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Algorithms in Invariant Theory

Second edition

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Preface

The aim of this monograph is to provide an introduction to some fundamental problems, results and algorithms of invariant theory. The focus will be on the three following aspects:

(i) *Algebraic algorithms* in invariant theory, in particular algorithms arising from the theory of Gröbner bases;
(ii) *Combinatorial algorithms* in invariant theory, such as the straightening algorithm, which relate to representation theory of the general linear group;
(iii) *Applications* to projective geometry.

Part of this material was covered in a graduate course which I taught at RISC-Linz in the spring of 1989 and at Cornell University in the fall of 1989. The specific selection of topics has been determined by my personal taste and my belief that many interesting connections between invariant theory and symbolic computation are yet to be explored.

In order to get started with her/his own explorations, the reader will find exercises at the end of each section. The exercises vary in difficulty. Some of them are easy and straightforward, while others are more difficult, and might in fact lead to research projects. Exercises which I consider “more difficult” are marked with a star.

This book is intended for a diverse audience: graduate students who wish to learn the subject from scratch, researchers in the various fields of application who want to concentrate on certain aspects of the theory, specialists who need a reference on the algorithmic side of their field, and all others between these extremes. The overwhelming majority of the results in this book are well known, with many theorems dating back to the 19th century. Some of the algorithms, however, are new and not published elsewhere.

I am grateful to B. Buchberger, D. Eisenbud, L. Grove, D. Kapur, Y. Lakshman, A. Logar, B. Mourrain, V. Reiner, S. Sundaram, R. Stanley, A. Zelevinsky, G. Ziegler and numerous others who supplied comments on various versions of the manuscript. Special thanks go to N. White for introducing me to the beautiful subject of invariant theory, and for collaborating with me on the topics in Chapters 2 and 3. I am grateful to the following institutions for their support: the Austrian Science Foundation (FWF), the U.S. Army Research Office (through MSI Cornell), the National Science Foundation, the Alfred P. Sloan Foundation, and the Mittag-Leffler Institute (Stockholm).

Ithaca, June 1993 — Bernd Sturmfels

Preface to the second edition

Computational Invariant Theory has seen a lot of progress since this book was first published 14 years ago. Many new theorems have been proved, many new algorithms have been developed, and many new applications have been explored. Among the numerous interesting research developments, particularly noteworthy from our perspective are the methods developed by Gregor Kemper for finite groups and by Harm Derksen on reductive groups. The relevant references include

Harm Derksen, Computation of reductive group invariants, Advances in Mathematics 141, 366–384, 1999;
Gregor Kemper, Computing invariants of reductive groups in positive characteristic, Transformation Groups 8, 159–176, 2003.

These two authors also co-authored the following excellent book which centers around the questions raised in my chapters 2 and 4, but which goes much further and deeper than what I had done:

Harm Derksen and Gregor Kemper, Computational invariant theory (Encyclopaedia of mathematical sciences, vol. 130), Springer, Berlin, 2002.

In a sense, the present new edition of "Algorithms in Invariant Theory" may now serve the role of a first introductory text which can be read prior to the book by Derksen and Kemper. In addition, I wish to recommend three other terrific books on invariant theory which deal with computational aspects and applications outside of pure mathematics:

Karin Gatermann, Computer algebra methods for equivariant dynamical systems (Lecture notes in mathematics, vol. 1728), Springer, Berlin, 2000;
Mara Neusel, Invariant theory, American Mathematical Society, Providence, R.I., 2007;
Peter Olver, Classical invariant theory, Cambridge University Press, Cambridge, 1999.

Graduate students and researchers across the mathematical sciences will find it worthwhile to consult these three books for further information on the beautiful subject of classical invariant theory from a contempory perspective.

Berlin, January 2008 Bernd Sturmfels

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