# Online algorithms for finding distinct substrings with length and multiple prefix and suffix conditions 

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#### Abstract

Let two static sequences of strings $P$ and $S$, representing prefix and suffix conditions respectively, be given as input for preprocessing. For the query, let two positive integers $k_{1}$ and $k_{2}$ be given, as well as a string $T$ given in an online manner, such that $T_{i}$ represents the length- $i$ prefix of $T$ for $1 \leq i \leq|T|$. In this paper we are interested in computing the set $a n s_{i}$ of distinct substrings $w$ of $T_{i}$ such that $k_{1} \leq|w| \leq k_{2}$, and $w$ contains some $p \in P$ as a prefix and some $s \in S$ as a suffix. More specifically, the counting problem is to output $\left|a n s_{i}\right|$, whereas the reporting problem is to output all elements of ans $_{i}$, for each iteration $i$. Let $\sigma$ denote the alphabet size, and for a sequence of strings $A,\|A\|=\sum_{u \in A}|u|$. Then, we show that after $O((\|P\|+\|S\|) \log \sigma)$-time preprocessing, the solutions for the counting and reporting problems for each iteration up to $i$ can be output in $O\left(\left|T_{i}\right| \log \sigma\right)$ and $O\left(\left|T_{i}\right| \log \sigma+\left|a n s_{i}\right|\right)$ total time. The preprocessing time can be reduced to $O(\|P\|+\|S\|)$ for integer alphabets of size polynomial with regard to $\|P\|+\|S\|$. Our algorithms have possible applications to network traffic classification.


Keywords: pattern matching • counting algorithm • suffix array • suffix tree.

## 1 Introduction

Pattern matching has long been a central topic in the field of string algorithms [4], leading to various applications, including DNA analysis in bioinformatics $[7,13]$ as well as packet classification $[3,6,16,22$ and anti-spam email filtering in network security (17].

In this paper, we propose algorithms for counting and reporting distinct substrings of an online text $T$ that have some $p \in P$ as a prefix and some $s \in S$ as a suffix, and whose length is within the interval $\left[k_{1} . . k_{2}\right]$, where $P$ and $S$ are static sequences of strings given as input for preprocessing, and integers $k_{1}, k_{2}$ and the characters of $T$ are given as query. A similar yet different problem where patterns are given in the form of a pair of a prefix and a suffix condition, i.e. $p \Sigma^{*} s$ patterns, rather than a pair of sequences where one is of prefixes and the other of suffixes, is well-studied as the followed-by problem [2, 14 or the Dictionary Recognition with One Gap $(D R O G)[1,12,21$ problem. The problem of this paper is also of importance with possible applications in network traffic classification: All the application signatures in 20], for example, can be expressed as an instance of our problem, as discussed in Section 4 and demonstrated in Appendix A. Note that while traffic classification via these signatures was shown to be highly accurate, there are still cases of false positives. When analyzing which patterns give false positives, we may be interested in which patterns match the signatures, in which case the distinct condition of our problem helps prevent wasting computation time on repeated occurrences of each pattern. Another possible application is in computing the distinct substrings of an online text $T$ whose lengths are at least $k$ and belong to a ( $k, r$ )-TTSS language [19]; the words of length $\geq k$ in a $(k, r)$ $T T S S$ language defined by the 4-tuple $\left(\overline{I_{k}}, F_{k}, T_{k, r}, g\right)$ are the words that have some element of $I_{k}$ as a prefix, some element of $F_{k}$ as a suffix, and includes, for each $t \in T_{k, r}$, at most $g(t)$ occurrences of $t$ as substring. Here, the elements of $I_{k}$ and $F_{k}$ are strings of length $k-1$ and the elements of $T_{k, r}$ are strings of length between 1 to $k$ inclusive, and $g$ is a function that projects $T_{k, r} \rightarrow\{0,1, \cdots, r-1\}$. A direct application of our algorithms can check whether $w$ fulfills the prefix and suffix condition, while the condition of restricted segments, i.e. the number of occurrences of $T_{k, r}$ can also be considered by implementing the following modification: for each iteration $i$, maintain the minimum start-index res of the suffix of $T_{i}$ that meets the condition of restricted segments, and use it to exclude any suffixes longer than $T_{i}[$ res..i] from the solution.

Our proposed algorithms take $O((\|P\|+\|S\|) \log \sigma)$ preprocessing time, while processing $T$ itself in an online manner and outputting the solutions up to iteration $i$ takes $O\left(\left|T_{i}\right| \log \sigma\right)$ and $O\left(\left|T_{i}\right| \log \sigma+\left|a n s_{i}\right|\right)$ cumulative time for the counting and reporting problems respectively, using $O\left(\left|T_{i}\right|+\|P\|+\|S\|\right)$ working space. Here, $T_{i}$ denotes the length- $i$ prefix of $T,\|P\|$ and $\|S\|$ denote the total length of strings in $P$ and $S$ respectively, $\sigma$ is the alphabet size, $\left|a n s_{i}\right|$ is the number of substrings reported for each $T_{i}$, and cumulative time refers to the total amount of running time up to iteration $i$, as opposed to the running time of only iteration $i$. In addition, the preprocessing time can be reduced to $O(\|P\|+\|S\|)$ in the case of integer alphabets of size polynomial in $\|P\|+\|S\|$.

Also note that, the problems addressed in this paper differ from those of [11], in which a different set of solution strings is output for each $p \in P$ where each solution must have that specific element of $P$ as a prefix, unlike the problem in this paper where only a single set of solution strings is output, where its elements can have any $p \in P$ as a prefix. Also, in [11] there is only one suffix
condition and no length condition, and the algorithm is offline w.r.t. $T$, unlike in this paper.

## 2 Preliminaries and definitions

### 2.1 Strings

Let $\Sigma$ be an alphabet of size $\sigma$. An element of the set $\Sigma^{*}$ is a string. The length of a string $w$ is denoted by $|w|$. The empty string is denoted by $\varepsilon$. That is, $|\varepsilon|=0$. For a string $w=p t s, p, t$, and $s$ are called a prefix, substring, and suffix of $w$, respectively. A prefix $p$ (resp. suffix $s$ ) of a string $w$ is called a proper prefix (resp. proper suffix) of $w$ if $|p|<|w|$ (resp. $|s|<|w|)$. For a string $w, w[i]$ denotes the $i$-th symbol of $w$ for $1 \leq i \leq|w|$, and $w[i . . j]$ denotes the substring $w[i] w[i+1] \cdots w[j]$ for $1 \leq i \leq j \leq|w|$. For a sequence $S$ of strings, let $\|S\|=\sum_{u \in S}|u|$.

### 2.2 Suffix array and LCP array

The suffix array [15] of a string $w$ is a lexicographically sorted array of suffixes of $w$, where each suffix is represented by its start-index. The LCP array is an auxiliary array commonly used alongside the suffix array, that stores the length of the longest common prefix of each adjacent pair of suffixes in the suffix array. More specifically, if $S A$ and $L C P$ are the suffix array and LCP array of the same string $w$, for $x \in[2 . .|w|], L C P[x]$ is the length of longest common prefix of the suffixes $w[S A[x] . .|w|]$ and $w[S A[x-1] . .|w|]$. In this paper, we will use suffix arrays for some strings, in which we denote by $L C P$ the LCP array of the same string of the suffix array being discussed. It is well-known that the suffix array and LCP array of a string $w$ can be built in $O(|w|)$ time for integer alphabets of polynomial size in $|w|[8,9,10]$, and in $O(|w| \log \sigma)$ time for general ordered alphabets 23.

### 2.3 The problems

The problems considered in the paper are as follows.
Definition 1 (Online substring counting and reporting problem with distinctness, multiple prefixes, multiple suffixes and length range conditions). Given two sequences of strings $P=\left(p_{1}, \cdots, p_{n}\right)$ and $S=\left(s_{1}, \cdots, s_{m}\right)$, two integers $k_{1}$ and $k_{2}$, and a string $T$ given in an online manner (i.e., $T_{0}=\varepsilon$ and for each iteration $i=1, \ldots,|T|$, the $i$-th character is appended to $T_{i-1}$ to form $T_{i}$ ), let ans ${ }_{i}$ denote the set of distinct substrings of $T_{i}$ that have some $p \in P$ as a prefix, some $s \in S$ as a suffix, and whose length falls within the interval [ $k_{1} . . k_{2}$ ].

The counting problem. On each iteration $i$, output $\left|a_{i}\right|$.
The reporting problem. On each iteration $i$, output ans ${ }_{i} \backslash a n s_{i-1}$.
This paper excludes the empty string $\varepsilon$ from the solutions.

## 3 Algorithm

### 3.1 Sketch of algorithm

In this section, we describe the general idea of our algorithm. During each iteration $i$, we need either to compute the size of $a n s_{i} \backslash a n s_{i-1}$ to add it to the counting solution, or to report all its elements. All elements of $a n s_{i} \backslash a n s_{i-1}$ must be suffixes of $T_{i}$, and thus for the suffix condition, $T_{i}$ itself must have some element of $S$ as a suffix; otherwise clearly $a n s_{i} \backslash a n s_{i-1}=\emptyset$ and there is no need to output a solution for the current iteration. Thus, let us consider the case where $T_{i}$ has at least one element of $S$ as a suffix, and call the shortest of them $s$. To help keep track of which suffixes of $T_{i}$ fulfill the prefix condition, let us maintain a linked list pList that contains in increasing order, all distinct indices $j$ such that there is an element of $P$ that occurs in $T_{i}$ with start-index $j$. Clearly, the elements of pList represent a bijection to the suffixes of $T_{i}$ that have an element of $P$ as a prefix, which are candidates for elements of the solution set $a n s_{i} \backslash a n s_{i-1}$. Specifically, for all $j \in p L i s t$, the string $u=T_{i}[j . . i] \in a n s_{i} \backslash a n s_{i-1}$ iff $u$ fulfills all the following conditions:
(a) $u$ has $s$ as a suffix.
(b) $u$ does not occur in $T_{i-1}$.
(c) $|u| \geq k_{1}$.
(d) $|u| \leq k_{2}$.

Here, (a) $u$ has $s$ as a suffix iff $|u| \geq|s|$, and (b) $u$ does not occur in $T_{i-1}$ iff $|u|>\left|l r s_{i}\right|$, where $l r s_{i}$ denotes the longest repeating suffix of $T_{i}$, i.e. the longest suffix of $T_{i}$ that occurs at least twice in $T_{i}$. Thus, conditions (a) to (c) set a lower bound for the length of suffixes of $T_{i}$ corresponding to elements of $p L i s t$ that can be a solution while condition (d) sets an upper bound. If we visualize pList horizontally as shown in Figure 1, conditions (a) to (c) exclude some elements from the right while condition (d) excludes some element from the left.


Fig. 1. A visualized example of $p$ List.

Take the maximum among the number of elements excluded by conditions (a) to (c) and denote it by excludeRight, and denote the number of elements excluded by condition (d) by excludeLeft. Then, $\left|a n s_{i} \backslash a n s_{i-1}\right|=\max (0,|p L i s t|-$ excludeLeft - excludeRight), giving us the solution for the counting problem. For
the reporting problem, let us maintain start, a pointer to the smallest element in pList not excluded by condition (d). Then report all elements traversed by starting at start and moving to the right $\left|a n s_{i} \backslash a n s_{i-1}\right|-1$ times.

Example 1. The example in Figure 1 occurs when $T_{i}=\operatorname{coldcocoaold,~} P=$ (cave, coco, cocoa, d, oao, old), $S=\left(\right.$ aold, oaold), $k_{1}=3$, and $k_{2}=8$. Then, $p$ List $=(2,4,5,8,10,12) .10,12$ correspond to old, d which are shorter than $s=$ aold, and thus excluded by condition (a). $l r s_{i}=o l d$ and thus condition (b) also excludes 10 and 12 , while condition (c) excludes only 12 which corresponds to d. Therefore, excludeRight $=\max (2,2,1)=2$. Meanwhile, condition (d) excludes 2 and 4 which correspond to oldcocoaold and dcocoaold and so excludeLeft $=$ 2.Thus, we have that $\left|a n s_{i} \backslash a n s_{i-1}\right|=\max (0,6-2-2)=2$. For the reporting solution, we have that start points to 5 . Traversing 2 elements starting from 8 gives us 5 and 8, each corresponding to cocoaold and oaold, exactly the elements of $a n s_{i} \backslash a n s_{i-1}$.

### 3.2 Removing redundant elements

We say that $p_{k} \in P$ is redundant iff there exists $p_{k^{\prime}}$ of $P$ s.t. either $p_{k}$ has $p_{k^{\prime}}$ as a proper prefix, or $p_{k}=p_{k^{\prime}} \wedge k>k^{\prime}$. Similarly, $s_{k} \in S$ is redundant iff there exists $s_{k^{\prime}}$ of $S$ s.t. either $s_{k}$ has $s_{k^{\prime}}$ as a proper suffix, or $s_{k}=s_{k^{\prime}} \wedge k>k^{\prime}$.

It is not hard to see why they are called redundant; when multiple copies of the same string exist in $P$, keeping only one copy suffices, and when $p_{k} \in P$ has $p_{k^{\prime}} \in P$ as a proper prefix, the strings that have $p_{k}$ as a prefix is a subset of strings that have $p_{k^{\prime}}$ as a prefix, and thus the solution remains the same even if we delete $p_{k}$ from $P$. The same can be said for redundant elements of $S$.

As one part of the preprocessing, we rebuild $P$ and $S$ so that the redundant elements are deleted. First, we describe how to rebuild $P$. Let $P_{\text {seq }}=$ $\$ p_{1} \$ p_{2} \$ \cdots \$ p_{n} \$$, where $\$ \notin \Sigma$ and $\$ \prec c$ for all $c \in \Sigma$, and let $S A$ be the suffix array of $P_{\text {seq }}$.

Then, each $S A[x]$ for $x \in[2 . . n+1]$ corresponds to the start-index of $\$ p$ in $P_{\text {seq }}$, for some $p \in P$. Starting from $x=2$, output the corresponding $p$, namely the unique $p \in P$ s.t. $\$ p \$$ occurs on index $S A[x]$. Then, increment $x$ (at least once) until we have that $L C P[x]<|\$ p|$, i.e. until we find an index that corresponds to $\$ p^{\prime}$ where $p^{\prime} \in P$ does not have $p$ as a prefix. Output the element of $P$ corresponding to the new $x$, then again increment $x$ in the same manner. Repeat this until $x>n+1$, and we have that all non-redundant elements of $P$ are output.

Other than the construction of the suffix array and LCP array, clearly this takes $O(\|P\|)$ time, and the same method can be used to compute non-redundant elements of $S$ : Let $S^{-1}=\left(s_{1}^{-1}, \ldots, s_{m}^{-1}\right)$ be the sequence of reversed elements of $S$, then apply the above algorithm to $S^{-1}$ and reverse each string in the output to get the non-redundant elements of $S$. Thus, both $P$ and $S$ are rebuilt to exclude redundant elements in $O(\|P\|+\|S\|)$ time, in addition to the construction time of the suffix array and LCP array, which depends on the alphabet.

Example 2. Let $P=(\mathrm{abc}, \mathrm{ab}, \mathrm{acc}, \mathrm{ab}, \mathrm{cab})$. Then, $P_{\text {seq }}=\$ \mathrm{abc} \$ \mathrm{ab} \$ \mathrm{acc} \$ \mathrm{ab} \$ \mathrm{cab} \$$ and we have the table as shown in Figure $2, x=2$ corresponds to the occurrence of $\$ \mathrm{ab}$, which occurs on $P_{\text {seq }}$ on index $S \overrightarrow{A[2]}=5$. Thus, ab is determined to be non-redundant, and we have that $|\$ \mathrm{ab}|=3$, so increment $x$ until we have that $L C P[x]<3$. This skips over $x=3,4$, correctly determining their corresponding elements of $P$, namely the second ab and abc to be redundant. When $x=5$, $L C P[x]=2<3$ and so the corresponding $p=\operatorname{acc}$ is output. Similarly, for $x=6$, $L C P[x]=1<4$ and thus cab is output. Afterwards, $x$ is incremented beyond the interval $[2 . . n+1]$ and thus the algorithm terminates and the non-redundant elements (ab, acc, cab) are output.


Fig. 2. The table for $P_{s e q}=\$ a b c \$ a b \$ a c c \$ a b \$ c a b$

For the rest of the paper, we will assume that the above preprocessing is done and thus $P=\left(p_{1}, \ldots, p_{n}\right)$ and $S=\left(s_{1}, \ldots, s_{m}\right)$ from this point refer to the rebuilt sequences that have no redundant elements.

### 3.3 Detecting $P$ and $S$ occurrences

As discussed, on each iteration $i$ we need to detect whenever an element of $S$ occurs as a suffix of $T_{i}$. Additionally, we will also need to detect when an element of $P$ occurs as a suffix of $T_{i}$, in order to maintain $p L i s t$. This can be done by building an Aho-Corasick automaton for $P$ and $S$ separately. Constructing both automata takes $O((\|P\|+\|S\|) \log \sigma)$ preprocessing time in general, and $O(\|P\|+\|S\|)$ time in the case of integer alphabets of size polynomial with regard to $\|P\|+\|S\|\left[5\right.$. Running each automaton up to iteration $i$ takes $O\left(\left|T_{i}\right| \log \sigma+\right.$ $o c c$ ) cumulative time, where occ is the number of occurrences detected. Here, the occurrences of elements of $P$ in $T_{i}$ must have distinct start-indices, as two
occurrences with a shared start-index imply that one of them is redundant. Similarly, occurrences of elements of $S$ must have distinct end-indices and thus occ $\in O\left(\left|T_{i}\right|\right)$ for both automata, and so the cumulative running time becomes $O\left(\left|T_{i}\right| \log \sigma\right)$.

### 3.4 Maintaining pList

To maintain $p L i s t$, whenever some $p \in P$ is detected to occur as a suffix of $T_{i}$, its start-index $j$ needs to be added to $p L i s t$ while maintaining the increasing order. Doing this naively would take $O(|p L i s t|)=O\left(\left|T_{i}\right|\right)$ time for every insertion which gives quadratic time overall, so a more efficient scheme is necessary.

During any iteration $i$, the elements of $P$ that occur as suffixes of $T_{i}$ are detected in decreasing order of length, because we use Aho-Corasick automaton. Let $p$ be such an element detected, and $j$ be the start-index of its occurrence, i.e. $j=i-|p|+1$. We need to add $j$ into $p$ List so that the increasing order of its elements are maintained.

To do that, we need to find the minimum $j^{\prime}$ among the current elements of $p$ List such that $j^{\prime}>j$. Here, $j^{\prime}$ being an element of $p$ List implies that $j^{\prime}$ corresponds to an occurrence of $p^{\prime} \in P$ starting at $j^{\prime}$ and ending at some $i^{\prime} \leq i$. We can see that in fact $i^{\prime}<i$, for if $i^{\prime}=i, j^{\prime}$ was added to $p$ List in the current iteration $i$ before $j$, while $j^{\prime}>j \wedge i^{\prime}=i$ implies $\left|p^{\prime}\right|<|p|$, contradicting the fact that the Aho-Corasick automaton detects the occurrences in decreasing order of length. Thus, $j^{\prime}>j$ and $i^{\prime}<i$, meaning the occurrence of $p^{\prime}$ falls completely within $p[2 . .|p|-1]$.

Our scheme is then as follows: We precompute, for each $p_{k} \in P$, the minimum value $y$ such that there is some $p_{k^{\prime}} \in P$ that occurs in $p_{k}\left[2 . .\left|p_{k}\right|-1\right]$ on startindex $y+1$. If there is no such $p_{k^{\prime}}$, then let $y=\infty$. Then, we will store the values on the array successorOffset that maps each $p_{k} \in P$ to its corresponding $y$.

Additionally, maintain also an array $p$ Array such that $p$ Array $[j]$ points to the element of $p L i s t$ whose value is $j$ if it exists, or null otherwise. Then, whenever some $p_{k} \in P$ occurs as a suffix of $T_{i}$ with start-index $j$, we can just add $j$ into $p L i s t$ exactly before the element pointed to by pArray $[j+$ successorOffset $[k]]$. Clearly, once successorOffset is computed, adding each element of pList takes only constant time and thus maintaining pList and pArray takes cumulative $O\left(\left|T_{i}\right|\right)$ time, as the number of elements of pList for any given iteration $i$ is bounded by $\left|T_{i}\right|$.

Computing successorOffset. Let $P_{\text {concat }}=p_{1} \$ p_{2} \$ \cdots \$ p_{n} \$$. Note that it differs from $P_{\text {seq }}$ not only with regard to the positioning of $\$$, but also in that redundant elements of $P$ are not included. For $k \in[1 . . n]$, let $P L[k]$ denote the start-index of $p_{k}$ in $P_{\text {concat }}$, i.e. $P L[k]=1+\sum_{k^{\prime} \in[1 . . k)}\left(\left|p_{k^{\prime}}\right|+1\right)$. For $j \in\left[1 . .\left|P_{\text {concat }}\right|\right]$, let $P I[j]$ be the index of the element of $P$ covering index $j$ in $P_{\text {concat }}$. That is, $P I[j]=k$ where $j \in\left[P L[k] . . P L[k]+\left|p_{k}\right|-1\right]$ if such $k$ exists, otherwise $P I[j]=$ null. See Figure 3 for an example.

Next, we construct the suffix array $S A P$ of $P_{\text {concat }}$. We can then compute successorOffset. The general idea is that for each $p_{k}$, we find all its occurrences


Fig. 3. Example of $P L$ and $P I$.
using the suffix array, and when the occurrence falls within some $p_{k^{\prime}} \in P$, we update successorOffset $\left[k^{\prime}\right]$. A more detailed description is as follows:

- Initialize successorOffset $[k]=\infty$ for all $p_{k} \in P$.
- Using the LCP array, find the subinterval $[\ell . . r]$ in $S A P$ that corresponds to occurrences of $p_{k}$, i.e. each of $S A P[x]$ for all $x \in[\ell . . r]$ corresponds to some start-index of occurrences of $p_{k}$ in $P_{\text {concat }}$.
- For each such occurrence, whenever it falls within $p_{k^{\prime}}\left[2 . .\left|p_{k^{\prime}}\right|-1\right]$ for some $p_{k^{\prime}} \in P$, then assign to successorOffset $\left[k^{\prime}\right]$ the minimum value between itself and the offset distance $S A P[x]-P L\left[k^{\prime}\right]$. Formally, for all $x \in[\ell . . r]$, if $P_{\text {concat }}[S A P[x]+|p|] \neq \$$, assign to successorOffset $[P I[S A P[x]]]$ the following value:

$$
\begin{equation*}
\min (\text { successorOffset }[P I[S A P[x]]], S A P[x]-P L[P I[S A P[x]]]) \tag{1}
\end{equation*}
$$

Computing $P_{\text {concat }}$ and $P I$ trivially takes $O(\|P\|)$ time. To compute the subinterval corresponding to occurrences of $p_{k}$, simply find the longest subinterval $[\ell . r]$ of $S A P$ that includes the index $S A P^{-1}[P L[k]]$ and $L C P[x] \geq\left|p_{k}\right|$ for all $x \in[\ell+1 . . r]$. This takes linear time w.r.t. to the subinterval length, which all adds up to the total number of occurrences of elements of $P$ in $P_{\text {concat }}$. As no occurrence may share a start-index, it is bounded by $\left|P_{\text {concat }}\right| \in O(\|P\|)$. Thus, this preprocessing takes $O(\|P\|)$ time, in addition to the time required to compute $S A P$ and $L C P$ which depends on the alphabet.

### 3.5 Computing excludeLeft, excludeRight, and start

As discussed, the algorithm runs the Aho-Corasick automaton for $S$ to check whether there is some $s \in S$ that occurs as a suffix of $T_{i}$, for each iteration $i$. In case such $s$ exists, all of the computations below are performed, otherwise only maintaining start is necessary.
excludeRight

Exclusion by $s$. When $s \in S$ occurs as a suffix of $T_{i}$, the suffix $u=T_{i}[j . . i]$ for each $j \in p$ List is excluded from the solution iff $j>i-|s|+1$. That is, $u$ fails to meet condition (a) of Section 3.1 iff $j$ corresponds to an occurrence of $p \in P$ in $s[2 . .|s|]$. Thus, if we preprocess the number of occurrences of elements of $P$ that occur within $s[2 . .|s|]$ for each $s \in S$, we can compute the number of elements of $p$ List excluded by $s$ in constant time.

Example 3. In the case shown by Example 1 the elements 10 and 12 are excluded by $s$, which we can obtain from the fact that there are two occurrences of elements of $P$ within $s[2 . .|s|]=$ old; one each of old and d.

A suffix array-based approach that is similar to what we used to compute successorOffset can be used here. Let

$$
P S_{\text {concat }}=p_{1} \$ \cdots p_{n} \$ s_{1}\left[2 . .\left|s_{1}\right|\right] \$ \cdots s_{m}\left[2 . .\left|s_{m}\right|\right]
$$

and construct its suffix array $S A P S$. Define an array $S I$ such that $S I[j]=k$ when $j$ belongs to the part made up by $s_{k}\left[2 . .\left|s_{k}\right|\right]$ in $P S_{\text {concat }}$, similar to $P I$. Then, for all $p \in P$, compute the subinterval [ $\ell . . r$ ] in SAPS corresponding to suffixes of $P S_{\text {concat }}$ that start with $p$, again using $S A P S^{-1}$ and LCP arrays. For all $x \in[\ell . . r]$, then increment $s P C o u n t[S I[S A P S[x]]]$ by one, where $s P C o u n t$ is an array that maps each $s \in S$ to the number of occurrences of elements of $P$ that occur within $s[2 . .|s|]$. Naturally, sPCount initially maps all elements of $S$ to zero before the counts are incremented.

The total of size of subintervals is bounded by the number of occurrences of elements of $P$ in $P S_{\text {concat }}$, which is $O\left(\left|P S_{\text {concat }}\right|\right)=O(\|P\|+\|S\|)$. Thus, computing successorOffset takes $O(\|P\|+\|S\|)$ time, in addition to the construction time of SAPS and LCP which depends on the alphabet. After the preprocessing, the number of elements of $p L i s t$ excluded by $s$ when $s$ occurs as a suffix of $T_{i}$ can be computed in constant time by simply referring to sPCount $[s]$.

Exclusion by lrs $_{i}$. It is known that Ukkonen's algorithm 23 maintains the locus of $l r s_{i}$ during each iteration $i$ where it builds the suffix tree of $T_{i}$, and thus we can compute its start-index $i-\left|l r s_{i}\right|+1$ by simply running Ukkonen's algorithm. Then, clearly $j \in p$ List is excluded by condition (b) of Section 3.1] iff $j \geq$ $i-\left|l r s_{i}\right|+1$. Furthermore, the start-index is non-decreasing between iterations, i.e. $i-\left|l r s_{i}\right|+1 \geq(i-1)-\left|l r s_{i-1}\right|+1$ for any iteration $i$. Thus, we can always maintain the count $\left|\left\{j \in p L i s t \mid j \in\left[i-\left|l r s_{i}\right|+1 . . i\right]\right\}\right|$ for each $i$ as follows:

- During some iterations, $i-\left|l r s_{i}\right|+1>(i-1)-\left|l r s_{i-1}\right|+1$ which we will know from Ukkonen's algorithm. In that case, for each $j \in\left[(i-1)-\left|l r s_{i-1}\right|+\right.$ $\left.2 . . i-\left|l r s_{i}\right|+1\right]$ such that $j \in p L i s t$, decrement the count by one.
- Whenever a new element $j$ is added to $p$ List such that $j \in i-\left|\operatorname{lrs}_{i}\right|+1$, increment the count.

We can check whether $j \in p$ List in constant time for any $j$ using $p$ Array, and both the left end $i-\left|l r s_{i}\right|+1$ and right end $i$ of the interval only ever increases and is within 1 to $i$, so the above method takes cumulative $O\left(\left|T_{i}\right|\right)$ time, dominated by the runtime of Ukkonen's algorithm which is cumulative $O\left(\left|T_{i}\right| \log \sigma\right)$ time.

Exclusion by $k_{1}$. The same approach can be used to find the number of elements of $p$ List excluded by $k_{1}$ : we maintain the number of $j \in p$ List such that $i-j+1<$ $k_{1} \Leftrightarrow j \in\left[i-k_{1}+2 . . i\right]$ for each iteration $i$. Since both endpoints of this interval can only increase and are always between 1 to $i$ inclusive, maintaining the count can be done in cumulative $O\left(\left|T_{i}\right|\right)$ time.

## excludeLeft and start

Maintaining excludeLeft. Similarly, excludeLeft is the number of $j \in p$ List such that $i-j+1>k_{2} \Leftrightarrow j \in\left[1 . . i-k_{2}\right]$, and the same approach can be used to maintain this count in $O\left(\left|T_{i}\right|\right)$ time.

Maintaining start. We can easily maintain start so that it points to the minimum element of $p$ List of value at least $i+1-k_{2}$ in cumulative $O\left(\left|T_{i}\right|\right)$ time as follows:

- Initialize start to null.
- During each iteration $i$, if start is not null and start $<i+1-k_{2}$, then let start point to the next element in pList.
- Whenever a new element $j$ is added to $p$ List such that $j \geq i+1-k_{2}$, if start $=$ null or start $>j$, let start point to $j$.


### 3.6 Summarizing the algorithm

In the preprocessing, suffix arrays and LCP arrays are constructed and used to remove redundant elements of $P$ and $S$, as well as compute successorOffset. Additionally, Aho-Corasick automata for $P$ and $S$ are built. For general ordered alphabets, this preprocessing takes $O((\|P\|+\|S\|) \log \sigma)$ time, with the construction time for Aho-Corasick automata and suffix arrays being the bottleneck. In the case of integer alphabets of size polynomial w.r.t. $\|P\|+\|S\|$, the construction times, and consequently the whole preprocessing time, can be reduced to $O(\|P\|+\|S\|)$ time.

For the query processing time up to any iteration $i$, excludeLeft, excludeRight, and start are computed in $O\left(\left|T_{i}\right| \log \sigma\right)$ cumulative time, giving us the solution for the counting problem. For the reporting problem, we traverse a total of $\left|a^{2} s_{i}\right|$ elements in $p$ List, giving us $O\left(\left|T_{i}\right| \log \sigma+\left|a n s_{i}\right|\right)$ cumulative time, assuming the solution strings are output in the form of index pairs.

Additionally, all the data structures used require only $O\left(\left|T_{i}\right|+\|P\|+\|S\|\right)$ total working space. Thus, we have the following results.

Theorem 1. There are algorithms that solve the counting and solving problems from Definition 1 for general ordered alphabets, such that after $O((\|P\|+$ $\|S\|) \log \sigma)$ preprocessing time, the solutions are output for each iteration up to $i$ in $O\left(\left|T_{i}\right| \log \sigma\right)$ time for the counting problem, and $O\left(\left|T_{i}\right| \log \sigma+\left|a n s_{i}\right|\right)$ time for the reporting problem, using $O\left(\left|T_{i}\right|+\|P\|+\|S\|\right)$ total working space.

Corollary 1. The preprocessing time in Theorem 1 can be reduced to $O(\|P\|+$ $\|S\|)$ time in case of integer alphabets of size polynomial with regard to $\|P\|+\|S\|$.

## 4 Applying the algorithm for traffic classification

We show in Appendix A the input sets that match each of the application signatures described in 20 .

Note that the signatures shown in 20 are generally characterized in the format of $p$ followed by $s$, where $p \in P$ and $s \in S$ for sets or lists $P$ and $S$. In general, this differs from our problem in that occurrences of $p$ and $s$ overlapping should not be counted as a match. For example, ab followed by bc means abc should not be counted as a match, while our algorithms do count this as a match. Nevertheless, such matches, which would be erroneous in the context of implementing the signatures, do not occur with the input sets listed in Appendix A, as the elements of $P$ and $S$ simply cannot overlap. For example, with Gnutella signatures each element of $P$ ends with the character :, which no element of $S$ contains, so no string $w \in \Sigma^{*}$ exists such that has some $p \in P$ as prefix, $s \in S$ as suffix, and $p$ and $s$ overlap (i.e. $|w|<|p|+|s|$ ). Note also that this problem also differs from the followed-by problem of [2, 14], which does share the intolerance of such overlaps, but differs in that the inputs are given as pairs of $p$ and $s$ rather than pair of sets or lists $P$ and $S$. Naively solving the pair-of-sets problem using algorithms for the pairs of $p$ and $s$ problem would take $|P| \times|S|$ queries, as we need one query for each pair of $p \in P$ and $s \in S$. This is clearly inefficient for large $|P|$ and $|S|$, and hence the necessity remains for our proposed algorithms.

## 5 Conclusion and future work

In this paper, we proposed online algorithms for counting and reporting all distinct substrings of an online text $T$ that has some $p \in P$ as a prefix, some $s \in S$ as a suffix, and whose length is within the interval $\left[k_{1} . . k_{2}\right.$ ], where $P$ and $S$ are static sequences of strings given as input for preprocessing, while positive integers $k_{1}, k_{2}$ and the characters of $T$ are given as query. Our algorithms take $O((\|P\|+$ $\|S\|) \log \sigma$ ) preprocessing time for general ordered alphabets, which is reduced to $O(\|P\|+\|S\|)$ time for integer alphabets of size polynomial w.r.t. $\|P\|+\|S\|$. The computation up to the $i$-th character of $T$ takes $O\left(\left|T_{i}\right| \log \sigma\right)$ cumulative time for the counting problem, and $O\left(\left|T_{i}\right| \log \sigma+\left|a n s_{i}\right|\right)$ cumulative time for the reporting problem. Furthermore, we have shown that it has possible applications in traffic classification, by showing that all of the application signatures in 20 can be represented as input sets of our proposed problems.

A few problems remain to be considered as future work:

- As the discussion in Section 4 implies, solving the problem where the prefix and suffix strings are not allowed to overlap, i.e. substrings are in the form of $p \Sigma^{k} s$, where $k \in\left[k_{1} . . k_{2}\right], p \in P, s \in S$, while retaining the distinctness condition as well as that the input sets be given as pairs of lists $P$ and $S$, can be useful in case we have a signature implemented with $P, S$ such that there does exist a string $w$ such that $w$ has $p \in P$ as prefix, $s \in S$ as suffix and $|w|<|p|+|s|$, and we want to exclude such $w$ from matches. Is it possible to devise an algorithm that solve this problem efficiently?
- In practice, how do the running times of our algorithms compare to the signature implementations used in 20]?


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## A Appendix

Below, we show the input sets that match the each of the application signatures described in 20 .

Table 1. Input sets corresponding to application signatures

| Application | List of $P$ elements | List of $S$ elements | $k_{1}$ | $k_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| Gnutella | User - Agent :, UserAgent :, Server : | LimeWire, BearShare, Gnucleus, MorpheusOS, XoloX, MorpheusPE, gtkgnutella, Acquisition, Mutella-0.4.1, MyNapster, Mutella0.4.1, Mutella-0.4, Qtella, AquaLime, NapShare, Comeback, Go, PHEX, SwapNut, Mutella-0.4.0, Shareaza, Mutella-0.3.9b, Morpheus, FreeWire, Openext, Mutella-0.3.3, Phex | 1 | $\infty$ |
| eDonkey | 0xe3 (in hex) | (the packet length) | 5-byte long |  |
| DirectConnect | \$MyNick, \$Lock, \$Key, \$Direction, \$GetListLen, \$ListLen, \$MaxedOut, \$Error, \$Send, \$Get, \$FileLength, \$Canceled, \$HubName, \$ValidateNick, \$ValidateDenide, $\$$ GetPass, \$MyPass, \$BadPass, \$Version, \$Hello, \$LogedIn, \$MyINFO, \$GetINFO, \$GetNickList, \$NickList, $\$ 0 \mathrm{pList}, \$ \mathrm{To}$, \$ConnectToMe, \$MultiConnectToMe, \$RevConnectToMe, \$Search, \$MultiSearch, \$SR, \$Kick, \$OpForceMove, \$ForceMove, \$Quit |  | 1-5 | $\infty$ |
| BitTorrent | the 20-byte string where the first byte is 19 ( $0 \times 13$ ) and the next 19 bytes are the string 19BitTorrent protocol |  | 20-byte long |  |
| Kazaa | GET, HTTP | X - Kazaa | 1 | $\infty$ |

