Feature-Aided SMC-PHD Filter for Nonlinear Multi-target Tracking in Cluttered Environments

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Abstract. The Sequential Monte Carlo Probability Hypothesis Density (SMC-PHD) filter is a permissive multi-target tracker, performing state estimation through particle filtering with implicit data association. This filter is thus effective even in presence of clutter and nonlinear dynamics, while remaining tractable for real-time applications due to its computationally efficient data association process. Sensors are sometimes capable of sensing target features, which add up to kinematic measurements, e.g. range and bearing.
In this paper, the adaptive Feature-Aided-SMC-PHD filter is designed, making use of feature information to increase the SMC-PHD's estimation performance with respect to clutter, detection probability and location precision. As suspected, further differentiating targets from clutter led to greater sample degeneracy, especially as the detection probability drops. An adaptive sampling scheme was hence developed in order to relax this phenomenon. A radar application is considered in this study to validate this paper's approach using Monte Carlo simulations.

Keywords: multi-target tracking, feature-aided tracking, particle filter, probability hypothesis density, SMC-PHD, degeneracy

1 Introduction

In Multi-Target Tracking (MTT), sensor and detection algorithm qualities (precision, false alarm rate, etc.) can affect the simultaneous estimation of the number and states of targets. In particular, MTT is a key technology for sports player tracking [1] or drone surveillance [2], albeit those applications may imply erratic target motions and cluttered measurements.

As part of MTT, a data association algorithm, associating measurements to tracks, can identify targets from clutter. Instead of deterministically selecting the measurements to use for state estimation, probabilistic association is often more effective [3]. For intance, Multiple Hypothesis Trackers (MHT) yield good performance by considering all possible associations, leading combinatorial explosion though. This is mitigated in the Joint Probabillistic Data Association (JPDA) filter (and its many variants) which considers only a time step's hypotheses. Using JPDA, targets tend to coalesce. Rather than evaluating possible hypotheses, Probability Hypothesis Density (PHD) filters are computationally efficient in that they fuse all observations at once without requiring target labeling. This also explains a certain clutter-resilience. Additionally, SMC-PHD filter is compliant with nonlinear target dynamics. This framework is hence well suited to the challenging tracking conditions considered in this paper [4], namely highly nonlinear target dynamics, arbitrary multi-target distribution, clutter and low detection probabilities.

The main contribution of this paper is a set of implementation guidelines to supplement the canonical SMC-PHD filter with target feature observations in order to increase both tracking performance and persistence. We augment the posterior's PHD and the likelihood, and mitigate the arising degeneracy issue by introducing a feature-aided adaptive sampling mechanism, thereby improving track persistence.

In this paper, related work is reviewed in section 2. Section 3 contains background knowledge and problem formulation. In section 4 we present a strategy for augmenting the SMC-PHD filter with target features and highlight the occurrence of degeneracy in critical conditions. As a consequence, an adaptive sampling scheme is proposed in section 5. Filter performance is evaluated and discussed through numerical simulations in section 6.

2 Related work

2.1 Feature-aided data association

Integrating target features into data association often results in better tracking performance. A target feature can be a signature (target-inherent) or some measurable parameter related to the target or its behaviour [5].

The Signal-to-Noise Ratio (SNR) was used to refine JPDA's association probability in [6, 7] in order to better differentiate targets from clutter. In passive sonar applications, [8] proposed a gating process based on bearing and frequency attributes. Unlike in [6], this modification alleviates track coalescence rather than clutter. JPDA is from far the most common technique for feature-aided data association, as will testify the use of High Resolution Range (HRR) [9] or wavelets feature extraction [10] in radar applications. In these examples, feature and kinematic measurements are independent. Instead of increasing the dimension of the state vector, features are used separately as a way to identify, probabilistically, targets from clutter and other targets. The state estimator runs independently and would require another strategy to include target features.

Leveraging target doppler and down-range extent measurements, the Feature-Aided Gaussian Mixture PHD filter (GM-PHD) in [11] outperformed the canonical one: using features to associate tracks from an iteration to another. GM-PHD's update generates as many additional Gaussian components as there are measurements, leading to pruning and merging routines [12]. GM-PHD's 2D-assignment process (post-data association) has hence a greater impact on the overall state estimation process than the SMC-PHD's. SMC-PHD does not need this assignment step for state estimation, although this might help to produce a better sampling algorithm.

Feature extraction remains nonetheless very challenging; [13] warns on possible sudden variations. Mechanical vibrations or rotations induce frequency modulation for instance, which can fool a frequency-based data associator.

2.2 Target loss in particle PHD filters

Although SMC-PHD filters perform well in cluttered environments [14], estimating the multi-target posterior density with a weighted particle set is prone to sample degeneracy and impoverishment [14–16]. The former occurs when a few particles concentrate most of the weight (distribution with sharp peaks and negligible weights elsewhere), often caused by importance sampling. Degeneracy is typically fought via resampling when the Effective Sample Size (ESS) drops under a threshold [17]. Dropping too many particles with low weights during resampling causes particle impoverishment: the filter cannot recover the multi-target distribution over that region. Theoretical results on the stability of SMC methods are presented in [18, 19].

A particle flow guided by a (stochastic) propagation equation alleviates degeneracy [15, 20]. This method guides the particle distribution towards the posterior thanks to PDEs, this can be expensive to compute and truncation error or approximations such as local linearity and Gaussianity [20] can jeopardise the distribution estimation quality. Gaussian Particle Flow Importance Sampling (GPFIS) [21] is an interesting add-on to particle flow filters, an optimal importance sampling scheme is found to reduce efficiently degeneracy.

Authors of [22] propose an Equivalent-Weights particle filter in which an additional proposal density enables the weights to remain non-negligible with respect to each other. Statistical consistency is lost through this process though.

Another trend is using Markov Chain Monte Carlo methods (MCMC) [23, 24], replacing the resampling scheme by an MCMC-based sampling routine: the challenge lies in identifying effective kernels [25].

3 Problem Formulation

3.1 Multi-target representation and simulation setup

From a set of N_k targets with states $\mathbf{X}_k = \{\tilde{\mathbf{x}}_k^i\}_{i=1}^{N_k}$, a multi-target measurement set $\mathbf{Z}_k = \{\tilde{\mathbf{z}}_k^j\}_{j=1}^{M_k}$ of M_k observations (from one sensor) is produced at time-step k. Targets follow a Markov transition model $p(\mathbf{x}_k|\mathbf{x}_{k-1})$.

A target's kinematic state is denoted $\mathbf{x}_k = [x, v_x, y, v_y, \omega]^T$, producing a range $r = \sqrt{x^2+y^2}$ and bearing $\theta = \arctan 2(y, x)$ resulting in the kinematic observation $\mathbf{z}_k = \left[r, \theta\right]^T + \mathbf{w}_k$, with a probability of detection p_D. The observation noise $\mathbf{w}_k \sim \mathcal{N}(.; 0; diag([\sigma_r^2, \sigma_\theta^2]^T))$ depends on range and bearing standard deviations σ_r and σ_θ.

The observations $\tilde{\mathbf{z}}_k^j = [\mathbf{z}_k^j, \mathbf{z}_{f_k}^j]$ are the concatenation of a kinematic and a feature measurement. Similarly, a target state is decomposed as $\tilde{\mathbf{x}}_k^i = [\mathbf{x}_k^i, \mathbf{x}_{f_k}^i]$.

3.2 Target features

We identify two target feature types of interest: those whose measurements differ significantly when target- or clutter-originated, or those differentiating targets from each other. Therefore, two features are considered here: the SNR and the down-range extent (DRE), fulfilling the former and the latter purposes respectively. These features are consistent with a radar application, yet the selection logic is applicable to other fields.

On one hand, the SNR refers to a ratio between the signal power and the noise power at a certain location, it is kinematics-independent. We adopt the Rayleigh model as in [6, 26], and as a result, the probability $p_{snr}(a_k^j)$ for a measurement with SNR amplitude a_k^j to be target-originated is given by:

$$p_{snr}(a_k^j) = 1 - a_k^j . \exp\left(-\frac{(a_k^j)^2}{2} \right) \quad , \quad a_k^j \geq 0 \tag{1}$$

On the other hand, the DRE can be seen as a target's depth from the sensor's point of view: this feature provides additional information on the target (length) and its motion [5, 11]. DRE can help differentiating targets when these are spatially close from one another.

3.3 Background: SMC-PHD filter and assumptions

The SMC-PHD filter [27] sequentially performs five operations: particle initialization, then prediction, update, resampling and clustering. This filter relies on:

Assumption 1 *the targets are independent from one another, each generating a maximum of one observation per scan;*

Assumption 2 *clutter and target birth distributions are Poisson and target-independent, targets survive according to a Bernoulli process.*

4 Feature-aided SMC-PHD for increased nonlinearity and clutter resilience

In this paper, we propose the FA-SMC-PHD filter, leveraging feature observations in order to increase the tracking performance despite high clutter rates and nonlinear target dynamics. In this section, two modifications of the update module are presented based on features: a refinement of the weights $\{w_{k|k}^i\}_{i=1}^{L_k}$ of the posterior's PHD (associated to L_k particles), and the probabilistic integration of feature observations into the likelihood $g_k(\mathbf{z}_k|\mathbf{x}_k)$.

We anticipate two major drawbacks to these changes. Sharpening the peaks in the multi-target distribution using feature information can cause degeneracy. For this reason, as the probability of detection decreases, sample impoverishment is to be expected. Intermediate results are presented in section 6.2. The adaptive techniques described in section 5 will mitigate these drawbacks.

4.1 Augmenting the PHD of the posterior with feature likelihood

At this stage, we only consider kinematic-independent signature-like target features (e.g. SNR), i.e. those from which we can compute the (feature) likelihood $p_f(\cdot)$ that the measurement $\mathbf{z}^j_{f_k}$ was target-generated (by opposition to clutter).

A first approach consists in augmenting the multi-target posterior density with feature likelihoods. These are designed specifically to reject clutter, particle weights will decrease proportionally to their likelihood to represent a target.

$$w^i_{k|k} \leftarrow w^i_{k|k}\, p_f(\mathbf{x}^i_{f_k}) \tag{2}$$

This method requires to distribute the feature measurements' likelihoods over to all particles though. To this end we compute an interpolation function evaluated at the particles' locations. A multivariate Gaussian mixture model is proposed here, weighed by the feature likelihoods $p_f(\mathbf{z}^j_{f_k})$ associated to their mean location $\mathbf{z}^j_k$ and covariance matrix $\boldsymbol{\Sigma} = diag([\sigma_r^2, \sigma_\theta^2]^T)$, evaluated at $\mathbf{x}^i_k$.

Remark 1. Due to the scaling factors $\{p_f(\mathbf{x}^i_{f_k})\}_{i=1}^{L_k}$, the weights need to be normalized in order for their sum to remain unchanged.

Remark 2. This process cannot thus provide a better cardinality estimation. Ultimately, this feature-informed weight refinement causes the least relevant particles to be removed through resampling, which increases robustness to clutter.

4.2 Feature-aided likelihood

In order to refine the likelihood, we propose to multiply it by the feature likelihood, assuming both are independent from each other:

$$g_k(\tilde{\mathbf{z}}^j_k|\mathbf{x}^i_k) = g_k(\mathbf{z}^j_k|\mathbf{x}^i_k).p_f(\mathbf{z}^i_{f_k}) \tag{3}$$

Thus, the likelihood will drop faster as the measurements are probabilistically closer to be clutter- than target-originated.

However, the effects of this refined likelihood on the multi-target distribution are not as straightforward as in section 4.1, the formulation of the posterior's PHD in the update operator is strongly nonlinear with respect to the likelihood.

Remark 3. Unlike in the posterior augmentation proposed in section 4.1, normalizing the weights is not necessary, nor need the particles to be attributed features. Likelihood augmentation is hence computationally more effective and has the potential of improving cardinality estimation.

5 Adaptive FA-SMC-PHD: an adaptive sampling scheme to mitigate degeneracy

When a target is misdetected for too many time-steps, the number of particles tracking it shrinks until a critical size is reached, causing the death of a particle batch and the loss of a target.

Our contribution to alleviating this degeneracy issue is twofold: exploring the state-space where lost targets are expected to be located and propagating a slowly decreasing feature density to these locations (used only as extra sources for particle feature likelihood interpolation), thus introducing greater inertia in the filter without affecting cardinality estimation. Each technique involves a problem-dependent hyperparameter whose value lies in $]0, 1[$. The parameters were tuned using a *grid search*, although more advanced hyperparameter optimization algorithms could be investigated [28].

5.1 Adaptive importance sampling mechanism

The rationale for this first mitigation proposal is very intuitive as it consists in exploring the state-space even after losing a target, so its track can be recovered once the target is re-observed. This can be seen as importance sampling for both the lost and expected tracks. Unbounded exploration would result in unreasonable computational cost and tracking error though, due to clutter.

In other terms, this adaptive sampling mechanism will use a distribution represented as the predicted locations and error covariance matrices of the tracks at time $k-1$ and the tracks lost over the last m_o time steps. This boils down to wondering for how long a target can be unobserved, m_o should therefore reflect and limit the risk of degeneracy, depending on the detection probability.

Let $M_1, \ldots, M_{n_H}$ be a sequence of iid Bernoulli random variables such that an observation is missed with probability $\mathbb{P}(M_k = 1) = 1 - p_D$ and is produced with $\mathbb{P}(M_k = 0) = p_D$, according to Assumption 1, and let L_{n_H} be the random variable associated to the longest success run over a time horizon n_H. We parameterize m_o by the level of risk τ considered acceptable, so that $\mathbb{P}(L_{n_H} \geq m_o) \leq \tau$. A closed form for this probability was given in [29], although for simplicity, only the probability that a target remains unobserved over the next m_o time steps will be considered. This probability does not depend on any time horizon ($n_H = m_o$) and is simply given by $\mathbb{P}(L = m_o) = (1 - p_D)^{m_o}$, resulting in the suggestion of a value for m_o given the hyperparameter τ ($\lfloor \cdot \rceil$ being the nearest integer function):

$$m_o = \left\lfloor \frac{\log \tau}{\log (1 - p_D)} \right\rceil \tag{4}$$

Although we limit exploration, tracking error is expected to increase as more of the cluttered area is covered. The next proposal in section 5.2 remedies this.

5.2 Introducing artificial feature measurements

In order for the adaptive sampling mechanism to be truly effective with respect to tracking error, the filter needs to keep on tracking a target even when it is not observed.

Introducing artificial kinematic measurements where targets are expected to be would result in poor state estimation, because some artificial observations would be wrong and persistent, but more importantly these would modify the cardinality of the multi-target distribution.

Instead, we propose *artificial feature measurements*, using their kinematic counterpart (location) as a label in order for them to be correctly distributed to the particles. Unlike kinematic measurements feature measurements only modify the multi-target distribution's shape, by increasing the weights of particles of interest and decreasing others. In addition, feature likelihood (alternatively, feature amplitude) of artificial measurements must decrease over time as these propagate in order not to create and track ghosts. This also translates the decreasing reliability of artificial measurements originated by lost tracks over time. We hence impose all features from past time steps to decrease with a constant rate α_k, parameterized by the proportion $\beta_{k-m_o:k}$ of the feature likelihood (or amplitude) remaining after m_o time steps, β being a second tuning parameter.

$$\alpha_k = (\beta_{k-m_o:k})^{\frac{1}{m_o}} \tag{5}$$

6 Numerical Simulations

6.1 Simulation setup

An extensive study has been pursued, with 100 Monte Carlo runs performed for each parameter configuration. Similarly to [30], a scenario has been designed in order to evaluate the filters' performance with respect to the following requirements. An experiment, scenario running with multi-target estimation performance evaluation, is shown in `https://youtu.be/cbgKf4Zpb3I`.

– **Multiple targets:** 5 targets (fixed) with coordinated-turn (CT) dynamics [31], evolving independently within the sensor's field of view (FOV);
– **Targets crossing:** this may induce track coalescence or swapping;
– **Nonlinear motions:** the targets have different speeds and turn rates, and highly nonlinear motions, e.g. target maneuvers, have been obtained by adding a zero-mean Gaussian noise to the trajectories.

The same groundtruth is used for all simulations. The scanning period is $\Delta_T = 1s$ for a total run-time of 100 time-steps. The transition model is characterized by a Gaussian process noise, with $\sigma_{\nu_x} = \sigma_{\nu_y} = 15m/s^2$ and $\sigma_\omega = 2\frac{\pi}{180}rad/s$.

We consider a range-and-bearing sensor, typically a radar, centered on the 2D plane's origin, and its FOV is defined by a range $r_{max} = 2500m$ and an azimuth $\theta_{max} = \pi$. Radars are known to acquire cluttered measurements, herein assumed to be uniformly distributed over the FOV (as is the detection probability p_D), modeled as a Poisson Point Process (PPP) with intensity λ_{FA} false alarms per scan, and a surviving probability $p_S = 1$. Measurement noise is characterized by $\sigma_r = 10m$ and $\sigma_\theta = \frac{\pi}{180}rad$, associated to the measurement model in section 3.1. As for the SMC-PHD filter, the birth model of reference is a commonly-used Poisson point process at the initial targets' locations.

At each time-step, $\rho = 300$ new particles are generated per expected target. Systematic resampling is used, with the roughening strategy proposed in [32] in order to reduce slightly the risk of sample impoverishment.

For performance evaluation, we use the Optimal Sub-Pattern Assignment (*OSPA*) [33], combining state and cardinality error estimations with respect to the ground truth, with an Euclidean cut-off distance of 100 and a sensitivity of 1. For cardinality estimation and analysis (exclusively) we use the Single Integrated Air Picture (SIAP-C) [34].

6.2 Intermediate performance of the FA-SMC-PHD

To begin with, we studied the impact of posterior and likelihood feature-aided enhancements. The simulation results shown in Figure 1 highlight the difficulty of the simple FA-SMC-PHD to perform well with highly cluttered measurements.

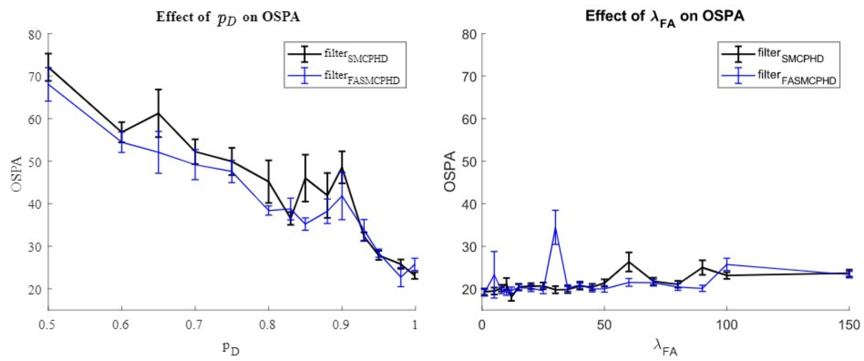

Fig. 1. Tracking performance (OSPA) with respect to the detection probability (with $\lambda_{FA} = 100$) and the clutter rate (with $p_D = 99\%$) respectively.

The gain in tracking performance, with respect to the canonical SMC-PHD filter, as the clutter rate increases, is insignificant (-2% of relative difference). This intermediate result is somewhat counter-intuitive as the FA-SMC-PHD filter is indirectly fed with knowledge about the objects it is sensing, whereas the canonical SMC-PHD has none. In fact, this is a side effect of degeneracy, as anticipated and explained in section 4. This may also slow down the Monte Carlo convergence, which would explain the peaks in Figures 1 and 3 in low-cluttered areas (which are not the primary focus of this study).

The main issue jeopardizing this tracker's results is the consequence of missed observations. When introducing feature information, the process of indirectly lowering the PHD in cluttered regions in favor of target-influenced areas is significantly accelerated, as this differentiation is blind in the canonical filter, but feature-informed in the FA-SMC-PHD. Though not all particles are destroyed immediatly, there is an obvious lack of intertia within SMC-PHD filters, and this is particularly true for the FA-SMC-PHD.

Although the FA-SMC-PHD already results in a 7.3% performance increase with respect to detection probability (Figure 1), systematic target loss can easily be identified on the left figure, typically around $p_D \approx 95\%$.

6.3 Alleviating degeneracy using the adaptive sampling mechanism

As suggested in Figure 2, the ESS of the FA-SMC-PHD is 15% to 35% below the SMC-PHD's, taken as reference, in relative difference. This translates by a higher potential for degeneracy. Thus the adaptive sampling mechanism improves the filter's ESS, which, combined with the enhancements proposed in section 4, results in comparable ESS levels.

The challenge of alleviating degeneracy in these difficult tracking conditions is not yet completely overcome. However this issue has been sufficiently mitigated for the final filter to yield far better performance than the SMC-PHD.

On a side note, the adaptive FA-SMC-PHD is better at estimating the system's cardinality, for low detection probabilities it even tends to overestimate it whereas the reference filter underestimates it.

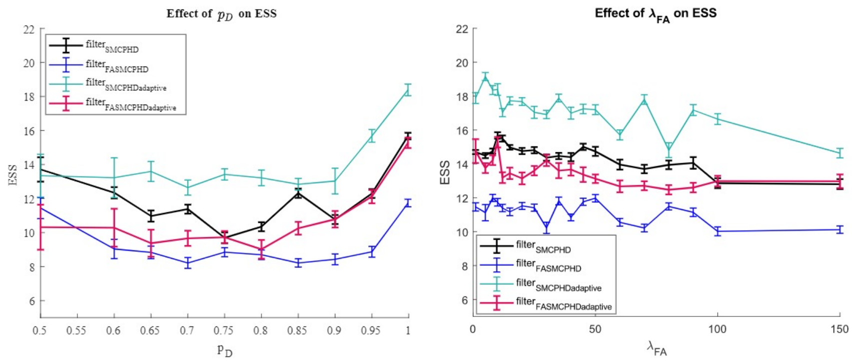

Fig. 2. Effective sample size with respect to the detection probability (with $\lambda_{FA} = 10$) and the clutter rate (with $p_D = 99\%$).

6.4 Adaptive FA-SMC-PHD performance assessment

The final adaptive FA-SMC-PHD filter outperforms the canonical SMC-PHD filter for high clutter rates ($\lambda_{FA} \geq 50$), with a 14.4% increase in tracking accuracy. When the level of clutter is low, the SMC-PHD performs better; this is caused by the adaptive sampling mechanism exploring more of the state-space than necessary. This study focuses on cases where the clutter rate is high though.

Furthermore, although this cannot be shown here due to paper length limit, the adaptive FA-SMC-PHD filter's ability to handle missed detection gained an

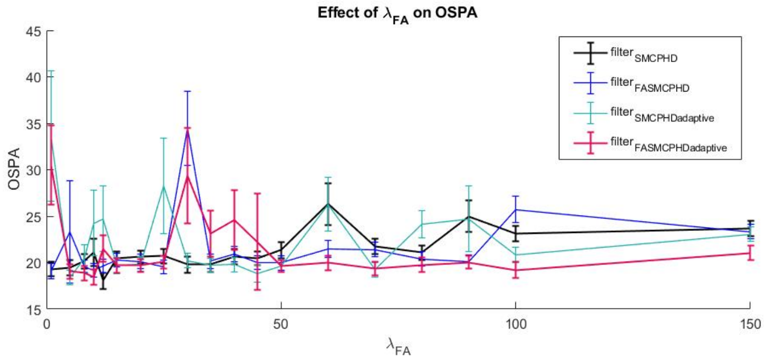

Fig. 3. Clutter resilience assessment: tracking performance with respect to the clutter rate (with $p_D = 99\%$).

average 7.3% in state estimation performance. Moreover, the reliability also increased, as the standard deviation of the OSPA with respect to clutter decreased by 33.8%. Taking the absolute cardinality estimation error, the proposed algorithm was 50.4% better at estimating the number of targets within the FOV.

7 Conclusions

In this paper, an adaptive FA-SMC-PHD filter was proposed, providing a significant increase in clutter resilience, despite the canonical SMC-PHD's natural effectiveness in this regard. In extreme conditions, cardinality estimation performance was also greatly improved, together with reliability.

Moreover, by decomposing the particle-PHD filter in atomic components, identifying opportunities for sensor feature information to be integrated, and analyzing the effect of the multiple enhancement combinations on state estimation performance, cardinality estimate, track coalescence and swapping, and degeneracy, we shed light on SMC-PHD's main flaws. In particular, the degeneracy issue is problematic in this filter; not because of the classical sample impoverishment in particle filters, but due to a lack of data when resampling, and eventually updating. We proposed a way to limit the risk of degeneracy by introducing inertia through feature propagation and adaptive sampling. Generic methods have been given, then applied, allowing future studies to design upon such a tracker.

Future work will focus on better handling this filter's degeneracy and further develop the approaches mentioned in this paper, the adaptive sampling scheme in particular. Therefore more Monte Carlo experiments will also be carried out. These enhancement proposals, which are not domain-specific by design, will be assessed upon other benchmarks (e.g. [31]), and confronted to filters fulfilling similar purposes such as a feature-aided GM-PHD or particle flow SMC-PHD filter, in order to validate the generalizability of these methods.

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