substitution. See rule TL-PROG. The remaining reduction rules are standard.

We write $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^* V$ to denote that under substitution Φ_{m} , expression E reduces to the value V in a finite number of steps. We write $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^k V$ to denote that under substitution Φ_{m} , expression E reduces to V within at most k steps. This means we might need fewer than k steps but k are clearly sufficient. If there is no such V for any number of steps, we say that E is irreducible w.r.t. Φ_{m} , written $irred(\Phi_{\mathsf{m}}, E)$.

5 Dictionary-Passing Translation

We formalize the dictionary-passing translation of FG to TL. The translation rules are split over two figures. Figure 4 covers methods, programs and some expressions. Figure 5 covers structural subtyping and type assertions. The translation rules are guided by type checking the FG program. The gray shaded parts highlight target terms that are generated. If these parts are ignored, the translation rules are effectively equivalent to the FG type checking rules [7]. We assume that conditions FG1-4 hold as well.

We use the following conventions. We assume that each FG variable x translates to the TL variable X. For each structure t_S we introduce a TL constructor K_{t_S} . For each interface t_I we introduce a TL constructor K_{t_I} . In the translation, a source value of an interface type t_I translates to an interface-value tagged by K_{t_I} . The interface-value contains the underlying structure value and a dictionary consisting of the set of methods as specified by the interface. For each method declaration $\mathbf{func}\ (x\ t_S)\ mM\ \{\mathbf{return}\ e\}$ we introduce a TL variable X_{m,t_S} , thereby relying on FG4 which guarantees that m and t_S uniquely identify this declaration. We write Δ to denote typing environments where we record the types of FG variables. The notation [n] is a short-hand for the set $\{1,\ldots,n\}$.

5.1 Translating programs, methods and expressions

The translation of programs and methods boils down to the translation of expressions involved. Rule TD-METHOD translates a specific method declaration, rule TD-PROG collects all method declarations and also translates the main expression. See Figure 4.

```
Convention for mapping source to target terms
                    x \rightsquigarrow X \quad t_S \rightsquigarrow K_{t_S} \quad t_I \rightsquigarrow K_{t_I} \quad \text{func } (x \ t_S) \ mM \ \{\text{return } e\} \rightsquigarrow X_{m,t_S}
                                                       FG Environment \Delta ::= \{\} \mid \{x : t\} \mid \Delta \cup \Delta
 \overline{D} \vdash_{\mathsf{meth}} \mathsf{func} (x \ t_S) \ m(\overline{x \ t}) \ t \leadsto E
                                                                                                                                            Translating method declarations
\frac{\text{distinct}(x,\overline{x}^n) \qquad \langle \overline{D}, \{x:t_S, \overline{x_i:t_i}^n\} \rangle \vdash_{\text{exp}} e:t \leadsto E}{\overline{D} \vdash_{\text{meth}} \text{ func } (x\;t_S)\; m(\overline{x}\;\overline{t}^n)\; t \; \{\text{return } e\} \leadsto \lambda X. \lambda(\overline{X}^n).E}
   \vdash_{\mathsf{prog}} P \leadsto Prog
                                                                                                                                                                         Translating programs
                                                                                \langle \overline{D}, \{\} \rangle \vdash_{\mathsf{exp}} e : t \leadsto E
                           \overline{D} \vdash_{\mathsf{meth}} D'_i \leadsto E_i
                                                                                               D_i' = \text{func } (x_i \ t_{Si}) \ m_i M_i \ \{ \text{return } e_i \}
                                     (for all i \in [n], where \overline{D'}^n are the func declarations in \overline{D})
                                        \vdash_{\mathsf{prog}} \overline{D} \; \mathsf{func} \; \mathsf{main}()\{ \_=e \} \leadsto \mathsf{let} \; \overline{X_{m_i,t_{S_i}} = E_i}^n \; \mathsf{in} \; E
   \langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : t \leadsto E
                                                                                                                                                                   Translating expressions
                                                                                \frac{\text{TD-VAR}}{(x:t) \in \Delta}
\frac{\langle \overline{D}, \Delta \rangle \vdash_{\text{exp}} x:t \leadsto X}{\langle \overline{D}, \Delta \rangle}
                TD-STRUCT
                 \frac{\text{type } t_S \text{ struct } \{\overline{f} \ \overline{t}^n\} \in \overline{D} \qquad \langle \overline{D}, \Delta \rangle \vdash_{\text{exp}} e_i : t_i \leadsto E_i \qquad (\text{for all } i \in [n])}{\langle \overline{D}, \Delta \rangle \vdash_{\text{exp}} t_S \{\overline{e}^n\} : t_S \leadsto K_{t_S} (\overline{E}^n)}
                                        TD-ACCESS
                                         \langle \overline{D}, \varDelta \rangle \vdash_{\mathsf{exp}} e : t_S \leadsto E \qquad \mathbf{type} \ t_S \ \mathbf{struct} \ \{ \overline{f \ t}^n \} \in \overline{D}
                                             \langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e.f_i : t_i \leadsto \mathsf{case} \ E \ \mathsf{of} \ K_{t_S} \ (\overline{X}^n) \to X_i
                   {\tt TD\text{-}CALL\text{-}STRUCT}
                                                                            m(\overline{x} \ \overline{t}^n) \ t \in \mathsf{methods}(\overline{D}, t_S)
                    \frac{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : t_S \leadsto E}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e_i : t_i \leadsto E_i \quad (\mathsf{for all } i \in [n])}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e.m(\overline{e}^n) : t \leadsto X_{m,t_S} \ E(\overline{E}^n)}
           TD-CALL-IFACE
           \langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e.m(\overline{e}^n) : t \leadsto \mathsf{case} \ E \ \mathsf{of} \ K_{t_I} \ (X, \overline{X}^q) \to X_j \ X \ (\overline{E}^n)
```

Fig. 4. Translation of methods, programs and expressions

Fig. 5. Translation of structural subtyping and type assertions

The translation rules for expressions are of the form $\langle \overline{D}, \Delta \rangle \vdash_{\sf exp} e : t \leadsto E$ where \overline{D} refers to the sequence of FG declarations, Δ refers to type binding of local variables, e is the to be translated FG expression, t its type and E the resulting target term. Departing from FG's original typing rules [7], the translation rules are non-syntax directed due the structural subtyping rule TD-SUB defined in Figure 5. We could integrate this rule via the other rules but this would make all the rules harder to read. Hence, we prefer to have a separate rule TD-SUB.

We now discuss the translations rules for the expression forms in Figure 4. (The remaining expression forms are covered in Figure 5, to be explained in the next section.) Rule TD-VAR translates variables and follows our convention that x translates to X. Rule TD-STRUCT translates a structure creation. The translated field elements E_i are collected in a tuple and tagged via the constructor K_{t_S} . Rule TD-ACCESS uses pattern matching to capture field access in the translation.

Method calls are dealt with by rules TD-CALL-STRUCT and TD-CALL-IFACE. Rule TD-CALL-STRUCT covers the case that the receiver e is of the structure type t_S . The first precondition guarantees that an implementation for this specific method call exists. (See Figure 2 for the auxiliary methods.) Hence, we can assume that we have available a corresponding definition for X_{m,t_S} in our translation. The method call then translates to applying X_{m,t_S} first on the translated receiver E, followed by the translated arguments collected in a tuple (\overline{E}^n) .

Rule TD-CALL-IFACE assumes that receiver e is of interface type t_I , so e translates to interface-value E. Hence, we pattern match on E to access the underlying value and the desired method in the dictionary. We assume that the order of methods in the dictionary corresponds to the order of method declarations in the interface. The preconditions guarantee that t_I provides a method m as demanded by the method call, where j denotes the index of m in interface t_I .

5.2 Translating structural subtyping and type assertions

Rule TD-SUB deals with structural subtyping and yields an interface-value constructor derived via rules TD-CONS-STRUCT-IFACE and TD-CONS-IFACE-IFACE in Figure 5. These rules correspond to the structural subtyping rules in Figure 2 but additionally yield an interface-value constructor.

The preconditions in rule TD-CONS-STRUCT-IFACE check that structure t_S implements the interface t_I . This guarantees the existence of method definitions X_{m_i,t_S} . Hence, we can construct the desired interface-value.

The preconditions in rule TD-CONS-IFACE check that t_I 's methods are a superset of u_I 's methods. This is done via the total function $\pi:\{1,\ldots,q\}\to\{1,\ldots,n\}$ that matches each (wanted) method in u_I against a (given) method in t_I . We use pattern matching over the t_I 's interface-value to extract the wanted methods. Recall that dictionaries maintain the order of method as specified by the interface.

Type assertions e.(u) are dealt with in rule TD-ASSERT and translate to an interface-value destructor. In the static semantics of FG there are two cases to consider. Both cases assume that the expression e is of some interface type t_I .

The first case asserts the type of a structure and the second case asserts the type of an interface. Asserting that a structure is of the type of another structure is not allowed in FG, because such a type assertion would never succeed.

Rule TD-DESTR-IFACE-STRUCT deals with the case that we assert the type of a structure t_S . If t_S does not implement the interface t_I , the assertion can never be successful. Hence, we find the precondition $\overline{D} \vdash_{\mathsf{FG}} t_S <: t_I$. We pattern match over the interface-value that represents t_I to check the underlying value matches t_S and extract the value. It is possible that some other value has been used to implement the interface-value that represents t_I . In such a case, the pattern match fails and we experience run-time failure.

Rule TD-DESTR-IFACE-IFACE deals with the case that we assert the type of an interface u_I on a value of type t_I . The outer case expression extracts the value Y underlying interface-value t_I (this case never fails). We then check if we can construct an interface-value for u_I via Y. This is done via an inner case expression. For each structure t_{Sj} implementing u_I , we have a pattern clause Cls_j that matches against the constructor $K_{t_{Sj}}$ of the structure and then constructs an interface-value for u_I . There are two reasons for run-time failure here. First, Y (used to implement t_I) might not implement u_I ; that is, none of the pattern clauses Cls_j match. Second, $\overline{[Cls]}$ might be empty because no receiver at all implements u_I . This case is rather unlikely and could be caught statically.

6 Properties

We wish to show that the dictionary-passing translation preserves the dynamic behavior of FG programs. To establish this property we make use of (binary) logical relations [16,20]. Logical relations express that related terms behave the same. We say that source and target terms are equivalent if they are related under the logical relation. The goal is to show that FG expressions and target expressions resulting from the dictionary-passing translation are equivalent.

For example, in FG the run-time value associated with an interface type is a structure that implements the interface whereas in our translation each interface translates to an interface-value. To establish that a structure $t_S\{\overline{v}\}$ and an interface-value $K_{t_I}(V,\overline{V})$ are equivalent w.r.t. some interface t_I we need to require that

- (Struct-I-Val-1) $t_S\{\overline{v}\}$ and V are equivalent w.r.t. t_S , and
- (Struct-I-Val-2) method definitions for receiver type t_S are equivalent to \overline{V} .

Because signatures in method specifications of an interface may refer to the interface itself, there may be cyclic dependencies that then result in well-foundness issues of the definition of logical relations. To solve this issue we include a step index [1]. We explain this technical point via the example in Figure 1. We will write $e \approx E \in [\![t]\!]_k$ to denote that FG expression e and TL expression E are in a logical relation w.r.t. the FG type e, where e is the step index. Similarly, func e is the step index of the following and a TL value e are in a logical relation w.r.t. the FG method declaration and a TL value e are in a logical relation w.r.t. the FG method specification e.

$$\begin{array}{c} \mathbb{R} & \mathbb{E} \in \mathbb{E} \mathbb{I} \mathbb{I}_{\mathbb{K}}^{[\overline{D},\Phi_m)} \\ \mathbb{R} & \mathbb{E} D\text{-REL-EXP} \\ \forall k_1 < k, k_2 < k, v, V. (k-k_1-k_2) > 0 \wedge \overline{D} \vdash_{\mathsf{FG}} e \overset{k_1}{\longrightarrow} v \wedge \Phi_m \vdash_{\mathsf{TL}} E \overset{k_2}{\longrightarrow} V) \\ & \Rightarrow v \approx V \in \mathbb{E} \mathbb{I}_{\mathbb{K}}^{[\overline{D},\Phi_m)} \\ \mathbb{E} & \mathbb{E} \in \mathbb{E} \mathbb{E} \mathbb{I}_{\mathbb{K}}^{[\overline{D},\Phi_m)} \\ \mathbb{E} & \mathbb{E} \in \mathbb{E} \mathbb{E} \mathbb{E} \mathbb{E} \mathbb{E} \\ \mathbb{E} & \mathbb{E} \in \mathbb{E} \mathbb{E} \mathbb{E} \mathbb{E} \\ \mathbb{E} & \mathbb{E} \otimes \mathbb{E} \mathbb{E} \mathbb{E} \mathbb{E} \\ \mathbb{E} & \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \mathbb{E} \mathbb{E} \\ \mathbb{E} & \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} & \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} & \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} \otimes \mathbb{E} \\ \mathbb{E} \otimes \mathbb{E}$$

Consider the FG expression $Int\{1\}$ from example in Figure 1. When viewed at type Eq, our translation yields the interface-value K_{Eq} (K_{Int} 1, eq_{Int}). We need to establish $Int\{1\} \approx K_{Eq}$ (K_{Int} 1, eq_{Int}) $\in [Eq]_{k_1}$.

- (1) $\operatorname{Int}\{1\} \approx K_{Eq}$ (K_{Int} 1, eq_{Int}) $\in [\![Eq]\!]_{k_1}$
 - if (2) $Int\{1\} \approx K_{Int} \ 1 \in [Int]_{k_2}$ and
 - (3) func (x Int) eq(y Eq) bool $\{\text{return}\,e\} \approx \text{eq}_{\text{Int}} \in [\![\text{eq(y Eq) bool}]\!]_{k_3}$ where $k_2 < k_1, k_3 < k_1$

$$\begin{split} \text{if (4)} \ \forall v_1 \approx V_1 \in \llbracket \texttt{Int} \rrbracket_{k_4}, v_2 \approx V_2 \in \llbracket \texttt{Eq} \rrbracket_{k_4}. \\ \langle x \mapsto v_1, y \mapsto v_2 \rangle e \approx \mathsf{eq}_{\texttt{Int}} \ \ V_1 \ \ V_2 \in \llbracket \texttt{bool} \rrbracket_{k_4} \text{where } k_4 \leq k_3 \end{split}$$

Following (Struct-I-Val-1) and (Struct-I-Val-2), (1) holds if we can establish (2) and (3). (2) is easy to establish. (3) holds if we can establish (4). (4) states that for equivalent inputs the respective method definitions are equivalent as well. Without the step index, establishing . \approx . \in [Eq] would reduce to establishing . \approx . \in [Eq]. We are in a cycle. With the step index, . \approx . \in [Eq] $_{k_1}$ reduces to . \approx . \in [Eq] $_{k_4}$ where $k_4 < k_1$. The step index represents the number of reduction steps we can take and will be reduced for each reduction step. Thus, we can give a well-founded definition of our logical relations.

Figure 6 gives the step-indexed logical relations to relate FG and TL terms. Rule RED-REL-EXP relates FG and TL expressions. The expressions are in a relation assuming that the resulting values are in a relation where we impose a step limit on the number of reduction steps that can be taken. We additionally find parameters \overline{D} and Φ_{m} as FG and TL expressions refer to method definitions.

Rule RED-REL-STRUCT is straightforward. Rule RED-REL-IFACE has been motivated above. We make use of the following helper function to lookup up the method definition for a specific pair of method name and receiver type.

$$\frac{\text{func }(x\ t_S)\ mM\ \{\text{return }e\}\in \overline{D}}{\text{methodLookup}(\overline{D},(m,t_S))=\text{func }(x\ t_S)\ mM\ \{\text{return }e\}}$$

Rule RED-REL-METHOD covers method definitions. Rule RED-REL-VB ensures that the substitutions from free variables to values are related. Rule RED-REL-DECLS ensures that our labeling for the translation of method definitions is consistent.

A fundamental property of step-indexed logical relations is that if two expressions are in a relation for k steps then they are also in a relation for any smaller number of steps.

Lemma 1 (Monotonicity). Let $e \approx E \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and $k' \leq k$. Then, we find that $e \approx E \in [\![t]\!]_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Proof. By induction over the derivation $e \approx E \in [t]_k^{\langle \overline{D}, \Phi_m \rangle}$. Case RED-REL-EXP:

$$\frac{\forall k_1 < k, k_2 < k, v, V.(k - k_1 - k_2 > 0 \land \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} v \land \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^{k_2} V)}{\Longrightarrow v \approx V \in \llbracket t \rrbracket_{k - k_1 - k_2}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}}$$

$$e \approx E \in \llbracket t \rrbracket_{k}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$$

If either e or E is irreducible, $e \approx E \in [t]_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ holds immediately because the universally quantified statement in the premise holds vacuously.

Otherwise, we find $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} v$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^{k_2} V$ for some k_1 and k_2 . If $k' - k_1 - k_2 \leq 0$, $e \approx E \in [\![t]\!]_{k'}^{\sqrt{D}, \Phi_{\mathsf{m}}\rangle}$ holds again immediately.

Otherwise, by induction applied on the premise of rule RED-REL-EXP we find that $v \approx V \in [\![t]\!]_{k'-k_1-k_2}^{\langle \overline{D}, \Phi_m \rangle}$ and we are done for this case.

Case RED-REL-STRUCT:

$$\frac{\text{type }t_S \text{ struct } \{\overline{f}\ \overline{t}^n\} \in \overline{D} \qquad \forall i \in [n].v_i \approx V_i \in [\![t_i]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}}{t_S \{\overline{v}^n\} \approx K_{t_S} \ (\overline{V}^n) \in [\![t_S]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}}$$

Follows immediately by induction.

Case RED-REL-IFACE:

$$\begin{split} V &= K_{u_S} \ \overline{V'} \\ (1) \ \forall k_1 < k.v \approx V \in \llbracket u_S \rrbracket_{k_1}^{\overline{D}, \varPhi_{\mathsf{m}} \rangle} \quad \text{methods}(\overline{D}, t_I) = \{\overline{mM}^n\} \\ \underline{(2) \ \forall k_2 < k, i \in [n]. \text{methodLookup}(\overline{D}, (m_i, u_S)) \approx V_i \in \llbracket m_i M_i \rrbracket_{k_2}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle} \\ v \approx K_{t_I} \ (V, \overline{V}^n) \in \llbracket t_I \rrbracket_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle} \end{split}$$

Consider the first premise (1). If there exists $k_1 < k'$ then $v \approx V \in \llbracket u_S \rrbracket_{k_1}^{\langle \overline{D}, \Phi_m \rangle}$. Otherwise, this premise holds vacuously. The same argument for $k_2 < k'$ applies to the second premise (2). Hence, $v \approx K_{t_I} (V, \overline{V_i}^n) \in \llbracket t_I \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle}$.

A similar monotonicity result applies to method definitions and declarations. Monotonicity is an essential property to obtain the following results.

Interface-value constructors and destructors preserve equivalent expressions via logical relations as stated by the following results.

Lemma 2 (Structural Subtyping versus Interface-Value Constructors). Let $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E_1 \text{ and } \vdash_{\mathsf{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ and } e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$ Then, we find that $e \approx E_1 E_2 \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

Lemma 3 (Type Assertions versus Interface-Value Destructors). Let $\overline{D} \vdash_{\mathsf{iDestr}} t \searrow u \leadsto E_1 \text{ and } \vdash_{\mathsf{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ and } e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. Then, we find that $e.(u) \approx E_1 \ E_2 \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Based on the above we can show that target expressions resulting from FG expressions and target methods resulting from FG methods are equivalent.

Lemma 4 (Expression Equivalence). Let $\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} \underline{e} : t \leadsto E \text{ and } \Phi_{\mathsf{v}}, \Phi_{\mathsf{v}}, \Phi_{\mathsf{m}} \text{ such that } \langle \overline{D}, \Phi_{\mathsf{m}}, \Delta \rangle \vdash_{\mathsf{red-rel}}^k \Phi_{\mathsf{v}} \approx \Phi_{\mathsf{v}} \text{ and } \vdash_{\mathsf{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ for some } k.$ Then, we find that $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{v}}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

Lemma 5 (Method Equivalence).

Let $\overline{D} \vdash_{\mathsf{meth}} \mathsf{func}\ (x\ t_S)\ m(\overline{x}\ t^n)\ t\ \{\mathsf{return}\ e\} \leadsto \lambda X.\lambda(\overline{X}^n).E.$ Then, we find that $\vdash_{\mathsf{red}-\mathsf{rel}}^k \overline{D} \approx \varPhi_{\mathsf{m}}\ where\ \varPhi_{\mathsf{m}}(X_{m,t_S}) = \lambda X.\lambda(\overline{X}^n).E$ for any k.

The lengthy proofs of the above results are given in the appendix.

From Lemmas 4 and 5 we can derive our main result that the dictionary-passing translation preserves the dynamic behavior of FG programs.

Theorem 1 (Program Equivalence). Let $\vdash_{\mathsf{prog}} \overline{D}$ func $\mathsf{main}()\{ _ = e \} \leadsto \mathsf{let} \ \overline{X_{m_i,t_{S_i}}} = \overline{E_i}^n \ \mathsf{in} \ E \ where \ we \ assume \ that \ e \ has \ type \ t. \ Then, \ we \ find \ that \ e \approx E \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle} \ for \ any \ k \ where \ \Phi_{\mathsf{m}} = \langle \overline{X_{m_i,t_{S_i}}} \mapsto \overline{E_i}^n \rangle.$

Proof. Follows from Lemmas 4 and 5.

Our main result also implies that our translation is coherent. Recall that the translation rules are non-syntax directed because of rule TD-SUB. Hence, we could for example insert an (albeit trivial) interface-value constructor resulting from $\overline{D} \vdash_{\mathsf{iCons}} t_I <: t_I \leadsto E$. Hence, there might be different target terms for the same source term. Our main result guarantees that all targets obtained preserve the meaning of the original program.

7 Related Work and Conclusion

The dictionary-passing translation is well-studied in the context of Haskell type classes [24]. A type class constraint translates to an extra function parameter, constraint resolution provides a dictionary with the methods of the type class for this parameter. In our translation from Featherweight Go [7], dictionaries are not supplied as separate parameters because FG does not support parametric polymorphism. Instead, a dictionary is always passed as part of an interface-value, which combines the dictionary with the concrete value implementing the interface. Thus, interface-values can be viewed as representations of existential types [13,10,23]. How to adapt our dictionary-passing translation scheme to FG extended with parametric polymorphism (generics) is something we plan to consider in future work.

In the context of type classes it is common to show that resulting target programs are well-typed. For example, see the work by Hall and coworkers [8]. Typed target terms in this setting require System F and richer variants depending on the kind of type class extensions that are considered [19]. Our target terms are untyped and we pattern match over constructors to check for "runtime types". For example, see rule TD-DESTR-IFACE-STRUCT in Figure 5. There are various ways to support dynamic typing in a typed setting. For example, we could employ GADTs as described by Peyton Jones and coworkers [9]. A simply-typed first order functional language with GADTs appears then to be sufficient as a typed target language for Featherweight Go. This will require certain adjustments to our dictionary-passing translation. We plan to study the details in future work.

Another important property in the type class context is coherence. Bottu and coworkers [3] make use of logical relations to state equivalence among distinct target terms resulting from the same source type class program. Thanks to our main result Theorem 1, we get coherence for free. We believe it is worthwhile

to establish a property similar to Theorem 1 for type classes. We could employ a simple denotational semantics for source type class programs such as [21,14] which is then related to target programs obtained via the dictionary-passing translation. This is something that has not been studied so far and another topic for future work.

Method dictionaries bear some resemblance to virtual method tables (vtables) used to implement virtual method dispatch in object-oriented languages [5]. The main difference between vtables and dictionaries is that there is a fixed connection between an object and its vtable (via the class of the object), whereas the connection between a value and a dictionary may change at runtime, depending on the type the value is used at. Dictionaries allow access to a method at a fixed offset, whereas vtables in the presence of multiple inheritance require a more sophisticated lookup algorithm [2].

Subtyping for interfaces in Go is based purely on width subtyping, there is no support for depth subtyping [15]: a subtype might provide more methods than the super-interface, but method signatures must match invariantly. Method dispatch in Go is performed only on the receiver of the method call. Multi-dispatch [18,4] refers to the ability to dispatch on multiple arguments, but this approach turns out to be difficult in combination with structural subtyping [11].

To summarize the results of the paper at hand: we defined a dictionary-passing translation from Featherweight Go to a untyped lambda calculus with pattern matching. The compiler for the full Go language [22] employs a similar dictionary-passing approach. We proved that the translation preserves the dynamic semantics of Featherweight Go, using step-indexed logical relations.

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A Proofs for Properties Stated in the Main Text

A.1 Monotonicity for Method Definitions and Declarations

Lemma 6 (Monotonicity 2). Let func $(x t_S) mM$ {return e} $\approx V \in \llbracket mM \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ and $k' \leq k$. Then, we find that func $(x t_S) mM$ {return e} $\approx V \in \llbracket mM \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle}$.

Proof. Follows immediately by observing the premise of rule RED-REL-METHOD.

Lemma 7 (Monotonicity 3). Let $\vdash_{\mathsf{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ and $k' \leq k$. Then, we find that $\vdash_{\mathsf{red-rel}}^{k'} \overline{D} \approx \Phi_{\mathsf{m}}$.

Proof. Follows via Lemma 6.

A.2 Lemma 2

Proof. We show that $e \approx E_1$ $E_2 \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ by making use of the following auxiliary statement.

Let $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E \text{ and } \vdash_{\mathsf{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ and } v \approx V \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$ Then, we find that $v \approx E$ $V \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

Suppose $k_1 < k$ and $k_2 < k$ and $k - k_1 - k_2 > 0$ and (1) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} v$ for some v and (2) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^{k_2} V$ for some V. Based on the assumption that $e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and via rule RED-REL-EXP we conclude that $v \approx V \in \llbracket t \rrbracket_{k-k_1-k_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Via the auxiliary statement we conclude that (3) $v \approx E_1 \ V \in \llbracket u \rrbracket_{k-k_1-k_2}^{\langle \overline{D}, \Phi_m \rangle}$. Via rule RED-REL-EXP making use of (1), (2) and (3) we conclude that $e \approx E_1 \ E_2 \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ and we are done.

Proof of auxiliary statement.

We have to show that for all $k_2 < k$ where $k - k_2 > 0$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \ V \longrightarrow^{k_2} V'$ we have that $v \approx V' \in [\![t]\!]_{k-k_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

We perform a case analysis of the derivation for $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E$ and label the assumptions (1) $\vdash_{\mathsf{red-rel}}^k \overline{D} \approx \varPhi_{\mathsf{m}}$ and (2) $v \approx V \in \llbracket t \rrbracket_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$ for later reference.

Case TD-CONS-STRUCT-IFACE:

$$\frac{\text{type }t_I \text{ interface } \{\overline{S}\} \in \overline{D} \qquad \text{methods}(\overline{D}, t_S) \supseteq \overline{S} \qquad \overline{S} = \overline{mM}^n}{\overline{D} \vdash_{\mathsf{iCons}} t_S <: t_I \leadsto \lambda X. K_{t_I} \left(X, \overline{X_{m_i, t_S}}^n\right)}$$

Set $E = \lambda X.K_{t_I} (X, \overline{X_{m_i, t_S}}^n)$. Then, (3) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E V \longrightarrow^1 V'$ where $V' = K_{t_I} (V, \overline{X_{m_i, t_S}}^n)$.

From (1) and Lemma 7 we obtain (4) $\forall k_1 < k$. $\vdash_{\mathsf{red-rel}}^{k_1} \overline{D} \approx \Phi_{\mathsf{m}}$.

From (2) and Lemma 1 we obtain (5) $\forall k_2 < k.v \approx V \in [t_S]_{k_2}^{\langle \overline{D}, \Phi_m \rangle}$ (for this case $t = t_S$).

From (4), (5) and via rule RED-REL-IFACE we obtain (6) $v \approx V' \in [t_I]_k^{\langle \overline{D}, \Phi_m \rangle}$.

From (3), (6) and via rule RED-REL-EXP we obtain $v \approx E \ V \in [t_I]_k^{\langle \overline{D}, \Phi_m \rangle}$ and we are done for this case.

Case TD-CONS-IFACE-IFACE:

$$\begin{array}{c} \text{type } t_I \text{ interface } \{\overline{R_i}^n\} \in \overline{D} \\ \text{type } u_I \text{ interface } \{\overline{S_i}^q\} \in \overline{D} \qquad S_i = R_{\pi(i)} \quad \text{(for all } i \in [q]) \\ \overline{D} \vdash_{\mathsf{iCons}} t_I <: u_I \leadsto \lambda X. \mathsf{case} \, X \, \mathsf{of} \, K_{t_I} \, \left(X_{val}, \overline{X_i}^n\right) \to K_{u_I} \, \left(X_{val}, \overline{X_{\pi(i)}}^q\right) \end{array}$$

From (2) and for this case we can conclude via rule RED-REL-IFACE that (3) $V = K_{t_I} (V', \overline{V_i}^n)$ for some u_S, V' and $\overline{V_i}^n$ where $\operatorname{methods}(\overline{D}, t_I) = \{\overline{m_i M_i}^n\}$ and (4) $\forall k_1 < k.v \approx V' \in \llbracket u_S \rrbracket_{k_1}^{\langle \overline{D}, \Phi_m \rangle}$ and (5) $\forall k_2 < k, i \in [n]. \operatorname{methodLookup}(\overline{D}, (m_i, u_S)) \approx V_i \in \llbracket m_i M_i \rrbracket_{k_2}^{\langle \overline{D}, \Phi_m \rangle}$.

Set $E = \lambda X$.case X of $K_{t_I}(X_{val}, \overline{X_i}^n) \to K_{u_I}(X_{val}, \overline{X_{\pi(i)}}^q)$. Then, (6) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E V \longrightarrow^1 V''$ where (7) $V'' = K_{u_I}(V', \overline{V_{\pi(i)}}^q)$.

From (4), (5) and (7) via rule RED-REL-IFACE we obtain that (8) $v \approx V'' \in$ $[u_I]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

From (6), (8) and via rule RED-REL-EXP we obtain $v \approx E \ V \in [\![u_I]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and we are done.

A.3 Lemma 3

We first introduce an auxiliary statement.

Lemma 8. Let $e' \approx E' \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} e'$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^{k_2} E'$. Then, we find that $e \approx E \in \llbracket t \rrbracket_{k+k_1+k_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

Proof. If either e' or E' are irreducible the result follows immediately.

Otherwise, based on rule RED-REL-EXP we find that $\overline{D} \vdash_{\mathsf{FG}} e' \longrightarrow^{k'_1} v$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E' \longrightarrow^{k'_2} V \text{ and } v \approx V \in \llbracket t \rrbracket_{k-k'_1-k'_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

Based on the above, our assumptions and rule RED-REL-EXP we find that $e \approx E \in [t]_{k+k_1+k_2}^{\langle \overline{D}, \Phi_m \rangle}$ and we are done.

Here comes the proof of Lemma 3.

Proof. We perform a case analysis of the derivation $\overline{D} \vdash_{\mathsf{iDestr}} t \searrow u \leadsto E_1$ and label the assumptions (1) $\vdash_{\text{red-rel}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ and (2) $e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for later reference.

Case TD-DESTR-IFACE-STRUCT:

$$\frac{\text{type }t_I \text{ interface } \{\overline{S}^n\} \in \overline{D} \quad \overline{D} \vdash_{\mathsf{FG}} t_S <: t_I}{\overline{D} \vdash_{\mathsf{iDestr}} t_I \searrow t_S \leadsto \lambda X. \mathsf{case} \, X \, \mathsf{of} \, K_{t_I} \, \left(K_{t_S} \, \overline{Y}, \overline{X}^n\right) \to K_{t_S} \, \overline{Y}}$$

We set $E_1 = \lambda X$.case X of K_{t_I} $(K_{t_S} \overline{Y}, \overline{X}^n) \to K_{t_S} \overline{Y}$. From (2) and via rule RED-REL-EXP we conclude that for all $k_1 < k, k_2 < k, v$, $V \text{ where (3) } k - k_1 - k_2 > 0 \text{ and (4) } \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} v \text{ and (5) } \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \stackrel{\mathsf{def}}{\longrightarrow^{k_2}} V$ we have that (6) $v \approx V \in [t_I]_{k-k_1-k_2}^{\langle \overline{D}, \Phi_m \rangle}$.

From (6) and rule RED-REL-IFACE we conclude that (7) $V = K_{t_I} (K_{Rep_{u,c}}, V, \overline{V_i}^n)$ where $V = K_{u_S} \overline{V'}$ and for all (8) $k'_1 < k - k_1 - k_2$ we have that (9) $v \approx V \in$

Subcase $t_S \neq u_S$: Neither $e(t_S)$ nor E_1 E_2 are reducible and therefore we immediately can conclude that $e \approx E_1$ $E_2 \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ holds.

Subcase $t_S = u_S$: From (4) we conclude that (10) $\overline{D} \vdash_{\mathsf{FG}} e.(t_S) \longrightarrow^{k_1+1} v.$

From (5) and (7) we conclude that (11) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_1 E_2 \longrightarrow^{k_2+3} V$. There are three additional reduction steps as we have one extra lambda and two extra pattern match applications.

From (8) and (9) and the Monotonicity Lemma 1 we conclude that $v \approx V \in \llbracket t_s \rrbracket_{k-(k_1+1)-(k_2+3)}^{\langle \overline{D}, \Phi_m \rangle}$ and via rule RED-REL-EXP we obtain that $e \approx E_1$ $E_2 \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ and we are done for this case.

Case TD-DESTR-IFACE-IFACE:

$$\begin{array}{c} \text{type } t_I \text{ interface } \{\overline{S}^n\} \in \overline{D} \\ \overline{Cls} = [K_{t_S} \ \overline{Y} \to (E \ X_{val}) \mid \overline{D} \vdash_{\mathsf{iCons}} t_S <: u_I \leadsto E] \\ \overline{\overline{D}} \vdash_{\mathsf{iDestr}} t_I \searrow u_I \leadsto \lambda X. \mathsf{case} \ X \ \mathsf{of} \ K_{t_I} \ (X_{val}, \overline{X}^n) \to \mathsf{case} \ X_{rep} \ \mathsf{of} \ \overline{Cls} \end{array}$$

We set $E_1 = \lambda X$.case X of K_{t_I} $(X_{val}, \overline{X}^n) \to \text{case } X_{rep}$ of \overline{Cls} .

We apply similar reasoning as in case of TD-DESTR-IFACE-STRUCT.

From (2) and via rule RED-REL-EXP we conclude that for all $k_1 < k$, $k_2 < k$, v, V where (3) $k - k_1 - k_2 > 0$ and (4) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k_1} v$ and (5) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^{k_2} V$ we have that (6) $v \approx V \in \llbracket t_I \rrbracket_{k-k_1-k_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

From (6) and rule RED-REL-IFACE we conclude that (7) $V = K_{t_I} (V, \overline{V_i}^n)$ where

From (6) and rule RED-REL-IFACE we conclude that (7) $V = K_{t_I} (V, \overline{V_i}^n)$ where $V = K_{t_S} \overline{V'}$ and for all (8) $k'_1 < k - k_1 - k_2$ we have that (9) $v \approx V \in [t_s]_{k'_1}^{\langle \overline{D}, \Phi_m \rangle}$. We use here t_S (instead of u_S) to match the naming conventions in the premise of rule TD-DESTR-IFACE-IFACE.

Subcase $\overline{D} \vdash_{\mathsf{FG}} t_S <: u_I$ does not hold: Neither $e.(t_S)$ nor $E_1 E_2$ are reducible and therefore we immediately can conclude that $e \approx E_1 E_2 \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$ holds. Subcase $\overline{D} \vdash_{\mathsf{FG}} t_S <: u_I$ does hold: From (4) we conclude that (10) $\overline{D} \vdash_{\mathsf{FG}} e.(u_I) \longrightarrow^{k_I} v$.

From (5) and (7) we conclude that (11) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_1 E_2 \longrightarrow^{k_2+3} E V$ where (12) $\overline{D} \vdash_{\mathsf{iCons}} t_S <: u_I \leadsto E$. There are three additional reduction steps as we have one extra lambda and two extra pattern match applications. The upcast E has not been applied.

From (6) and (12) and Lemma 2 we obtain that (13) $v \approx E \ V \in \llbracket u_I \rrbracket_{k'_1}^{\langle \overline{D}, \Phi_m \rangle}$. Via the Monotonicity Lemma 1 and Lemma 8 we obtain that $e.(u_I) \approx E_1 \ E_2 \in \llbracket u_I \rrbracket_k^{\langle \overline{D}, \Phi_m \rangle}$ and we are done.

A.4 Lemma 4

Proof. By induction over the derivation $\langle \overline{D}, \Delta \rangle \vdash_{\sf exp} e : t \leadsto E$. We label the assumptions (1) $\langle \overline{D}, \varPhi_{\sf m}, \Delta \rangle \vdash_{\sf red-rel}^k \varPhi_{\sf v} \approx \varPhi_{\sf V}$ and (2) $\vdash_{\sf red-rel}^k \overline{D} \approx \varPhi_{\sf m}$ as well as the to be proven statement (3) $\varPhi_{\sf v}(e) \approx \varPhi_{\sf V}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \varPhi_{\sf m} \rangle}$ for some later reference.

Case TD-VAR:

$$\frac{(x:t) \in \Delta}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} x:t \leadsto X}$$

(3) follows immediately from (1).

Case TD-STRUCT:

$$\frac{\text{type } t_S \text{ struct } \{\overline{f_i \ t_i} n\} \in \overline{D} \qquad \langle \overline{D}, \Delta \rangle \vdash_{\text{exp}} e_i : t_i \leadsto E_i \quad (\text{for all } i \in [n])}{\langle \overline{D}, \Delta \rangle \vdash_{\text{exp}} t_S \{\overline{e_i}^n\} : t_S \leadsto K_{t_S} \left(\overline{E_i}^n\right)}$$

Suppose there exists $k_1 < k$ and $k_2 < k$ and v and V such that $(k-k_1-k_2 > 0)$ and (4) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{V}}(t_s\{\overline{e_i}^n\}) \longrightarrow^{k_1} t_s\{\overline{v_i}^n\}$ and (5) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(K_{t_S}(\overline{E_i}^n)) \longrightarrow^{k_2} K_{t_S}(\overline{V_i}^n)$ for $i \in [n]$.

From (4) and (5) we conclude that (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k'_1} v_i$ and (7) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \longrightarrow^{k'_2} V_i$ for $i \in [n]$ where we pick k'_1 and k'_2 such that $k'_1 < k_1$ and $k'_2 < k_2$ and all the subreductions yield some value.

By induction (8) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t_i \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for $i \in [n]$.

From (6), (7), (8) and via rule RED-REL-EXP we conclude that (9) $v_i \approx V_i \in [t_i]_{k-k_1'-k_2'}^{\sqrt{D},\Phi_{\mathsf{m}}\rangle}$ for $i \in [n]$.

From (9) and rule RED-REL-STRUCT we conclude that (10) $t_S\{\overline{v_i}^n\} \approx K_{t_S}(\overline{V_i}^n) \in [t_S]_{k-k_1'-k_2'}^{\langle \overline{D}, \Phi_m \rangle}$.

From (10) and Lemma 1 we conclude that (11) $t_S\{\overline{v_i}^n\} \approx K_{t_S}(\overline{V_i}^n) \in \mathbb{I}_{t_S}[\overline{V_i}^{\overline{D},\Phi_m}]$.

From (4), (5), (11) and via rule RED-REL-EXP we conclude that $\Phi_{\mathsf{v}}(t_s\{\overline{e_i}^n\}) \approx \Phi_{\mathsf{v}}(K_{t_S}(\overline{E_i}^n)) \in [\![t_S]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and we are done for this case. Case TD-ACCESS:

$$\frac{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : t_S \leadsto E \qquad \mathsf{type} \ t_S \ \mathsf{struct} \ \{ \overline{f_j \ t_j}^n \} \in \overline{D}}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e. f_i : t_i \leadsto \mathsf{case} \ E \ \mathsf{of} \ K_{t_S} \ (\overline{X_i}^n) \to X_i}$$

Similar reasoning as in case of TD-STRUCT.

Case TD-CALL-STRUCT:

$$\frac{m(\overline{x_i}\ \overline{t_i}^n)\ t\in \mathsf{methods}(\overline{D}, t_S)}{\langle \overline{D}, \Delta \rangle\ \vdash_{\mathsf{exp}}\ e: t_S \leadsto E \qquad \langle \overline{D}, \Delta \rangle\ \vdash_{\mathsf{exp}}\ e_i: t_i \leadsto E_i \quad (\mathsf{for\ all}\ i \in [n])}{\langle \overline{D}, \Delta \rangle\ \vdash_{\mathsf{exp}}\ e.m(\overline{e_i}^n): t \leadsto X_{m,t_S}\ E\ (\overline{E_i}^n)}$$

Suppose there exists $k_1 < k$ and $k_2 < k$ and v and V such that $(k-k_1-k_2 > 0$ and (4) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{V}}(e.m(\overline{e_i}^n)) \longrightarrow^{k_1} v$ and (5) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(X_{m,t_S} \ E\ (\overline{E_i}^n)) \longrightarrow^{k_2} V$.

From the assumptions and (4) we conclude that (4a) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^1 \langle x \mapsto \Phi_{\mathsf{v}}(e), \overline{x_i \mapsto \Phi_{\mathsf{v}}(e_i)}^n \rangle e'$ and (4b) $\overline{D} \vdash_{\mathsf{FG}} \langle x \mapsto \Phi_{\mathsf{v}}(e), \overline{x_i \mapsto \Phi_{\mathsf{v}}(e_i)}^n \rangle e' \longrightarrow^{k_1-1} v$ where (4c) **func** $(x \ t_S) \ m(\overline{x_i \ t_i}^n) \ t \ \{\underline{\mathbf{return}} \ e'\} \in \overline{D}.$

From (4) we conclude that (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k'_1} v'$ and (7) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k'_1} v_i$ for some v' and v_i for $i \in [n]$ where $k'_1 < k_1$. We pick again some large enough k'_1 such that all subreductions yields some value.

Similarly, from (5) we conclude that (8) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^{k'_2} V'$ and (9) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \longrightarrow^{k'_2} V_i$ for some V' and V_i for $i \in [n]$ where $k'_2 < k_2$.

By induction we have that (10) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and (11) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t_i \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for $i \in [n]$.

From (6), (8), (10) and via rule RED-REL-EXP we conclude that (12) $v' \approx V' \in [t_S]_{k-k_1'-k_2'}^{\langle \overline{D}, \Phi_m \rangle}$.

Similarly, from (7), (9), (11) and via rule RED-REL-EXP we conclude that (13) $v_i \approx V_i \in [\![t_i]\!]_{k-k_1'-k_2'}^{\langle\overline{D},\Phi_m\rangle}$ for $i \in [n]$.

From (4c), (12), (13), (3) and via rule RED-REL-METHOD we conclude that (14) $\langle x \mapsto v, \overline{x_i \mapsto v_i}^n \rangle e' \approx X_{m,t_S} \ V' \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k-k_1'-k_2'}^{\langle \overline{D}, \Phi_m \rangle}.$

Based on our choice of k'_1 and k'_2 we conclude that (15) $\overline{D} \vdash_{\mathsf{FG}} \langle x \mapsto v, \overline{x_i \mapsto v_i}^n \rangle e' \longrightarrow^{k_1 - k'_1} v$ and (16) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} X_{m,t_S} V'(\overline{V_i}^n) \longrightarrow^{k_2 - k'_2} V$. That is, with $k_1 - k'_1$ steps or less we reach v because k_1 is the overall number of steps required and k'_1 is the maximum number of one of the subcomputation steps. The same applies to $k_2 - k'_2$.

From (14), (15), (16) and via rule RED-REL-EXP we conclude that $v \approx V \in [\![t]\!]_{k-k_1-k_2}^{\overline{D},\Phi_{\mathrm{m}}\rangle}$ where we make use of the fact that $k-k_1'-k_1'-(k_1-k_1')-(k_2-k_2')=k-k_1-k_2$. Thus, we are done for this case.

Case TD-CALL-IFACE:

$$\begin{array}{c} \langle \overline{D}, \underline{\Delta} \rangle \vdash_{\mathsf{exp}} e : t_I \leadsto E \\ \mathbf{type} \ t_I \ \mathbf{interface} \ \{ \overline{S_i}^q \} \in \overline{D} \quad S_j = m(\overline{x_i \ t_i}^n) \ t \quad (\mathsf{for some} \ j \in [q]) \\ \langle \overline{D}, \underline{\Delta} \rangle \vdash_{\mathsf{exp}} e_i : t_i \leadsto E_i \quad (\mathsf{for all} \ i \in [n]) \\ \hline \langle \overline{D}, \underline{\Delta} \rangle \vdash_{\mathsf{exp}} e.m(\overline{e_i}^n) : t \leadsto \mathbf{case} \ E \ \mathbf{of} \ K_{t_I} \ (X_{val}, \overline{X_i}^q) \to X_j \ X_{val} \ (\overline{E_i}^n) \end{array}$$

Similar reasoning as in case of TD-CALL-STRUCT. We set

$$E' = \mathsf{case} \ E \ \mathsf{of} \ K_{t_I} \ (\underline{\ }, X_{val}, \overline{X_i}^q) \to X_j \ X_{val} \ (\overline{E_i}^n).$$

Suppose there exists $k_1 < k$ and $k_2 < k$ and v and V such that $(k-k_1-k_2 > 0$ and (4) $\overline{D} \vdash_{\mathsf{FG}} \varPhi_{\mathsf{V}}(e.m(\overline{e_i}^n)) \longrightarrow^{k_1} v$ and (5) $\varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} \varPhi_{\mathsf{V}}(E') \longrightarrow^{k_2} V$.

From the assumptions and (4) we conclude that (4a) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^1 \langle x \mapsto \Phi_{\mathsf{v}}(e), \overline{x_i} \mapsto \overline{\Phi_{\mathsf{v}}(e_i)}^n \rangle e' \text{ and } (4b) \overline{D} \vdash_{\mathsf{FG}} \langle x \mapsto \Phi_{\mathsf{v}}(e), \overline{x_i} \mapsto \overline{\Phi_{\mathsf{v}}(e_i)}^n \rangle e' \longrightarrow^{k_1-1} v \text{ where } (4c) \text{ func } (x t_S) \ m(\overline{x_i} \ \overline{t_i}^n) \ t \text{ {return } } e' \} \in \overline{D}.$

From (4) we conclude that (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k'_1} v'$ and (7) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k'_1} v_i$ for some v' and v_i for $i \in [n]$ where $k'_1 < k_1$. We pick again some large enough k'_1 such that all subreductions yields some value.

Similarly, from (5) we conclude that (8) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^{k'_2} V'$ and (9) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \longrightarrow^{k'_2} V_i$ for some V' and V_i for $i \in [n]$ where $k'_2 < k_2$.

By induction we have that (10) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and (11) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t_i \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for $i \in [n]$.

From (6), (8), (10) and via rule RED-REL-EXP we conclude that (12) $v' \approx V' \in [\![t_I]\!]_{k-k_1'-k_2'}^{\langle \overline{D}, \Phi_m \rangle}$.

Similarly, from (7), (9), (11) and via rule RED-REL-EXP we conclude that (13) $v_i \approx V_i \in [\![t_i]\!]_{k-k_1'-k_2'}^{\langle \overline{D}, \Phi_m \rangle}$ for $i \in [n]$.

From (12) and via RED-REL-IFACE we conclude that (13) $V' = K_{t_I} (V'', \overline{V_i'}^p)$ and $V'' = K_{t_S} \overline{V'''}$ and (14) $v' \approx V'' \in \llbracket t_S \rrbracket_{k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and (15) func $(x \, t_S) \, m(\overline{x_i \, t_i}^n) \, t$ {return e'} \approx

 $V_j' \in [m(\overline{x_i t_i}^n)]_{k''}^{\overline{D}, \Phi_m}$ for some k'' where $k'' < k - k_1' - k_2'$ and $j \in [q]$ is the same j as in the premise of rule TD-CALL-IFACE.

From (15) via rule RED-REL-METHOD and (14) and (13) plus the Monotonicity

Lemma 1 we conclude that (16) $\langle x \mapsto v', \overline{x_i \mapsto v_i}^n \rangle e' \approx V_j' \ V'' \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k''}^{\langle \overline{D}, \Phi_m \rangle}$. For concreteness, we can assume $k'' = k - k_1' - k_2' - 1$. Based on our choice of k_1' and k_2' we conclude that (17) $\overline{D} \vdash_{\mathsf{FG}} \langle x \mapsto v, \overline{x_i \mapsto v_i}^n \rangle e' \longrightarrow^{k_1 - k_1'} v$ and (18) $V_j' \ V'' \ (\overline{V_i}^n) \vdash_{\mathsf{TL}} V \longrightarrow^{k_2 - k_2' + 1}$. The argument is the same as in case of TD-CALL-STRUCT.

From (16), (17), (18) and via rule RED-REL-EXP we conclude that $v \approx V \in$ $[\![t]\!]_{k-k_1-k_2}^{\langle\overline{D},\Phi_{\rm m}\rangle}$ and we are done for this case.

Case TD-SUB:

$$\frac{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : t \leadsto E_2 \qquad \overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E_1}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : u \leadsto E_1 \ E_2}$$

By induction we obtain that (4) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E_2) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. From (3), (4) and Lemma 2 we obtain that $\Phi_{\mathsf{v}}(e) \approx E_1 \; \Phi_{\mathsf{V}}(E_2) \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

We have that $\Phi_{V}(E_1) = E_1$ and thus we are done for this case.

Case TD-ASSERT:

$$\frac{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e : u \leadsto E_2 \qquad \overline{D} \vdash_{\mathsf{iDestr}} u \searrow t \leadsto E_1}{\langle \overline{D}, \Delta \rangle \vdash_{\mathsf{exp}} e.(t) : t \leadsto E_1 \ E_2}$$

By induction we obtain that (4) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E_2) \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. From (3), (4) and Lemma 3 we obtain that $\Phi_{\mathsf{v}}(e).(t) \approx E_1 \; \Phi_{\mathsf{V}}(E_2) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

We have that $\Phi_{V}(E_1) = E_1$ and thus we are done for this case.

A.5Lemma 5

Proof. Based on rules RED-REL-DECLS and RED-REL-METHOD, for

func
$$(x \ t_S) \ m(\overline{x_i \ t_i}^n) \ t \ \{ \mathbf{return} \ e \} \in \overline{D}$$

we have to show that

$$\forall k' \leq k, v', V', \overline{v_i}^n, \overline{V_i}^n. (v' \approx V' \in \llbracket t_S \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle} \wedge (\forall i \in [n]. v_i \approx V_i \in \llbracket t_i \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle}))$$

$$\implies (1) \ \langle x \mapsto v', \overline{x_i \mapsto v_i}^n \rangle e \approx (X_{x, t_S} \ V') \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle}$$

We verify the result by induction on k.

Case k=1: We must perform several reductions on (X_{x,t_s}, V') $(\overline{V_i}^n)$ to obtain a value. Due to k = 1 the premise of rule RED-REL-EXP holds vacuously. Therefore, we can immediately establish (1).

Case $k \implies k+1$: Suppose $k' \le k+1$ and (2) $v' \approx V' \in [t_S]_{k'}^{\langle \overline{D}, \Phi_m \rangle}$ and (3) $v_{i} \approx V_{i} \in \llbracket t_{i} \rrbracket_{k'}^{\langle \overline{D}, \Phi_{m} \rangle} \text{ for some } v', V', v_{i}, V_{i} \text{ for } i \in [n].$ Suppose $\langle x \mapsto v', \overline{x_{i} \mapsto v_{i}}^{n} \rangle e$ and $(X_{x,t_{S}}, V')$ $(\overline{V_{i}}^{n})$ are reducible. Otherwise,

the result holds immediately.

We have to show that for (4) $\overline{D} \vdash_{\mathsf{FG}} \langle x \mapsto v', \overline{x_i \mapsto v_i}^n \rangle e \longrightarrow^{k_1} v''$ and (5) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (X_{x,t_S} V') (\overline{V_i}^n) \longrightarrow^{k_2} V''$ and $k+1-k_1-k_2>0$ we have that (6) $v'' \approx V'' \in \llbracket t \rrbracket_{k+1-k_1-k_2}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

From (5) we can conclude that (7) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (X_{x,t_S} V') (\overline{V_i}^n) \longrightarrow^1 \lambda X.\lambda(\overline{X_i}^n).E \longrightarrow^2$

 $\langle X \mapsto V', \overline{X_i \mapsto V_i}^n \rangle E \longrightarrow^{k'_2} V''$ where (8) $\underline{k_2} = k'_2 + 3$. By induction we have that (9) $\vdash^k_{\mathsf{red-rel}} \overline{D} \approx \Phi_{\mathsf{m}}$. From (2) and (3) and the Monotonicity Lemma 1 we find that (10) $v' \approx V' \in \overline{\mathbb{R}}$ $\llbracket t_S
rbracket^{\langle \overline{D}, \Phi_m \rangle}$ and (11) $v_i \approx V_i \in \llbracket t_i
rbracket^{\langle \overline{D}, \Phi_m \rangle}$ where $k'' \leq k$ for $i \in [n]$. By making use of (9), (10) and (11) we apply Lemma 4 on

$$\overline{D} \vdash_{\mathsf{meth}} \mathsf{func} (x \ t_S) \ m(\overline{x_i \ t_i}^n) \ t \ \{\mathsf{return} \ e\} \leadsto \lambda X. \lambda(\overline{X_i}^n). E$$

and thus obtain that (12) $\langle x \mapsto v', \overline{x_i \mapsto v_i}^n \rangle e \approx \langle X \mapsto V', \overline{X_i \mapsto \overline{V_i}^n} \rangle E \in$ $[\![t]\!]_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}.$

From (12), (4) and (7) via rule RED-REL-EXP we conclude that (11) $v'' \approx V'' \in$

From (8) we conclude that (12) $k + 1 - k_1 - k_2 = k - k_1 - k'_2 - 2$.

From (11), (12) and the Monotonicity Lemma 1 we conclude that $v'' \approx V'' \in$ $[\![t]\!]_{k-k_1-k_2'-2}^{\langle \overline{D},\Phi_{\mathsf{m}}\rangle}$ and we are done.