Quantified Degrees of Group Responsibility

Vahid Yazdanpanah(✉) and Mehdi Dastani

Utrecht University, Utrecht, The Netherlands
v.yazdanpanah@students.uu.nl, m.m.dastani@uu.nl

Abstract. This paper builds on an existing notion of group responsibility and proposes two ways to define the degree of group responsibility: *structural* and *functional* degrees of responsibility. These notions measure potential responsibilities of agent groups for avoiding a state of affairs. According to these notions, a degree of responsibility for a state of affairs can be assigned to a group of agents if, and to the extent that, the group of the agents have potential to preclude the state of affairs. These notions will be formally specified and their properties will be analyzed.

1 Introduction

The concept of responsibility has been extensively investigated in philosophy and computer science. Each proposal focuses on specific aspects of responsibility. For example, [1] focuses on the causal aspect of responsibility and defines a notion of graded responsibility, [2] focuses on the organizational aspect of responsibility, [3] argues that group responsibility should be distributed to individual responsibility, [4] focuses on the interaction aspect of responsibility and defines an agent's responsibility in terms of the agent's causal contribution, and [5] focuses on the strategic aspect of group responsibility and defines various notions of group responsibility. In some of these proposals, the concept of responsibility is defined with respect to a realized event "in past" while in other approaches it is defined as the responsibility for the realization of some event "in future". This introduces a major dimension of responsibility, namely backward-looking and forward-looking responsibility [6]. Backward-looking approaches reason about level of causality or contribution of agents in the occurrence of an already realized outcome while forward-looking notions are focused on the capacities of agents towards a state of affairs.

Although some of the existing approaches are designed to measure the degree of responsibility, they either constitute a backward-looking (instead of forward-looking) notion of responsibility [1], provide qualitative (instead of quantitative) levels of responsibility [7,8], or focus on individual (instead of group) responsibility [4]. To our knowledge, there is no forward-looking approach that could measure the degree of group responsibility quantitatively. Such notion would enable reasoning on the potential responsibility of an agent group towards a state of affairs in strategic settings, e.g., collective decision making scenarios. In this paper, we build on a forward-looking approach to group responsibility and

V. Dignum et al. (Eds.): COIN 2015, LNAI 9628, pp. 418–436, 2016.
DOI: 10.1007/978-3-319-42691-4_23

define two notions of responsibility degrees. The first concept is based on the partial or complete power of an agent group to preclude a state of affairs while the second concept is based on the potentiality of an agent group to reach a state where the agent group possesses the complete power to preclude the state of affairs. This results in a distinction between what we will call the "structural responsibility" versus the "functional responsibility" of an agent group. In our proposal, an agent group has the full responsibility, if it has an action profile to preclude the state of affairs. All other agent groups that do not have full responsibility, but may have contribution to responsible agent groups, will be assigned a partial degree of responsibility.

The paper is structured as follows. In Sect. 2 we provide a brief analysis of the concept of group responsibility from a power-based point of view. Section 3 presents the framework in which our proposed notions will be formally characterized. In Sects. 4 and 5 we introduce the notions that capture our conception of *degree of group responsibility* with respect to a given state of affairs and analyze their properties. Finally, concluding remarks and future work directions will be presented in Sect. 7.

2 Group Responsibility: A Power-Based Analysis

In order to illustrate our conception of group responsibility and the nuances in degrees of responsibility, we follow [1] and use a voting scenario to explain the degree of responsibility of agents' groups for voting outcomes. The voting scenario considers a small congress with ten members consisting of five Democrats (D), three Republicans (R), and two Greens (G). We assume that there is a voting in progress on a specific bill (B). Without losing generality and to reduce the combinatorial complexity of the setting, we assume that all members of a party vote either in favour of or against the bill B. Table 1 illustrates the eight possible voting outcomes. Note that in this scenario, six positive votes are sufficient for the approval of B. For example, row 4 shows the case where R and D vote against B and the bill is disapproved. For this case we say that the group RD votes against B. It should also be noted that our assumption reduces parties to individual agents with specific weights such that the question raises as why we use this party setting instead of a simple voting of three agents whose votes have different weights. The motivation is that this setting is realistic and makes the weighted votes of each agent (party) more intuitive.

Following [5] we believe that it is reasonable to assign the responsibility for a specific state of affairs to a group of agents if they jointly have the power to avoid the state of affairs[1]. According to [9], the preclusive power is the ability of a group to preclude a given state of affairs which entails that a group with preclusive power, has the potential but might not practice the preclusion of a given state of affairs. For our voting scenario, this suggests to assign responsibility to the group GR consisting of parties G and R since they can jointly

[1] See [5] for a detailed discussion on why to focus on avoiding instead of enforcing a state of affairs.

Table 1. Voting results

	G(2)	R(3)	D(5)	Result
0	−	−	−	×
1	−	−	+	×
2	−	+	−	×
3	−	+	+	✓
4	+	−	−	×
5	+	−	+	✓
6	+	+	−	×
7	+	+	+	✓

Table 2. War incidence

	Congress	President	War
0	−	−	×
1	−	+	×
2	+	−	×
3	+	+	✓

disapprove B. Note that the state of affair to be avoided can also be the state of affairs where B is disapproved. In this case, the group can be assigned the responsibility to avoid disproving B. Similarly, groups D, GD, RD, and GRD have preclusive power with respect to the approval of B as they have sufficient members (weights) to avoid the approval of B. Note that none of the other two groups, i.e., G and R, could preclude the approval of B independently. However, based on [5], the agent groups that consist of a smaller sub-group with preclusive power, must be excluded from the set of responsible groups. Hence, we consider GR and D as being responsible groups for the approval of B. The intuition for this concept of responsibility is supported by the fact that the lobby groups are willing (i.e., it is economically rational) to invest resources in parties that have the power to avoid a specific state of affairs.

We build on the ideas in [5] and propose two orthogonal approaches to capture our conception of *degree* of group responsibility towards a state of affairs. Our intuition suggests that the *degree* of responsibility of a group of agents towards a state of affairs should reflect the extent they structurally or functionally can contribute to the groups that have preclusive power with respect to the state of affairs. In the sequel, we will explain the conception of *degree of responsibility* according to the structural and functional approaches, and illustrate both approaches by means of our voting scenario example.

Our conception of *structural responsibility degree* is based on the following observation in the voting scenario. We deem that regarding the approval of B, although the groups G and R have no preclusive power independently, they nevertheless have a share in the composition of GR with preclusive power regarding the approval of B. Hence, we say that any group that shares members with responsible groups, should be assigned a degree of responsibility that reflects its proportional contribution to the groups with preclusive power. For example, group R with three members, has larger share in GR than the group G has. Therefore, we believe that the relative size of a group and its share in the groups with the preclusive power are substantial parameters in formulation of the notion of responsibility degree. In this case, the larger share of R in GR in comparison with the share of G in GR will be positively reflected in R's responsibility degree. These parameters will be explained in details later. We would like to emphasize that this concept of responsibility degree is supported by the fact that lobby groups do proportionally support political parties that can play a role in some key decisions. In a sense, the lobby groups consider political parties responsible for some decision and therefore they are willing to support the parties.

The second approach in capturing the notion of *functional responsibility degree* addresses the dynamics of preclusive power of a specific group. Suppose that the bill B was about declaration of the congress to the President (P) which enables P to start a war (Table 2). Roughly speaking, P will be in charge only after the approval of the congress. When we are reasoning at the moment when the voting is in progress in the congress, it is reasonable to assume that groups GR and D are responsible as they have preclusive power to avoid the war. Moreover, after the approval of B, the President P is the only group with preclusive power to avoid the war. Hence, we believe that although P alone would not have the preclusive power before the approval of B in the congress, it is rationally justifiable for an anti-war campaign to invest resources on P, even before the approval voting of the congress, simply because there exists possibilities where P will have the preclusive power to avoid the war. Accordingly, a reasonable differentiation could be made between the groups which do have the chance of acquiring the preclusive power and those they do not have any chance of power acquisition. This functional notion of responsibility degree addresses the eventuality of a state in which an agent group possesses the preclusive power regarding a given state of affairs.

Note that following [5], our notions of group responsibility are locally bounded as they will be defined with respect to some source state. Hence, a group might be responsible in a specific state and not responsible in the other states regarding a given state of affairs. Additionally, our proposed notions for responsibility degree have dependency to the global setting. In the voting scenario, the global setting that ten voters are situated in three parties of G (2 members), R (3 members) and D (5 members), is crucial for the responsibility degrees that are assigned to various groups. Any change in the global setting may alter the responsibility degree of various groups. For example, when two members of the Republican party secede from R and form a new Tea Party T, we face a different global setting, which in turn causes the responsibility degrees assigned to various groups to change. This is due to the fact that the new setting introduces new groups such as RGT with preclusive power regarding the approval of B. Our analysis is not limited to the voting scenarios, but can be applied to other situations as shown later in this paper.

3 Models and Preliminary Notions

The behaviour of a multi-agent system is often modelled by concurrent game structures (CGS) [10]. Such structures specify possible state of the system, agents' abilities at each state, and the outcome of concurrent actions at each state.

Definition 1 (Concurrent game structures [10]). *A concurrent game structure is a tuple $M = (N, Q, Act, d, o)$, where $N = \{1, ..., k\}$ is a nonempty finite set of agents, Q is a nonempty set of system states, Act is a nonempty and finite set of atomic actions, $d : N \times Q \to \mathcal{P}(Act)$ is a function that identifies the set of available actions for each agent $i \in N$ at each state $q \in Q$, and o is a deterministic and partial transition function that assigns a state $q' = o(q, \alpha_1, ..., \alpha_k)$ to a state q and an action profile $(\alpha_1, ..., \alpha_k)$ such that all k agents in N choose actions in the action profile respectively. An action profile $\bar{\alpha} = (\alpha_1, ..., \alpha_k)$ is a sequence that consists of actions $\alpha_i \in d(i, q)$ for all players in N. In case $o(q, \alpha_1, ..., \alpha_k)$ is undefined then $o(q, \alpha'_1, ..., \alpha'_k)$ is undefined for each action profile $(\alpha'_1, ..., \alpha'_k)$. For the sake of notation simplicity, $d(i, q)$ will be written as $d_i(q)$ and $d_C(q) := \prod_{i \in C} d_i(q)$.*

A *state of affairs* refers to a set $S \subseteq Q$, $\bar{S}$ denotes the set $Q \backslash S$, and $(\alpha_C, \alpha_{N \backslash C})$ denotes the action profile, where α_C is the actions of the agents in group C and $\alpha_{N \backslash C}$ denotes the actions of the rest of the agents. Following the setting of [5], we recall the definitions of *q-enforce*, *q-avoid*, *q-responsible* and *weakly q-responsible* (See [5] for details and properties of these notions).

Definition 2 (Agent groups: strategic abilities and responsibility [5]). *Let $M = (N, Q, Act, d, o)$ be a CGS, $q \in Q$ be a specific state, and S a state of affairs. We have the following concepts.*

1. *$C \subseteq N$ can q-*enforce *S in M iff there is a joint action $\alpha_C \in d_C(q)$ such that for all joint actions $\alpha_{N \backslash C} \in d_{N \backslash C}(q)$, $o(q, (\alpha_C, \alpha_{N \backslash C})) \in S$.*
2. *$C \subseteq N$ can q-*avoid *S in M iff for all $\alpha_{N \backslash C} \in d_{N \backslash C}(q)$ there is $\alpha_C \in d_C(q)$ such that $o(q, (\alpha_C, \alpha_{N \backslash C})) \in \bar{S}$.*
3. *$C \subseteq N$ is q-*responsible *for S in M iff C can q-enforce $\bar{S}$ and for all other $C' \subseteq N$ that can q-enforce $\bar{S}$, we have that $C \subseteq C'$.*
4. *$C \subseteq N$ is* weakly q-responsible *for S in M*[2] *iff C is a minimal group that can q-enforce $\bar{S}$.*

Considering the voting scenario from Sect. 2, groups GD, RD and GRD can q_s-enforce the approval of B while groups D, GR, GD, RD, and GRD can q_s-avoid the approval of B. In this scenario, q_s denotes the starting moment of the voting progress. Note that the notions of q-enforce and q-avoid correlate with the notions of, respectively, α-effectivity and β-effectivity in [11]. In this scenario, we have no q_s-responsible group for approval of B and two groups D and GR are weakly q_s-responsible for the approval of B. Note that the groups GD, RD, and GRD are not weakly q_s-responsible for the approval of B as they are not minimal.

The concept of (weakly) q-responsibility merely assigns responsibility to groups with preclusive power and considers all other groups as not being responsible. As we have argued in Sect. 2, we believe that responsibility can be assigned to all groups, even those without preclusive power, though to a certain degree

[2] In further references, "in M" might be omitted wherever it is clear from the context.

including zero degree. In order to define our notions of responsibility degree, we first introduce two notions of *structural power difference* and *power acquisition sequence*. Given an arbitrary group C, a state q, and a state of affair S, the first notion concerns the number of missing elements in C that when added to C makes it a (weakly) q-responsible groups for a S, and the second notion concerns a sequence of action profiles from given state q that leads to a state q' where C is (weakly) q'-responsible for S. According to the first notion, group C can gain preclusive power for S if supported by some additional members, and according to the second notion C can gain preclusive power for S in some potentially reachable state.

Let M be a multi-agent system, S a state of affairs in M, C an arbitrary group, and $\hat{C}$ be a (weakly) q-responsible group for S in M.

Definition 3 (Power measures). *We say that the structural power difference of C and $\hat{C}$ in $q \in Q$ with respect to S, denoted by $\Theta_q^{S,M}(\hat{C}, C)$, is equal to cardinality of $\hat{C} \backslash C$. Moreover, we say that C has a power acquisition sequence $\langle \bar{\alpha_1}, ..., \bar{\alpha_n} \rangle$ in $q' \in Q$ for S in M iff for $q_i \in Q$, $o(q_i, \bar{\alpha}_i) = q_{i+1}$ for $1 \leq i \leq n$ such that $q' = q_1$ and $q_{n+1} = q''$ and C is (weakly) q''-responsible for S in M.*

Consider the war approval declaration of the congress to the president (P) in Sect. 2. Here, we can see that the structural power difference of the group G and the weakly q_s-responsible group GR is equal to 3. Moreover, the singleton group P that is not responsible in q_s has the opportunity of being responsible for the war in states other than q_s. Note that power acquisition sequence does not necessarily need to be unique. If the group C is not (weakly) responsible in a state q, the existence of any power acquisition sequence with a length higher than zero implies that the group could potentially reach a state q' (from the current state of q) where C is (weakly) q'-responsible for S. This notion also covers the cases where C is already in a (weakly) responsible state where the minimum length of power acquisition sequence is taken to be zero. In this case, the group is already (weakly) q-responsible for S. For example, in the voting scenario, group D is weakly responsible for the state of affairs and therefore, the minimum length of a power acquisition sequence is zero. When we are reasoning in a source state q, the notion of *power acquisition sequence*, enables us to differentiate between the non (weakly) q-responsible groups that do have the opportunity of becoming (weakly) q'-responsible for a given state of affairs ($q \neq q'$) and those they do not. Moreover, we emphasize that the availability of a *power acquisition sequence* for an arbitrary group C from a source state q to a state q' in which C is (weakly) q-responsible for the state of affairs, does not necessitate the existence of an independent strategy for C to reach q' from q.

4 Structural Degree of Responsibility

Structural degree of responsibility addresses the preclusive power of a group for a given state of affairs by means of the *maximum* contribution that the group has in a (weakly) responsible group for the state of affairs. To illustrate the intuition behind this notion, consider again the voting scenario in the Sect. 2. If an anti-war campaign wants to invest its limited resources to prevent the bill start a war, we deem that it is reasonable to invest more on R than G, if the resources admit such a choice. Although neither R nor G could prevent the war individually, larger contribution of R in groups with preclusive power, i.e. GR and D, entitles R to be assigned with larger degree of responsibility than G. This intuition will be reflected in the formulation of *structural degree of responsibility*.

Definition 4 (Structural degree of responsibility). *Let $\mathbb{W}_q^{S,M}$ denote the set of all (weakly) q-responsible groups for state of affairs S in multi-agent system M, and $C \subseteq N$ be an arbitrary group. In case $\mathbb{W}_q^{S,M} = \varnothing$, the structural degree of q-responsibility of any C for S in M is undefined; otherwise, the structural degree of q-responsibility of C for S in M denoted $\mathcal{SDR}_q^{S,M}(C)$, is defined as follows:*

$$\mathcal{SDR}_q^{S,M}(C) = \max_{\hat{C} \in \mathbb{W}_q^{S,M}} (\{i \mid i = 1 - \frac{\Theta_q^{S,M}(\hat{C}, C)}{|\hat{C}|}\})$$

Intuitively, $\mathcal{SDR}_q^{S,M}(C)$ measures the highest contribution of a group C in a (weakly) q-responsible $\hat{C}$ for S. Hence, structural degree of responsibility is in range of $[0,1]$. In sequel, we write $\mathcal{SDR}_q^{S}(C)$ and $\mathbb{W}_q^{S}$ instead of $\mathcal{SDR}_q^{S,M}(C)$ and $\mathbb{W}_q^{S,M}$, respectively.

Proposition 1 (Full structural responsibility). *The structural degree of q-responsibility of group C for S is equal to 1 iff C is either a (weakly) q-responsible group for S or $C \supseteq \hat{C}$ such that $\hat{C}$ is (weakly) q-responsible for S.*

Proof. Follows directly from Definition 4 and definition of (weak) responsibility in [5]. □

Example 1. Consider again the voting scenario from Sect. 2 (Fig. 1). In this scenario, we have an initial state q_s in which all voters can use their votes in favour or against the approval of the bill B (no abstention or null vote is allowed). The majority of six votes (or more) in favour of B will be considered as the state of affairs consisting of states q_7, q_5 and q_3. This multi-agent system can be modelled as CGS $M = (N, Q, Act, d, o)$, where $N = \{1, ..., 10\}$, $Q = \{q_s, q_0, ..., q_7\}$, $Act = \{0, 1, wait\}$, $d_i(q_s) = \{0, 1\}$ and $d_i(q) = \{wait\}$ for all $i \in N$ and $q \in Q \backslash \{q_s\}$. Voters are situated in three parties such that $G = \{1, 2\}$, $R = \{3, 4, 5\}$ and $D = \{6, 7, 8, 9, 10\}$. For notation convenience, actions of party members will be written collectively in the action profiles, e.g., we write $(0, 1, 0)$ to denote the action profile $(0, 0, 1, 1, 1, 0, 0, 0, 0, 0)$. The outcome function is as illustrated in Fig. 1 (e.g., $o(q_s, (0, 0, 1)) = q_1$ is illustrated by the arrow from q_s to q_1). Moreover, the simplifying assumption that all party members vote collectively is implemented by $o(q_s, \bar{\alpha}') = q_s$ for all possible action profiles $\bar{\alpha}'$ in which party members act differently. We observe that the set of weakly q_s-responsible groups in this example is $\{GR, D\}$. Using Definition 4, the structural degree of q_s-responsibility of G will be equal to $max(\{2/5, 0/5\}) = 2/5$ and $\mathcal{SDR}_{q_s}^{S}(R) = 3/5$.

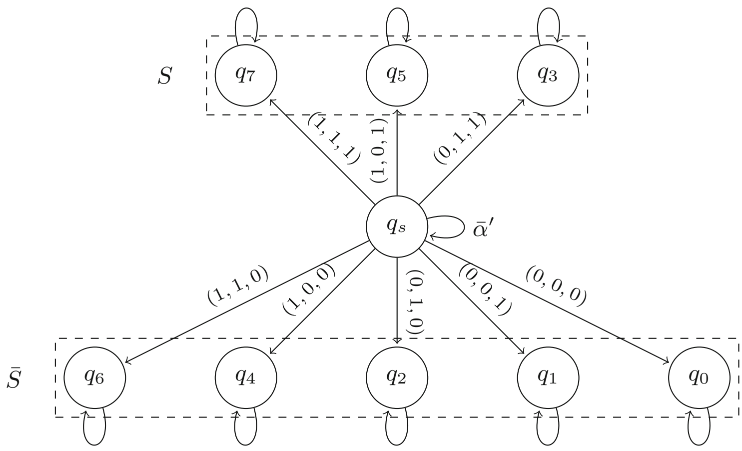

Fig. 1. Voting scenario

A similar calculation leads to the conclusion that the structural degree of q_s-responsibility for all (weakly) q_s-responsible groups, i.e., GR and D, and their super-sets is equal to 1. The structural degree of q_s-responsibility of empty group ($\varnothing$) is equal to 0 as the structural power difference of the empty group with all (weakly) q_s-responsible groups $\hat{C}$ is equal to the cardinality of $\hat{C}$.

A group C might share members with various (weakly) q-responsible groups, therefore the largest structural share of C in (weakly) q-responsible groups for S, will be considered to form the $\mathcal{SDR}_q^S(C)$. We would like to stress that our notions for responsibility degrees are formulated based on the maximum expected power of a group to preclude a state of affairs. While we believe that in legal theory, and with respect to its backward-looking approach, the minimum preclusive power of a group need be taken into account for assessing culpability, our focus as a forward-looking approach will be on maximum expected preclusive power of a group regarding a given state of affairs.

The following lemma introduces a responsibility paradox case in which our presented notion of *structural degree of responsibility* is not applicable as a notion for reasoning about responsibility of groups of agents.

Lemma 1 (Applicability constraint: responsibility paradox). *The empty group is (unique) q-responsible for S iff the structural degree of q-responsibility of all possible groups C for S is equal to 1.*

Proof. "$\Rightarrow$": Based on Proposition 1, if the empty group ($\varnothing$) is q-responsible for S, the structural degree of q-responsibility of the empty group and all its super-groups, i.e., all the possible groups, is equal to 1.

"$\Leftarrow$": According to Proposition 1, and because the empty group is only a super-group of itself, the premise entails that the empty group must be either a weakly q-responsible group for S or the unique q-responsible group for S.

Based on [5], if the empty group is weakly q-responsible for S, then it is the q-responsible group for S. $\square$

The common avoidability of S implies that the occurrence of S is impossible by means of any action profile in q. In other words, given the specification of a CGS model M, a state of affairs S and a source state q in M, no action profile $\bar{\alpha}$ leads to a state $q_s \in S$. Common avoidability of a state of affairs, correlates with the impossibility notion $\neg\Diamond S$ in modal logic [12]. An impossible state of affairs S in q, entitles all the possible groups to be "fully responsible". The impossibility of S neutralizes the space of groups with respect to their structural degree of q-responsibility for S. Therefore, we believe that in cases where the empty group is responsible for a given state of affairs, as S is impossible, full degree of structural responsibility of a group is not an apt measure, does not imply the preclusive power of any group, and hence, not an applicable reasoning notion for one who is willing to invest resources in the groups of agents that have the preclusive power over S. Note that in case the empty set is not responsible for S, its structural degree of responsibility is equal to 0 because its structural power difference with all (weakly) responsible groups $\hat{C}$ is equal to the cardinality of $\hat{C}$.

The next theorem illustrates a case in which a singleton group possesses the preclusive power over a state of affairs. The existence of such a *dictator* agent in a state q, polarizes the space of all possible groups with respect to their structural degree of q-responsibility for the state of affairs.

Theorem 1 (Polarizing dictatorship). *Let $\hat{C}$ be a singleton group, q an arbitrary state and S a possible state of affairs (in sense of Lemma 1). Then, $\hat{C}$ is a (unique) q-responsible group for S iff for any arbitrary group C, $\mathcal{SDR}_q^S(C) \in \{0,1\}$, where $\mathcal{SDR}_q^S\ (C \in I) = 1$ and $\mathcal{SDR}_q^S(C \in O) = 0$ for $I = \{C | C \supseteq \hat{C}\}$ and $O = \{C | C \not\supseteq \hat{C}\}$.*

Proof. "$\Rightarrow$": Based on Proposition 1, the structural degree of q-responsibility of any group $C \supseteq \hat{C}$ is equal to 1. In other cases, the structural degree of q-responsibility of $C \not\supseteq \hat{C}$ is equal to 0 because C shares no element with $\hat{C}$, which is the singleton (unique) q-responsible group for S.

"$\Leftarrow$": Here we have a partition $W = \{I, O\}$ of all possible groups. As S is not an impossible state of affair in sense of Lemma 1, the empty group is not q-responsible for S but has the structural degree of q-responsibility equal to 0; and therefore a member of O. I as a set of all groups with structural degree of responsibility equal to 1, is a non-empty set either; because there exists at least one group in I which is $\hat{C}$. Hence, $\mathcal{SDR}_q^S(\hat{C} \in I) = 1$ and necessarily there exists at least one non-empty weakly q-responsible group for S, i.e., $\mathbb{W}_q^S \neq \varnothing$. Accordingly, based on Proposition 1, and as $\hat{C}$ is a singleton, $\hat{C} \in \mathbb{W}_q^S$. Moreover, based on Proposition 1, we have that $\mathbb{W}_q^S \subseteq I$. As $\hat{C}$ is a subset of all groups in I, we conclude that $\hat{C} \subseteq \mathbb{W}_q^S$. Thus, $\hat{C}$ is a weakly q-responsible group and is a subset of all possible weakly q-responsible groups for S. Therefore, $\hat{C}$ is the unique singleton q-responsible group or the *q-dictator* for S. $\square$

Example 2 (Operating room scenario). Consider a surgery operation room where a patient is going to be operated. In this surgery operation a surgeon D, a surgeon assistant A and an anesthesiologist N are involved. In this scenario, each agent, i.e., D, A and N, can decide to perform her role in health-care delivery or to refuse. If the anesthesiologist chooses to refuse or if both the surgeon and the assistant decide to refuse, the patient will die. When all three agents choose to perform their tasks, the patient will recover in the state of *good health*. Finally, an exclusive refusal of the assistant or the surgeon, results in *medium health* or *infirm health*, respectively. This multi-agent scenario can be modelled as a CGS M, as shown in Fig. 2. This CGS is specified as $M = (\{D, A, N\}, \{q_s, q_1, q_2, q_3, q_4\}, \{perform, refuse, wait\}, d, o)$ where $d_i(q_s) = \{perform, refuse\}$ and $d_i(q) = \{wait\}$ for all $i \in \{D, A, N\}$ and $q \in \{q_1, q_2, q_3, q_4\}$. The outcome function o is shown in the Fig. 2, e.g. $o(q_s, (perform, refuse, perform)) = q_2$. The star $\star$ represents any available action, i.e. $\star \in \{perform, refuse\}$. In this example the weakly q_s-responsible groups for death of the patient (at state q_4) are DN and AN. Hence, the structural degree of q_s-responsibility of all possible groups, i.e., D, A, N, DA, DN, AN, and DAN, for q_4, could be measured based on their maximum contribution in DN and AN. Accordingly, the structural degree of q_s-responsibility of groups D, A, N and DA will be $1/2$. All groups of DN, AN and DAN have the structural degree of q_s-responsibility equal to 1 which reflects their preclusive power to avoid the death of P.

As our concept of group responsibility is based on the preclusive power of a group over a given state of affairs, the following monotonicity property shows that increasing the size of a group by adding new elements, does not have a negative effect on the preclusive power. This property, as formulated below, correlates with the monotonicity of power and power indices [13,14].

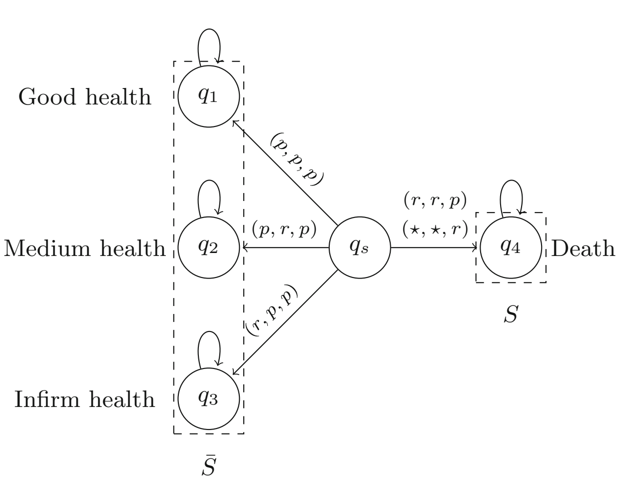

Fig. 2. Operating room scenario

Proposition 2 (Structural monotonicity). *Let C and C' be two arbitrary groups such that $C \subseteq C'$. If $\mathbb{W}_q^{S,M} \neq \varnothing$ then $\mathcal{SDR}_q^S(C) \leq \mathcal{SDR}_q^S(C')$.*

Proof. By definition, the structural degree of q-responsibility of C for S, in case it is not undefined in general, reflects the maximum share of C in all possible weakly q-responsible groups for S. Hence, as the structural degree of q-responsibility has a value in range $[0, 1]$, the elements in $C' \backslash C$ could have no negative effect on this degree. □

Note that the other way does not hold in general; because the structural degree of q-responsibility of the groups C and C', might be formulated based on their maximum contribution in two distinct weakly q-responsible groups. Consider the operating room scenario in Example 2. As presented, $\mathcal{SDR}_{q_s}^S(A) = 1/2 leq \mathcal{SDR}_{q_s}^S(DN) = 1$ but $A \nsubseteq DN$.

The following theorem shows that in case of existence of a unique nonempty q-responsible group for a state of affairs, the structural degree of q-responsibility of any group could be calculated cumulatively based on the degrees of disjoint subsets. In this case, for any two arbitrary groups C_1 and C_2, the summation of their structural degree of q-responsibility will be equal to the degree of the unified group.

Theorem 2 (Conditional cumulativity). *If there exists a nonempty (unique) q-responsible group for S, then for any arbitrary group C and partition $P = \{C_1, ..., C_n\}$ of C, we have $\sum_{i=1}^n \mathcal{SDR}_q^S(C_i) = \mathcal{SDR}_q^S(C)$.*

Proof. Suppose $\hat{C}$ is the q-responsible group for S. Then, as $\hat{C}$ is unique (See [5]), the structural degree of q-responsibility of any group $C_i \in P$, could be reformulated based on its contribution to $\hat{C}$. Thus, $\sum_{i=1}^n \mathcal{SDR}_q^S(C_i)$ is equal to $\sum_{i=1}^n \frac{|\hat{C} \cap C_i|}{|\hat{C}|}$. The whole equation is equal to $\frac{1}{|\hat{C}|} \sum_{i=1}^n |\hat{C} \cap C_i|$. Hence, as P is a partition of C, we have $\frac{|\hat{C} \cap C|}{|\hat{C}|}$ which is equal to $\mathcal{SDR}_q^S(C)$. □

5 Functional Degree of Responsibility

Functional degree of responsibility addresses the dynamics of preclusive power of a specific group with respect to a given state of affairs. We remind the example from Sect. 2 where the president will be in charge, regarding the war decision, only after the approval of the congress. It is our understanding that the existence of a sequence of action profiles that leads to a state where the president becomes responsible for the war decision rationalizes the investment of an anti-war campaign on the president, even before the approval of the congress.

The functional degree of responsibility of a group C in a state q will be calculated based on the notion of *power acquisition sequence* by tracing the number of necessary state transitions from q, in order to reach a state q' in which the group C is (weakly) q'-responsible for S. The length of a shortest power acquisition sequence form q to q', illustrates the *potentiality* of preclusive power of the group C. If two groups have the capacity of reaching a state in which they have the preclusive power over the state of affairs S, we say that the group which has the shorter path has a higher potential preclusive power and thus gets the larger functional degree of responsibility. Accordingly, a group which is already in a responsible state, has full potential to avoid a state of affairs. Hence, it will be assigned with maximum functional degree of responsibility equal to one.

Definition 5 (Functional degree of responsibility). *Let $\mathbb{P}_q^{S,M}(C)$ denote the set of all power acquisition sequences of group $C \subseteq N$ in q for S in M. Let also $\ell = \min_{k \in \mathbb{P}_q^{S,M}(C)} (\{i \mid i = length(k)\})$ be the length of a shortest power acquisition sequence. The functional degree of q-responsibility of C for S in M, denoted by $\mathcal{FDR}_q^{S,M}(C)$, is defined as follows:*

$$\mathcal{FDR}_q^{S,M}(C) = \begin{cases} 0 & \textit{if } \mathbb{P}_q^{S,M}(C) = \varnothing \\ \frac{1}{(\ell+1)} & \textit{otherwise} \end{cases}$$

The notion of $\mathcal{FDR}_q^{S,M}(C)$ is formulated based on the minimum length of power acquisition sequences, which taken to be 0 if C is a (weakly) q-responsible group for S. In such a case, C has already an action profile to avoid S in q. Hence, the functional degree of q-responsibility of C for S will be equal to 1. If no power acquisition sequence k does exist for C (i.e., $\mathbb{P}_q^{S,M}(C) = \varnothing$), then the minimum length of power acquisition sequences is taken to be ∞ such that the functional degree of q-responsibility of C for S becomes 0. In other cases $\mathcal{FDR}_q^{S,M}(C)$ will be strictly between zero and one. In sequel, we write $\mathcal{FDR}_q^{S}(C)$ and $\mathbb{P}_q^{S}(C)$ instead of $\mathcal{FDR}_q^{S,M}(C)$ and $\mathbb{P}_q^{S,M}(C)$, respectively.

Proposition 3 (Full functionality implies full responsibility). *Let $\hat{C}$ be a group, q an arbitrary state and S a given state of affairs. If $\mathcal{FDR}_q^{S}(\hat{C}) = 1$, then the structural degree of q-responsibility of $\hat{C}$ for S is equal to 1.*

Proof. According to Definition 5, only for (weakly) q-responsible groups C, $\mathcal{FDR}_q^{S}(C) = 1$. Hence, based on Proposition 1, for the group $\hat{C}$ with functional degree of q-responsibility equal to 1, we have that $\mathcal{SDR}_q^{S}(\hat{C}) = 1$. □

Note that the other side does not hold in general because $\mathcal{SDR}_q^{S}(C) = 1$ also includes the cases in which C is a proper super-set of a responsible group. For instance, consider the operating room scenario in Example 2. As presented, $\mathcal{SDR}_{q_s}^{S}(ADN) = 1$ but as it is not minimal, it is not weakly q_s-responsible for S. Hence, the functional degree of q_s-responsibility of ADN for S is not equal to one. In fact, $\mathcal{FDR}_{q_s}^{S}(ADN) = 0$ as there is no eventual state q' in which the group ADN is weakly q'-responsible for S.

Example 3 (War powers resolution). Consider again the voting scenario in the congress, as explained in Sect. 2; but now extended with a new *president* agent P. The decision of starting a war W should first be approved by a majority of the congress members (six votes or more in favour of W) after which the president makes the final decision. Hence, P has the preclusive power which is conditioned on the approval of the congress members. Moreover, we have a simplifying assumption that no party member acts independently and thus assume that all members of a party vote either in favor of or against the W. In this scenario, which is illustrated in Fig. 3, we have an initial state q_s in which all the congress members could use their votes in favour or against the approval of W (no abstention or null vote is allowed). In this example, W will be considered as the state of affairs consisting of states q_{11}, q_{12}, and q_{13}. This multi-agent scenario can be modelled by the CGS $M = (N, Q, Act, d, o)$, where $N = \{1, ..., 11\}$ (the first ten agents are the voters in the congress followed by the president), $Q = \{q_s, q_0, ..., q_{13}\}$, $Act = \{0, 1, wait\}$, $d_i(q_s) = \{0, 1\}$ for all $i \in \{1, ..., 10\}$, $d_{11}(q_s) = \{wait\}$, $d_i(q) = \{wait\}$ for all $i \in \{1, ..., 10\}$ and $q \in \{q_0, ..., q_{13}\}$, $d_{11}(r) = \{wait\}$ for $r \in (\{q_0, q_1, q_2, q_4, q_6\} \cup \{q_8, ..., q_{13}\})$, and $d_{11}(t) = \{0, 1\}$ for $t \in \{q_3, q_5, q_7\}$. The outcome function o is illustrated in Fig. 3 where for example $o(q_s, (1, 0, 0, \star)) = q_4$ in which the war W will not take place because of the disapproval of the congress ($\star$ represents any available action). For notation convenience, actions of party members will be written collectively in the action profiles, e.g., we write $(0, 1, 0, \star)$ to denote the action profile $(0, 0, 1, 1, 1, 0, 0, 0, 0, 0, \star)$. Moreover, the simplifying assumption that all party members vote collectively is implemented by $o(q_s, \bar{\alpha}') = q_s$ for all possible action profiles $\bar{\alpha}'$ in which at least one party member acts independently.

The set of all weakly q_s-responsible groups $\mathbb{W}^W_{q_s}$ consists of two groups of GR and D. These two are the minimal groups with the preclusive power over W in q_s. If an anti-war campaign wants to negotiate and invest its limited resources in order to avoid the war W, convincing any of groups in $\mathbb{W}^W_{q_s}$, can avoid the war. However, it is observable that convincing the president is also adequate. Although the president has no preclusive power in q_s over W, there exist some accessible states from q_s (i.e., q_3, q_5, and q_7), in which P is responsible for the state of affairs. This potential capacity of P, will be addressed by means of the introduced notion of *functional degree of responsibility.* Two weakly q_s-responsible groups GR and D, have the functional degree of q_s-responsibility of 1 for W because they already have sufficient power to avoid W in source state q_s. Groups $\varnothing$, G, R, D, GD, RD, and GRD are not (weakly) q_s-responsible for W and no power acquisition sequence exists for these groups. Accordingly, their functional degree of q_s-responsibility for W is 0. Groups PG, PR, PD, PGR, PGD, PRD and $PGRD$, have the potentiality of possessing the preclusive power in other states, i.e., q_3, q_5, and q_7, but none of them will be minimal group with preclusive power over W. Note that minimality is a requirement for being a (weakly) responsible group [5]. Hence, the functional degree of q_s-responsibility for all these groups will be 0. The group which has a chance of becoming a (weakly) responsible group in states other than q_s (i.e., q_3, q_5, and q_7) is P. In fact, the President is the (unique) responsible group for W in states q_3, q_5, and q_7. As the minimum length of power acquisition sequence for P is 1, the functional degree of q_s-responsibility of P for W is 1/2. Although, P has no

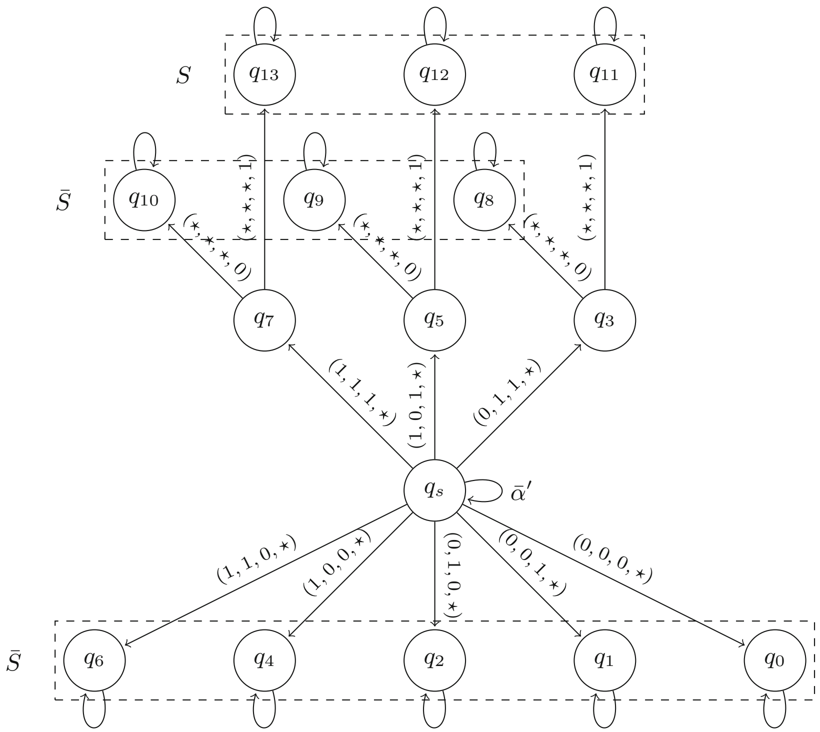

Fig. 3. War powers resolution

independent action profile to avoid W in q_s, there exists a *power acquisition sequence* for P through which P acquires the preclusive power over W.

The next proposition illustrates that through a shortest power acquisition sequence, the potentiality that the group is responsible for the state of affairs, increases strictly. This potential reaches its highest possible value where the group "really" has the preclusive power over the state of affairs as a (weakly) responsible group. Note that there is a one-to-one correspondence between any power acquisition sequence $P = \langle \bar{\alpha_1}, ..., \bar{\alpha_n} \rangle$ in q for a group C for S and the sequence of states $\langle q_1 = q, ..., q_{n+1} \rangle$ due to the deterministic nature of the action profiles $\bar{\alpha_i}$ for $1 \leq i \leq n$, i.e., $o(q_i, \bar{\alpha_i}) = q_{i+1}$ and $q = q_1$ and $q' = q_{n+1}$ and C is (weakly) q'-responsible for S. Hence, in the following, we write $P = \langle q_1, ..., q_{n+1} \rangle$ and interchangeably use it instead of $P = \langle \bar{\alpha_1}, ..., \bar{\alpha_n} \rangle$. Therefore, we simply refer to any state q_i as a state "in" the power acquisition sequence P.

Proposition 4 (Strictly increasing functionality). *Let $P = \langle q_1, ..., q_{n+1} \rangle$ ($n \geq 1$) be a power acquisition sequence in $q = q_1$ for a group C for S. Then, for any tuple of states (q_i, q_{i+1}), $1 \leq i \leq n$, $\mathcal{FDR}^S_{q_i}(C) < \mathcal{FDR}^S_{q_{i+1}}(C)$ iff P is a shortest power acquisition sequence in q for C for S.*

Proof. "$\Rightarrow$": Suppose the claim is false. Then, although the functional degree of responsibility of C for S is strictly increasing from q_1 to q_{n+1} in P, there exists a shorter power acquisition sequence $P' = \langle q'_1, ..., q'_{m+1} \rangle$ $(n > m \geq 0)$ in $q = q'_1$ for C for S. Note that as degrees are strictly increasing, for any states q_a and q_b in P $(q_a \neq q_b)$ we have that $\mathcal{FDR}^S_{q_a}(C) \neq \mathcal{FDR}^S_{q_b}(C)$. Both P and P' end in a state in which C is (weakly) responsible for S. Thus, for states q_{n+1} and q'_{m+1} we have that $\mathcal{FDR}^S_{q_{n+1} \in P}(C) = \mathcal{FDR}^S_{q'_{m+1} \in P'}(C) = 1$. If we trace back step by step through both sequences, the functional degree of responsibility of C for S is equal in corresponding states in P and P'. For example, for the states q_n and q'_m, we have that $\mathcal{FDR}^S_{q_n}(C) = \mathcal{FDR}^S_{q'_m}(C) = 1/2$ $(m \geq 1)$. By continuing the stepwise process of matching all states in P' with corresponding states in P, as number of states in P' is strictly less than P and both sequences start in same state of $q = q_1 = q'_1$, we reach the corresponding states q_{n+1-k} and q'_{m+1-k} for $0 \leq k \leq m$ where $\mathcal{FDR}^S_{q_{n+1-k}}(C) = \mathcal{FDR}^S_{q'_{m+1-k}}(C)$ and $q_{n+1-k} \neq q'_{m+1-k}$ and both states of q_{n+1-k} and q'_{m+1-k} are in P. This contradicts with the assumption that for any states q_a and q_b in P, if $q_a \neq q_b$, we have that $\mathcal{FDR}^S_{q_a}(C) \neq \mathcal{FDR}^S_{q_b}(C)$.

"$\Leftarrow$": Suppose the sequence P is a shortest power acquisition sequence in q for C for S. According to Definitions 3 and 5, the functional degree of q_i-responsibility of C for S must be formulated based on the sequence $P_i = \langle \bar{\alpha_i}, ..., \bar{\alpha_n} \rangle$ as a sub-sequence of P. Accordingly, length of P_i is equal to $\ell_i = n-i+1$. Hence, in each state q_{i+1}, the length of a shortest power acquisition sequence for C for S, ℓ_{i+1}, will be one unit shorter than ℓ_i. Finally, as $\ell \geq 0$, the functional degree of responsibility of C for S in each state q_{i+1} in P is strictly larger than in the state q_i in P. $\square$

The following propositions focus on the cases in which a group has partial degrees of functional and structural responsibility in a specific state. In former, we can reason about the degree of responsibility of the group in some states other than the current sate while in the latter, we can reason about the degree of responsibility of some other groups in the current state.

Proposition 5 (Global signalling of partial functional degree). *Let C be a group with functional degree of q-responsibility $1/k$ for S where k is a natural number. Then, it is guaranteed that there exists at least $k-1$ states $\hat{q}$ such that $\mathcal{FDR}^S_{\hat{q}}(C) > \mathcal{FDR}^S_q(C)$ and at least one state q' such that $\mathcal{FDR}^S_{q'}(C) = \mathcal{SDR}^S_{q'}(C) = 1$.*

Proof. According to Proposition 4, the functional degree of responsibility of C for S is strictly increasing during a shortest power acquisition sequence in q for C for S. This sequence passes $k-2$ states and reaches a state q'. Hence, the existence of at least $k-1$ states in which C has functional degree of responsibility larger than $1/k$ for S, and one state in which the functional and structural degree of responsibility of C is equal to 1 for S is guaranteed. $\square$

Note that based on Definition 5, the functional degree of responsibility could always be written in form of $1/k$ $(k \in \mathbb{N})$ unless it is equal to 0.

Proposition 6 (Local signalling of partial structural degree). *Let C be a group with structural degree of q-responsibility of k for S such that $0 < k < 1$. Then, there exists at least a group $\hat{C}$ with structural and functional degree of q-responsibility of* 1 *for S.*

Proof. Based on Definition 4, k is assigned to C based on its contribution in a (weakly) q-responsible group which has the structural and functional degree of q-responsibility of 1 for S. □

In general, the existence of a group $\hat{C}$ with the structural and the functional degree of q-responsibility of 1, could not guarantee the existence of a group with structural degree of q-responsibility of k such that $0 < k < 1$. As explained in Theorem 1, cases in which we have a singleton q-responsible group for S are counterexamples for such a claim.

6 Related Work

Presented notions for degree of group responsibility follow the responsibility notions in [5] and are in coherence with the concept of preclusive power in [9]. Our notion of *functional degree of responsibility* of an agent group is based on the minimum length of a sequence from a source state towards a state in which the agent group has power over a given state of affairs. This step-wise formulation was put forward by [1] in a quantified degree of responsibility as a backward-looking approach. However, [1] traces the steps in a causal network and studies the degree of causality, whereas we define our notions in strategic settings by means of a similar formulation. The other connection is to the [4] in which the notion of *avoidance potential* is central. There are two main differences between our approach and [4]. First, our notion of preclusion of a state of affairs is a property of a group, whereas in [4] the avoidance potential for a state of affairs is a property of a strategy of an individual agent. Second, the notion of preclusion in our case considers the power of a group while avoidance potential in [4] considers the probability of other agents to choose a strategy such that the strategy of the agent in question has no contribution to the establishment of the state of affairs.

As our degrees of group responsibility are based on quantifying the structural and functional potentials of agent groups in multi-agent systems, we would like to provide a brief comparison between our approach and the two well-known power indices, the Banzhaf index (with its related measure) [15], and the Shpley-Shubik index [16]. A main distinction is that both indices measure the power or contribution of individual agents in possible coalitions, rather than measuring the power of agent groups. The methodological difference between Banzhaf measure and our measure is that we formulate the degree of group responsibility based on the maximum contribution of an agent group to groups with preclusive power (structural degree) or minimum number of transitions that is necessary for a group to gain preclusive power (functional degree). This is different than the Banzhaf measure, where the main parameter is the probability that an agent would be the so called *swing player* with the ability to transform a "looser" group (of agents) to a "winner" group. Moreover, we focus on the ability of groups to preclude a state of affairs and base our notions on the potential of groups to *q-enforce* the complement of state of affairs (See Definition 2). This is different than "winner" groups in both the Banzhaf measure and the Shpley-Shubik index as they are the agent groups with the ability to determine the outcome which may be more related to the ability of agent groups to *q-enforce* a state of affairs (See *q-control* in [5]). Finally, in the Shpley-Shubik index, the order in which agents join a group plays an important role, which we believe is more relevant for the group/coalition formation process [17,18]. We stand before the group formation process, reason about all the possible agent groups, and assign them forward-looking degrees of group responsibility with respect to their potentials to avoid the materialization of a given state of affairs.

7 Conclusion and Future Work

In this paper, we proposed a forward-looking approach to measure the degree of group responsibility. The proposed notions can be used as a tool for analyzing the potential responsibility of agent groups towards a state of affairs. In our approach, full structural and functional degrees of responsibility towards a state of affairs are assigned to agent groups, if they can preclude the state of affairs. All other groups that may contribute to such responsible groups receive a partial structural degree of responsibility. Also, all other groups for which there exists a path to a state in which they possess the preclusive power receive a partial functional degree of responsibility. The structural degree of responsibility captures the responsibility of a group based on accumulated preclusive power of included agents while the functional degree of responsibility captures the responsibility of a group due to the potentiality of reaching a state in which it has the preclusive power.

We plan to apply our presented methodology for analyzing forward-looking responsibility to backward-looking responsibility. We believe that integrating the responsibility notions as proposed in [1,4] with our methodology could lead to a graded notion for backward-looking responsibility in strategic settings. In such extension, one could reason from a realized outcome state and assign a degree of blameworthiness to agent groups in liability determination principles from legal domain such as *contributory negligence*. In this paper we used concurrent game structure in its original form as we had a logical approach to formalize our two notions for degree of group responsibility. In an extended version, we plan to use probabilistic concurrent game structures to make our notions also applicable in probabilistic settings (See [19,20]). Finally, we aim at extending our framework with logical characterizations of the proposed notions based on the coalitional logic with quantification [5,21,22].

References

1. Chockler, H., Halpern, J.Y.: Responsibility and blame: a structural-model approach. J. Artif. Intell. Res. (JAIR) **22**, 93–115 (2004)
2. Grossi, D., Royakkers, L.M.M., Dignum, F.: Organizational structure and responsibility. Artif. Intell. Law **15**(3), 223–249 (2007)
3. Miller, S.: Collective moral responsibility: An individualist account. Midwest Stud. Philos. **30**(1), 176–193 (2006)
4. Braham, M., Van Hees, M.: An anatomy of moral responsibility. Mind **121**(483), 601–634 (2012)
5. Bulling, N., Dastani, M.: Coalitional responsibility in strategic settings. In: Leite, J., Son, T.C., Torroni, P., van der Torre, L., Woltran, S. (eds.) CLIMA XIV 2013. LNCS, vol. 8143, pp. 172–189. Springer, Heidelberg (2013)
6. van de Poel, I.: The relation between forward-looking and backward-looking responsibility. In: Vincent, N.A., van de Poel, I., van den Hoven, J. (eds.) Moral Responsibility. Library of Ethics and Applied Philosophy, vol. 27, pp. 37–52. Springer, Dordrecht (2011)
7. Sulzer, J.L.: Attribution of responsibility as a function of the structure, quality, and intensity of the event. Ph.D. thesis, University of Florida (1965)
8. Shaver, K.: The Attribution of Blame: Causality, Responsibility, and Blameworthiness. Springer, New York (2012)
9. Miller, N.R.: Power in game forms. In: Holler, M.J. (ed.) Power, Voting, and Voting Power, pp. 33–51. Springer, Würzburg (1982)
10. Alur, R., Henzinger, T.A., Kupferman, O.: Alternating-time temporal logic. J. ACM **49**(5), 672–713 (2002)
11. Pauly, M.: Logic for social software. Universiteit van Amsterdam (2001)
12. Kripke, S.A.: Semantical analysis of modal logic i normal modal propositional calculi. Math. Logic Q. **9**(5–6), 67–96 (1963)
13. Turnovec, F.: Monotonicity of power indices. In: Stewart, T., van den Honert, R. (eds.) Trends in Multicriteria Decision Making. Lecture Notes in Economics and Mathematical Systems, vol. 465, pp. 199–214. Springer, Heidelberg (1998)
14. Holler, M.J., Napel, S.: Monotonicity of power and power measures. Theor. Decis. **56**(1–2), 93–111 (2004)
15. Banzhaf III, J.F.: Weighted voting doesn't work: A mathematical analysis. Rutgers L. Rev. **19**, 317 (1964)
16. Shapley, L.S., Shubik, M.: A method for evaluating the distribution of power in a committee system. Am. Polit. Sci. Rev. **48**(03), 787–792 (1954)
17. Rahwan, T., Michalak, T.P., Elkind, E., Faliszewski, P., Sroka, J., Wooldridge, M., Jennings, N.R.: Constrained coalition formation. In: Burgard, W., Roth, D. (eds.) Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2011, San Francisco, California, USA, 7–11 August 2011. AAAI Press (2011)
18. Griffiths, N., Luck, M.: Coalition formation through motivation and trust. In: Proceedings of the Second International Joint Conference on Autonomous Agents and Multiagent Systems, AAMAS, 14–18 July 2003, Melbourne, Victoria, Australia, pp. 17–24. ACM (2003)
19. Chen, T., Lu, J.: Probabilistic alternating-time temporal logic and model checking algorithm. In: Lei, J. (ed.) Proceedings of the Fourth International Conference on Fuzzy Systems and Knowledge Discovery, FSKD 2007, 24–27 August 2007, Haikou, Hainan, China, vol. 2, pp. 35–39. IEEE Computer Society (2007)

20. Huang, X., Su, K., Zhang, C.: Probabilistic alternating-time temporal logic of incomplete information and synchronous perfect recall. In: Hoffmann, J., Selman, B. (eds.) Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence, 22–26 July 2012, Toronto, Ontario, Canada. AAAI Press (2012)
21. Ågotnes, T., van der Hoek, W., Wooldridge, M.: Quantified coalition logic. Synthese **165**(2), 269–294 (2008)
22. Pauly, M.: A modal logic for coalitional power in games. J. Log. Comput. **12**(1), 149–166 (2002)