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Intelligent Comparisons: Analytic Inequalities

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*To my wife Koula and my daughters
Angela and Peggy*

Preface

In this monograph we present the recent work of the past 5 years of the author on inequalities in real, functional and fractional analysis. It is the natural outgrowth of his related publications. Chapters are self-contained and can be read independently and several advanced courses can be taught out of this book. An extensive list of references is given per chapter.

The inequalities covered are diverse. A list of these follows:

Fractional Polya type integral inequality.

Univariate fractional Polya type integral inequalities.

Multivariate generalized fractional Polya type integral inequalities.

Balanced Canavati type fractional Opial inequalities.

Fractional representation formulae under initial conditions and fractional Ostrowski type inequalities.

Basic fractional integral inequalities.

Harmonic multivariate Ostrowski and Grüss type inequalities for several functions.

Fractional Ostrowski and Grüss type inequalities involving several functions.

Further interpretation of some fractional Ostrowski and Grüss type inequalities.

Multivariate fractional representation formula and Ostrowski type inequality.

Multivariate weighted fractional representation formulae and Ostrowski type inequalities.

Multivariate Lyapunov inequalities.

Ostrowski inequalities for semigroups.

Ostrowski inequalities for cosine and sine operator functions.

Hilbert-Pachpatte type inequalities for semigroups, cosine and sine operator functions.

Ostrowski and Landau inequalities for Banach space valued functions.

Multidimensional Ostrowski inequalities for Banach space valued functions.

Fractional representation formulae and right fractional inequalities.

Canavati fractional Ostrowski type inequalities.

Most general fractional representation formula for functions and implications.

Rational inequalities for integral operators under convexity.

Fractional integral inequalities involving convexity.

Vectorial inequalities for integral operators involving ratios of functions and convexity.

Vectorial splitting rational L_p inequalities for integral operators.

Separating rational L_p inequalities for integral operators.

Vectorial Hardy type fractional inequalities.

Vectorial fractional integral inequalities with convexity.

This book's results are expected to find applications in many areas of pure and applied mathematics, especially in ordinary and partial differential equations and fractional differential equations. As such this monograph is suitable for researchers, graduate students, and seminars of the above subjects, also to be in all science and engineering libraries.

The preparation of this book took place during 2014–2015 in Memphis, Tennessee, USA.

I would like to thank Prof. Alina Lupas, of University of Oradea, Romania, for checking and reading the manuscript.

Memphis, USA
April 2015

George A. Anastassiou

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