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The Topos of Music III: Gestures

Musical Multiverse Ontologies

Second Edition

 Springer

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Preface to the Second Edition

*Comprendre, c'est
attraper le geste
et pouvoir continuer*
Jean Cavallès [181, p. 186]

A major reason for a second edition of *The Topos of Music*—besides the simple fact that the first edition is now sold out—goes back to spring 2002, when I was completing its first edition, published in fall 2002. I was asked to give a talk in the MaMuX seminar of the IRCAM in Paris, to explain how I applied the mathematics of *The Topos of Music* to my free jazz improvisations.

While preparing my talk I realized that despite the presence of mathematical music theory the decisive generator of my instant compositions was the gestural deployment of formulas, the “action painting” of musical thoughts, not the abstract formulas in their static facticity. First and foremost this was a shocking insight in view of the forthcoming publication of the formulaic setup in *The Topos of Music*.

Fortunately, I knew from Hermann Hesse that “every end is a beginning”¹, which meant in my case that the end of a scientific development as traced in the book’s first edition initiated the next step: a music theory of gestures. It goes without saying that this new phase would not destroy the previous research, but incorporate it as the stratum of facticity in an extended ontology of embodiment, where facts are the output of processes and their gestural generators.

In the sequel, I discovered that I was far from being the first scholar and artist to discover the crucial role of gestures in music. For instance, free jazz pianist Cecil Taylor, music philosopher Theodor Wieselgrund Adorno, or lateral thinker Paul Valéry had clearly stressed the dancing essence of art, an insight that I had embodied in my own pianist’s art, but never understood on an intellectual level.

Of course, I could not be satisfied by the very existence of gesture philosophy or gestural practice, just as I could not accept traditional music and performance theory when I started my enterprise of mathematical music theory in 1978. The gesturally colored thoughts and actions needed a rigorous conceptualization in the same vein as my efforts before the first edition of *The Topos of Music*.

In 2002, I was in the privileged position to work in the multimedia division of Peter Stucki at the Institut für Informatik of the University of Zurich. I had excellent PhD students, and we could, with one of them, Stefan Müller, realize a first experimental software for the gestural representation of a pianist’s hand, a work presented at the ICMC conference in 2003 [772].

This experimental preliminary work was then taken as a point of departure for a mathematical theory of musical gestures. I presented this theory in a course in spring 2005 at École normale supérieure in Paris, a course that was later in 2007 taken as the material basis of my French book *La vérité de beau dans la musique* [718]. The first publication of a formally more evolved mathematical theory of musical gestures was

¹ Actually, he says that “Jeder Anfang ist ein Ende,” but the reverse is immediate.

written with co-author Moreno Andreatta in 2007 [720]. This date could be called the birthday of a valid mathematical theory of musical gestures.

Until the publication of this second edition of *The Topos of Music*, several important conceptual extensions of the mathematical theory of musical gestures, models of musical gestural processes, as well as a number of theorems have been published. The decade from date of birth to the present proved that the mathematical theory of musical gestures is an important added value to the theory described in the first edition of *The Topos of Music*.

We can however not state that this theory of gestures is in a complete state, quite the opposite is true: The coming years will reveal important news of theoretical as well as practical nature. So why did we make the decision to publish the present state of the art? The first argument is that the present state is rich enough to define concrete new directions, be it in music theory, such as harmony or counterpoint, be it in performance theory, or be it in the understanding of embodiment in the making of music. The second argument is that the present material, roughly 500 pages of new material, is ample enough to present a book's stature. And the third and very important argument is that we would like to communicate the state of the art in the spirit of Cavaillès: *Understanding is catching the gesture and being able to continue*. The co-authors of the gesture theory part, René Guitart, Jocelyn Ho, Alex Lubet, Maria Mannone, Matt Rahaim, and Florian Thalmann, are a wonderful confirmation of this philosophy. So let us continue!

Here is a summary of the new material and its authorship. Whenever I don't mention the author, it is my own contribution, all others are mentioned explicitly.

Until Part XIV, nearly everything is as in the book's first edition, refer to the preface of that edition (also included in this edition) for detailed summaries. The only new content—besides errata corrections—is Chapter 45 in Part XI, which is a shortened version of a paper [110] on a statistical analysis of Chopin's *Prélude* op. 28, No. 4, written with Jan Beran, Robert Goswitz, and Patrizio Mazzola.

Gesture theory starts with Part XV: Gesture Philosophy for Music. Chapter 57 gives an overview of philosophical aspects of gestures, including works by Jean-Claude Schmitt, Vilém Flusser, Michel Guérin, Adam Kendon, David McNeill, Juhani Pallasmaa, André Chastel, Émile Benveniste, Marie-Dominique Popelard, and Anthony Wall. In Chapter 58, we discuss the presemiotic approach to gestures in the French perspective of Maurice Merleau-Ponty, Gilles Deleuze, Jean Cavaillès, Charles Alunni, and Gilles Châtelet. Paul Valéry is also referenced in Section 59.4.

Chapter 59 deals with gestural aspects in cognitive science. After a review of Embodied AI and anthropology, Alex Lubet in Section 59.5 introduces us to gestural disability studies, focusing on two famous disabled pianists: Horace Parlan and Oscar Peterson (in his last years). Then in Section 59.6 Lubet reflects on perception of musical gesture as being inherently synaesthetic.

Chapter 60 concludes this part with a review of musical models of gesturality as proposed by Wolfgang Graeser, Theodor W. Adorno, Neil P. McAngus Todd, David Lewin, Robert Hatten, Marcelo Wanderley, Claude Cadoz, and Marc Leman.

Part XVI introduces the mathematics of gestures. Chapter 61 presents the mathematical concept of a gesture in a topological space and states the Diamond Conjecture, which deals with a hypothetical big space that unites algebraic and topological categories. Chapter 62 extends the theory from gestures in topological spaces to gestures in topological categories and introduces functorial gestures, i.e., functors on topological categories with values in the category of gestures, similar to functorial compositions in the previous theory.

Chapter 63 presents a generalized singular homology, where cubes are replaced by general hypergestures. Hypergesture homology applies to a gestural model of counterpoint and to a gestural refinement of performance stemma theory.

Chapter 64 presents—similar to Chapter 63—Stokes' Theorem for hypergestures. This theorem applies to problems in gestural modulation theory.

Chapters 65 and 66 discuss categories of local and global compositions, processes/networks, and gestures, together with their functorial relationships. This triple typology composition/process/gesture corresponds to the ontological dimension of embodiment with its three coordinates facts/processes/gestures.

In Sections 67.1–67.7 of Chapter 67, René Guitart develops a fascinating and demanding mathematical model of mathematical creativity, where thought is viewed as an algebra of gestures.

In Section 61.14, we, Maria Mannone and Guerino Mazzola, present a group-theoretical model of Georg Wilhelm Friedrich Hegel's initial discourse in his *Wissenschaft der Logik*, a model that applies to the Yoneda concept of creativity [726, Chapter 19.2]. We illustrate the method with an experimental composition by Mannone. This discussion extends over Sections 67.8–67.15 and completes Chapter 67.

Part XVII deals with concept architectures and software for musical gesture theory. Chapter 68 explains the denotator formalism for gestures over topological categories. Chapter 69 is a summary of the Java-based RUBATO[®] Composer software [739], written to sketch the framework of Chapters 70–74, where Florian Thalmann presents his gesture-oriented software component, the BigBang rubette. This discourse again follows the coordinates of the dimension of embodiment: facts, processes, gestures, which in this situation specify to: visualization and sonification of denotators (Chapter 71), BigBang's operation graph (Chapter 72), and gestural interaction and gesturalization (Chapter 73). In the final Chapter 74 of this part, Thalmann discusses musical examples.

Part XVIII is entitled *The Multiverse Perspective* because it opens up the relationship of gesture theory with string theory in theoretical physics. After a critical review of Hermann Hesse's *Glasperlenspiel* with regard to its gestural deficiencies, we, Mazzola and Mannone, develop the Euler-Lagrange formalism of world-sheets for musical gestures. This theory extends to functorial global gestures over global topological categories.

Part XIX is dedicated to applications of gesture theory to a number of musical themes.

Chapter 79 deals with singular gesture homology being applied to counterpoint.

Chapter 80 introduces a gestural restatement of modulation theory, applying in particular Stokes' Theorem for hypergestures.

Chapter 81 applies gesture theory to a gestural performance stemma theory.

Chapter 82 is written by Jocelyn Ho as a creative presentation of composition and analysis as embodied gestures in an inter-corporeal world. She presents two compositions, Toru Takemitsu's *Rain Tree Sketch II* and her own composition *Sheng* for piano, smartphones, and fixed playback.

Chapter 83 is Mannone's analysis and classification of a conductor's movements from the viewpoint of gesture theory.

Chapter 84 is a review of gestural aspects that were developed in *Flow, Gesture, and Spaces in Free Jazz* [721].

Chapter 85 is written by Matt Rahaim and presents the gestural approach to understanding Hindustani music in its vocal gesturality.

Chapter 86 is a first approach, written by Mannone, to a future theory of vocal gestures. The short addendum was written by Mazzola.

The Appendix has been enriched by additional complements on mathematics (Chapter J) plus complements on physics (Chapter K).

The Leitfaden III has been added to the original Leitfaden I & II for the gestural chapters.

The ToM_CD has been updated, containing now the present book's pdf *ToposOfMusic.pdf*. However the original CD is no longer added to the book, instead the ToM_CD can be downloaded from

www.encycloSPACE.org/special/ToM_CD.zip.

Concerning the division of the now very large book into parts, this is the split:

- Volume I: *Theory*, Prefaces and Table of Contents, Parts I to VII
- Volume II: *Performance*, Parts VIII to XIV
- Volume III: *Gestures*, Parts XV to XIX
- Volume IV: *Roots*, Appendices

My sincere acknowledgments go to my co-authors and to Springer's Ronan Nugent and Frank Holzwarth as well as to Birkhäuser's Thomas Hempfling.

Preface

*Man kann
einen jeden Begriff,
einen jeden Titel,
darunter viele Erkenntnisse gehören,
einen logischen Ort nennen.*
Immanuel Kant [519, p. B 324]

This book’s title subject, *The Topos of Music*, has been chosen to communicate a double message: First, the Greek word “topos” ($\tau\acute{o}\pi\omicron\varsigma$ = location, site) alludes to the logical and transcendental location of the concept of music in the sense of Aristotle’s [40, 1154] and Kant’s [519, p. B 324] topic. This view deals with the question of *where music is situated as a concept*—and hence with the underlying ontological problem: *What is the type of being and existence of music?* The second message is a more technical understanding insofar as the system of musical signs can be associated with the mathematical theory of *topoi*, which realizes a powerful synthesis of geometric and logical theories. It laid the foundation of a thorough geometrization of logic and has been successful in central issues of algebraic geometry (Grothendieck, Deligne), independence proofs and intuitionistic logic (Cohen, Lawvere, Kripke).

But this second message is intimately entwined with the first since the present concept framework of the musical sign system is technically based on topos theory, so the topos of music receives its topos-theoretic foundation. In this perspective, the double message of the book’s title in fact condenses to a unified intention: to unite philosophical insight with mathematical explicitness.

According to Birkhäuser’s initial plan in 1996, this book was first conceived as an English translation of my former book *Geometrie der Töne* [682], since the German original had suffered from its restricted access to the international public. However, the scientific progress since 1989, when it was written, has been considerable in theory and technology. We have known new subjects, such as the denotator concept framework, performance theory, and new software platforms for composition, analysis, and performance, such as RUBATO® or OpenMusic. Modeling concepts via the denotator approach in fact results from an intense collaboration of mathematicians and computer scientists in the object-oriented programming paradigm and supported by several international research grants.

Also, the scientific acceptance of mathematical music theory has grown since its beginnings in the late 1970s. As the first acceptance of mathematical music theory was testified to by von Karajan’s legendary Ostertagsymposium “Musik und Mathematik” in 1984 in Salzburg [383], so is the significantly improved present status of acceptance testified to by the Fourth Diderot Forum on Mathematics and Music [711] in Paris, Vienna, and Lisbon 1999, which was organized by the European Mathematical Society. The corresponding extension of collaborative efforts in particular entail the inclusion of works by other research groups in this

book, such as the “American Set Theory”, the Swedish school of performance research at Stockholm’s KTH, or the research on computer-aided composition at the IRCAM in Paris.

Therefore, as a result of these revised conditions, *The Topos of Music* appears as a vastly extended English update of the original work. The extension is visibly traced in the following parts which are new with respect to [682]: Part II exposes the theory of denotators and forms, part V introduces the topological theories of rhythms and motives, part VIII introduces the structure theory of performance, part IX deals with the expressive semantics of performance in the language of performance operators and stemmata (genealogical trees of successively refined performance), part X is devoted to the description of the RUBATO[®] software platform for representation, analysis, composition, and performance, part XI presents a statistical analysis of musical analysis, part XII concludes the subject of performance with an inverse performance theory, in fact a first formalization of the problem of music criticism.

This does however not mean that the other parts are just translations of the German text. Considerable progress has been made in most fields, except the last part XIV which reproduces the status quo in [682]. In particular, the local and global theories have been thoroughly functorialized and thereby introduce an ontological depth and variability of concepts, techniques, and results, which by far transcend the semiotically naive geometric approach in [682]. The present theory is as different from the traditional geometric conceptualization as is Grothendieck’s topos theoretic algebraic geometry from classical algebraic geometry in the spirit of Segre, van der Waerden, or Zariski.

Beyond this topos-theoretic generalization, the denotator language also introduces a fairly exceptional technique of circular concept constructions. This more precisely is rooted in Finsler’s pioneering work in foundations of set theory [322], a thread which has been rediscovered in modern theoretical computer sciences [5]. The present state of denotator theory rightly could be termed a Galois theory of concepts in the sense that circular definitions of concepts play the role of conceptual equations (corresponding to algebraic equations in algebraic Galois theory), the solutions of which are concepts instead of algebraic numbers.

Accordingly, the mathematical apparatus has been vastly extended, not only in the field of topos theory and its intuitionistic logic, but also with regard to general and algebraic topology, ordinary and partial differential equations, Pólya theory, statistics, multiaffine algebra and functorial algebraic geometry. It is mandatory that these technicalities had to be placed in a more elaborate semiotic perspective. However, this book does not cover the full range of music semiotics, for which the reader is referred to [703]. Of course, such an extension on the technical level has consequences for the readability of the theory. In view of the present volume of over 1300 pages, we could however not even make the attempt to approach a non-technical presentation. This subject is left to subsequent efforts. The critical reader may put the question whether music is really that complex. The answer is yes, and the reason is straightforward: We cannot pretend that Bach, Haydn, Mozart, or Beethoven, just to name some of the most prominent composers, are outstanding geniuses and have elaborated masterworks of eternal value, without trying to understand such singular creations with adequate tools, and this means: of adequate depth and power. After all, understanding God’s ‘composition’, the material universe, cannot be approached without the most sophisticated tools as they have been elaborated in physics, chemistry, and molecular biology.

So who is recommended to read this book? A first category of readers is evidently the working scientist in the fields of mathematical music theory, the soft- and hardware engineer in music informatics, but also the mathematician who is interested in new applications from the above fields of pure mathematics. A second category are those theoretical mathematicians or computer scientists interested in the Galois theory of concepts; they may discover interesting unsolved problems. A third category of potential readers are all those who really want to get an idea of what music is about, of how one may conceptualize and turn into language the “ineffable” in music for the common language. Those who insist on the dogma that precision and beauty contradict each other, and that mathematics only produces tautologies and therefore must fail when aiming at substantial knowledge, should not read such a book.

Despite the technical character of *The Topos of Music*, there are at least four different approaches to its reading. To begin with, one may read it as a philosophical text, concentrating on the qualitative passages, surfing over technical portions and leaving those paragraphs to others. One may also take the book as a dictionary for computational musicology, including its concept framework and the lists of musical objects

and processes (such as modulation degrees, contrapuntal steps) in the appendices. Observe however, that not all existing important lists have been included. For example, the list of all-interval series and the list of self-addressed chords are omitted, the reader may find these lists in other publications. Thirdly, the working scientist will have to read the full-fledged technicalities. And last, but not least, one may take the book as a source for ideas of how to go on with the whole subject of music. The GPL (General Public License²) software sources in the appended CD-ROM may support further development.

The prerequisites to a more in-depth reading of this book are these. Generally speaking, a good acquaintance with formal reasoning as mathematics (including formal logic) preconizes, is a *conditio sine qua non*. As to musicology and music theory, the familiarity with elementary concepts, like chords, motives, rhythm, and also musical notation, as well as a real interest in understanding music and not simply (ab)using it, are recommended. For the more computer-oriented passages, familiarity with the paradigm of object-oriented programming is profitable. We have not included the appendix on mathematical basics because it should help the reader get familiar with mathematics, but as an orientation in fields where the specialized mathematician possibly needs a specification of concepts and notation. The appendix was also included to expose the spectrum of mathematics which is needed to tackle the formal problems of computational musicology. It is by no means an overkill of mathematization: We have even omitted some non-trivial fields, such as statistics or Lambda calculus, for which we have to apologize.

There are different supporting instances to facilitate orientation in this book. To begin with, the table of contents and an extensive subject and name index may help find one's key-words. Further, following the list of contents, a *leitfaden* (on page xlix) is included for a generic navigation. Each chapter and section is headed by a summary that offers a first orientation about specific contents. Finally, the book is also available as a file `ToposOfMusic.pdf` with bookmarks and active cross-references in the appended CD-ROM (see page li for its contents). This version is also attractive because the figures' colors are visible only in this version.

In order to obtain a consistent first reading, we recommend chapters 1 to 5, and then appendix A: Common Parameter Spaces (appendix B is not mandatory here, though it gives a good and not so technical overview of auditory physiology). After that, the reader may go on with chapter 6 on denotators and then follow the outline of the *leitfaden* (see page xlix).

This book could not have been realized without the engaged support of nineteen collaborators and contributors. Above all, my PhD students Stefan Göller and Stefan Müller at the MultiMedia Laboratory of the Department of Information Technology at the University of Zurich have collaborated in the production of this book on the levels of the $\text{\LaTeX}$ installation, the final production of hundreds of figures, and the contributions sections 20.2 through 20.5 (Göller) and sections 47.3 through 47.3.6.2 (Müller). My special gratitude goes to their truly collaborative spirit.

Contributions to this book have been delivered by (in alphabetic order): By Carlos Agon, and Gérard Assayag (both IRCAM) with their precious Lambda-calculus-oriented presentation of the object-oriented programming principles in the composition software OpenMusic described in chapter 52, Moreno Andreatta (IRCAM) with an elucidating discourse on the American Set Theory in section 11.5.2 and section 16.3, Jan Beran (Universität Konstanz) with his contribution to the compositional strategies in his original composition [103] in section 11.5.1.1, as well as with his inspiring work on statistics as reported in chapters 43 and 44, Chantal Buteau (Universität and ETH Zürich) with her detailed review of chapter 22, Roberto Ferretti (ETH Zürich) with his progressive contributions to the algebraic geometry of inverse performance theory in sections 39.8 and 47.2, Anja Fleischer (Technische Universität Berlin) with her short but critical preliminaries in chapter 23, Harald Friepertinger (Universität Graz) with his 'killer' formulas concerning enumeration of finite local and global compositions in sections 11.4, 16.2.2 and appendix C.3.6, Jörg Garbers (Technische Universität Berlin) with his portation of the RUBATO[®] application to Mac OS X, as documented in the screenshots in chapters 40, 41, Werner Hemmert (Infineon) with a very up-to-date presentation of room acoustics in section A.1.1.1 and auditory physiology in appendix B.1 (we would have loved to include more of his knowledge), Michael Leyton (DIMACS, Rutgers University) with a formidable cover figure entitled "Dark Theory", a beautiful subtitle to this book, as well as with innumerable discussions around time and its reduction to symmetries as presented in chapter 48, Emilio Lluís Puebla (UNAM, Mexico City)

² A legal matter file is contained in the book's CD-ROM, see page li.

with his unique and engaged promotion and dissipation of mathematical music theory on the American continent, especially also in the preparation and critical review of this book, Mariana Montiel Hernandez (UNAM, Mexico City) with her critical review of the theory of circular forms and denotators in section 6.5 and appendix G.2.2.1, Thomas Noll (Technische Universität Berlin) with his substantial contributions to the functorial theory of compositions, and for his revolutionary rebuilding of Riemann's harmony and its relations to counterpoint, Joachim Stange-Elbe (Universität Osnabrück) with a very clear and innovative description of his outstanding RUBATO[®] performance of Bach's contrapunctus III in the *Art of Fugue* in sections 42.2 through 42.4.3, Hans Straub with his adventurous extensions of classical cadence theory in section 26.2.2 and his classification of four-element motives in appendix O.4, and, last but not least, Oliver Zahorka (Out Media Design), my former collaborator and chief programmer of the NeXT RUBATO[®] application, which has contributed so much to the success of the Zürich school of performance theory. To all of them, I owe my deepest gratitude and recognition for their sweat and tears.

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Vulpera, June 2002

Guerino Mazzola

Volume III Contents

Part XV Gesture Philosophy for Music

56	The Topos of Gestures	843
57	Gesture Philosophy: Phenomenology, Ontology, and Semiotics	845
57.1	A Short Recapitulation of Musical Ontology	845
57.1.1	Ontology: Where, Why, and How	845
57.1.2	Ontology: Facts, Processes, and Gestures	846
57.2	Jean-Claude Schmitt's Historiographic and Philosophical Treatise "La raison des gestes dans l'Occident médiéval"	846
57.2.1	Comments	847
57.3	Vilém Flusser's <i>Gesten: Versuch einer Phänomenologie</i>	848
57.3.1	A Short Introduction to Flusser's Essay	848
57.3.2	The Semiotic Neurosis	848
57.4	Michel Guérin's <i>philosophie des gestes</i>	850
57.4.1	The Essay's Structure	850
57.4.2	Gestural Ontology and Four Elementary Gestures	850
57.5	Flusser and Guérin: Some Consequences	851
57.6	A Program	852
57.6.1	Circularity	852
57.7	The Semiotic Gesture Concept of Adam Kendon and David McNeill	853
57.7.1	Comments	855
57.8	Juhani Pallasmaa and André Chastel: The Thinking Hand in Architecture and the Arts	855
57.9	Benveniste, Popelard, Wall	856
58	The French Presemiotic Approach	859
58.1	Maurice Merleau-Ponty	860
58.2	Francis Bacon and Gilles Deleuze	860
58.3	Jean Cavallès and Charles Alunni	861
58.4	Gilles Châtelet	862
59	Cognitive Science	867
59.1	Embodiment	867
59.1.1	Embodiment Science	868
59.2	Neuroscience	871
59.2.1	Embodied AI	873

59.3	Anthropology	874
59.4	Dance	874
59.5	Disabled Gestures Versus Gestures Disabled: Parlan's Versus Peterson's Pianism	875
59.5.1	Performative Gestures: Disabled Jazz Pianists	876
59.5.2	Horace Parlan: Disabled Gestures	877
59.5.3	Parlan with Bass (and Drums)	877
59.5.4	Parlan with Rhythm Section	878
59.5.5	Parlan as Soloist	879
59.5.6	Parlan's Duets with Archie Shepp	880
59.5.7	Disabled Gestures	881
59.5.8	Gestures Disabled: Oscar Peterson	882
59.5.9	Conclusion	884
59.6	Aristotle, Blind Lemon Jefferson, and Vilayanur S. Ramachandran Walk into a Bar	885
59.6.1	Introduction	885
59.6.2	Division by (Almost) Zero: Many Blind Bluesmen but Few Blind Blues	885
59.6.3	Seeing Blind Blues: Gesture, Flow, Circuitry, and Amplification	886
59.6.4	Epilogue: Puns as Gestures	887
60	Models from Music	889
60.1	Wolfgang Graeser	890
60.2	Adorno, Wieland, Sessions, Clynes	890
60.2.1	Theodor Wiesengrund Adorno	891
60.2.2	Renate Wieland	893
60.2.3	Roger Sessions	894
60.2.4	Manfred Clynes	895
60.3	Johan Sundberg and Neil P. McAngus Todd	896
60.4	David Lewin and Robert S. Hatten	898
60.5	Marcelo Wanderley and Claude Cadoz, Rolf Inge Godøy and Marc Leman	901

Part XVI Mathematics of Gestures

61	Fundamental Concepts and Associated Categories	907
61.1	Introduction	907
61.2	Towards a Musical String Theory	909
61.3	Initial Investigations: Diagrams of Curves	910
61.4	Modeling a Pianist's Hand	912
61.4.1	The Hand's Model	912
61.4.2	Transforming Abstract Note Symbols into Symbolic Gestures	912
61.4.3	From Symbolic Hand Gestures to Physical Gestures	913
61.5	The Mathematical Definition of Gestures	914
61.6	Hypergestures	915
61.6.1	Spatial Hypergestures	917
61.7	Categorically Natural Gestures	918
61.8	Connecting to Algebraic Topology: Hypergestures Generalize Homotopy	919
61.9	Gestoids	922
61.9.1	The Fundamental Group, Klumpenhouwer Networks, and Fourier Representation	924
61.10	Gabriel's Spectroids and Natural Formulas	925
61.10.1	Solutions of Representations of Natural Formulas by Local Networks	927

61.11	The Tangent Category	927
61.12	The Diamond Conjecture	929
61.13	Topos Logic for Gestures	930
61.14	The Escher Theorem for Hypergestures	931
61.14.1	Hypergestures and the Escher Theorem for Fux Counterpoint	931
61.14.2	Rebecca Lazier's Vanish: Lawvere, Escher, Schoenberg	933
62	Categories of Gestures over Topological Categories	937
62.1	Gestures over Topological Categories	939
62.1.1	The Categorical Digraph of a Topological Category	940
62.1.2	Gestures with Body in a Topological Category	940
62.1.3	Varying the Underlying Topological Category	942
62.2	From Morphisms to Gestures	942
62.2.1	Diagrams as Gestures	944
62.2.2	Gestures in Factorization Categories	944
62.2.3	Extensions from Homological Algebra Are Gestures	945
62.2.4	The Bicategory of Gestures	945
62.2.5	Entering the Diamond Space	946
62.3	Diagrams in Topological Groups for Gestures	947
62.4	Gestural Interpretation of Modulations in Beethoven's op.106/Allegro	950
62.4.1	Recapitulation of the Results from Section 28.2	951
62.4.2	The Modulation B_b -major $\rightsquigarrow$ G -major Between Measure 31 and Measure 44	952
62.4.3	Lewin's Characteristic Gestures Identified?	955
62.4.4	Modulation E_b -major $\rightsquigarrow$ D -major/ B -minor from W to W^*	957
62.4.5	The Fanfare	958
62.5	Conclusion for the Categorical Gesture Approach	960
62.6	Functorial Gestures: General Addresses	961
62.7	Yoneda's Lemma for Gestures	962
62.8	Examples from Music	963
62.8.1	Collections of Acoustical Waves	963
62.8.2	Collections of Spectral Music Data	964
62.8.3	MIDI-Type ON-OFF Transformations	964
63	Singular Homology of Hypergestures	965
63.1	An Introductory Example	965
63.2	Chain Modules for Singular Hypergestural Homology	967
63.3	The Boundary Homomorphism	968
64	Stokes' Theorem for Hypergestures	973
64.1	The Need for Stokes' Theorem for Hypergestures	973
64.2	Almost Regular Manifolds, Differential Forms, and Integration for Hypergestures	973
64.2.1	Locally Almost Regular Manifolds	974
64.2.2	Differential Forms	975
64.2.3	Integration	975
64.3	Stokes' Theorem	976
65	Local Facts, Processes, and Gestures	979
65.1	Categories of Local Compositions	979
65.2	Categories of Local Networks	980
65.3	Categories of Local Gestures	982
65.3.1	Local Gestures on Topological Categories of Points	982

65.4	Connecting Functors	984
65.5	Hypernetworks and Hypergestures	985
65.5.1	Escher Theorems	985
65.6	Singular Homology of Hypernetworks and Hypergestures	985
66	Global Categories	987
66.1	Categories of Global Compositions	987
66.1.1	Simplicial Methods	988
66.2	Classification of Global Compositions	989
66.3	Non-interpretable Global Compositions	990
66.4	Categories of Global Networks	990
66.4.1	Non-interpretable Global Networks	991
66.5	Categories of Global Gestures	993
66.6	Globalizing Topological Categories: Categorical Manifolds	993
66.7	Globalizing Skeleta	996
66.8	Functorial Global Gestures	998
67	Mathematical Models of Creativity	1001
67.1	Forewarning: Invention of Gestures in Mathematics	1001
67.1.1	Thinking Exactness, Like a Rolling Mind	1001
67.1.2	Thought as an Algebra of Gestures	1002
67.2	Method and Objects, Summarily Explained: I—Preamble	1003
67.2.1	Prelude to a Discourse of a Method: “Caminos”, “Aletheia”, Irreverence	1003
67.2.2	Our Posture	1015
67.3	Method and Objects, Summarily Explained: II—Data	1021
67.3.1	Simple Objects, Structures and Invariants in Mathematics	1021
67.3.2	Complete Frameworks, Computations and Representations	1029
67.4	Creativity in Mathematics: Gestures in Historical Contexts	1031
67.4.1	Creativity: Phenomenology, Psychology and Skills, and Life	1031
67.4.2	Determination of Mathematics as a History of Its Gestures	1036
67.4.3	Invention in the Art of Mathematics	1044
67.5	On the Mathematical Invention of Coordinations	1046
67.5.1	Emergence of Coordinations	1047
67.5.2	Arrows	1050
67.5.3	Bodies, Implicit Surfaces, Abstract Relations	1055
67.5.4	Sketches	1056
67.6	Pulsation in the Living Process of Invention Among Shapes	1058
67.6.1	Productions: Objects and Relations, Problems, Pulsation	1058
67.6.2	Creativity in the Mathematical World Seen as a Living System of Shapes, in a Categorical Framework	1060
67.7	Conclusion: Categorical Presentation of Pulsations	1064
67.8	The Hegel Group Action on a Critical Concept’s Walls	1066
67.9	Introduction	1066
67.10	The Hegel Concept Group $\mathcal{G}$	1067
67.10.1	Hegel’s Initial Thought Movement in <i>Wissenschaft der Logik</i>	1067
67.10.2	The Implicit Group Structure	1070
67.10.3	The Conceptual Box Structure	1072
67.11	The $\mathcal{G}$ Action on the Yoneda Model of Creativity	1073
67.12	The Hegel Body $\mathcal{B}$ in the Concept Architecture of Forms and Denotators	1073
67.13	The Usage of $\mathcal{G}$ for the Dynamics of Creativity	1074
67.13.1	Two Preliminary Examples	1074
67.13.2	The Challenge: Creating a Spectrum of Conceptual Extensions	1075

67.13.3	Escher's Theorem for Beethoven's Fanfare in the " <i>Hammerklavier</i> " Sonata op. 106	1075
67.13.4	The Rotation $S@N$ as a Driving Creative Force in the Incipit of Liszt's <i>Mephisto</i> Walzer No.1	1076
67.14	An Experimental Composition	1078
67.15	Still More Symmetries? Future Developments	1080

Part XVII Concept Architectures and Software for Gesture Theory

68	Forms and Denotators over Topological Categories	1085
68.1	The General Topos—Theoretical Framework	1085
68.1.1	The category TopCat of Small Topological Categories	1085
68.2	Forms and Denotators	1086
68.3	Mathematics of Objects, Structures, and Concepts	1087
68.4	Galois Theory of Concepts	1087
68.4.1	Introduction	1088
68.4.2	Form Semiotics	1089
68.4.3	The Category of Form Semiotics	1092
68.4.4	Galois Correspondence of Form Semiotics	1093
69	The Rubato Composer Architecture	1095
69.1	The Software Architecture	1096
69.2	The Rubette World	1097
69.2.1	Rubettes for Counterpoint	1097
69.2.2	Rubettes for Harmony	1098
69.2.3	MetroRubettes	1099
70	The BigBang Rubette and the Ontological Dimension of Embodiment	1101
71	Facts: Denotators and Their Visualization and Sonification	1103
71.1	Some Earlier Visualizations of Denotators	1103
71.1.1	Göller's PrimaVista Browser	1103
71.1.2	Milmeister's ScorePlay and Select2D Rubettes	1105
71.2	An Early Score-Based Version of BigBang	1106
71.2.1	The Early BigBang Rubette's View Configurations	1107
71.2.2	Navigating Denotators	1111
71.2.3	Sonifying Score-Based Denotators	1111
71.3	BigBangObjects and Visualization of Arbitrary $Mod^{\circledast}$ Denotators	1111
71.3.1	A Look at Potential Visual Characteristics of Form Types	1112
71.3.2	From a General View Concept to BigBang Objects	1114
71.3.3	New Visual Dimensions	1115
71.4	The Sonification of BigBangObjects	1116
71.5	Examples of Forms and the Visualization of Their Denotators	1117
71.5.1	Some Set-Theoretical Structures	1117
71.5.2	Tonal and Transformational Theory	1119
71.5.3	Synthesizers and Sound Design	1121
72	Processes: BigBang's Operation Graph	1127
72.1	Temporal BigBangObjects, Object Selection, and Layers	1128
72.1.1	Selecting None and Lewin's Transformation Graphs	1128
72.1.2	The Temporal Existence of BigBangObjects	1129
72.1.3	BigBang Layers	1131

72.2	Operations and Transformations in BigBang	1132
72.2.1	Non-transformational Operations	1132
72.2.2	Transformations	1136
72.3	BigBang's Process View	1138
72.3.1	Visualization of Processes	1138
72.3.2	Selecting States and Modifying Operations	1139
72.3.3	Alternative and Parallel Processes	1139
72.3.4	Structurally Modifying the Graph	1141
72.3.5	Undo/Redo	1141
73	Gestures: Gestural Interaction and Gesturalization	1143
73.1	Formalizing: From Gestures to Operations	1143
73.1.1	Modes, Gestural Operations, and the Mouse	1144
73.1.2	Affine Transformations and Multi-touch	1148
73.1.3	Dynamic Motives, Sound Synthesis, and Leap Motion	1149
73.1.4	Recording, Modifying Operations and MIDI Controllers	1151
73.2	Gesturalizing and the Real BigBang: Animated Composition History	1152
73.2.1	Gesturalizing Transformations	1152
73.2.2	Gesturalizing Other Operations	1154
73.2.3	Using Gesturalization as a Compositional Tool	1154
74	Musical Examples	1157
74.1	Some Example Compositions	1157
74.1.1	Transforming an Existing Composition	1157
74.1.2	Gesturalizing and Looping with a Simple Graph	1158
74.1.3	Drawing UPIC-like Motives and Transforming	1159
74.1.4	Drawing Time-Slices	1160
74.1.5	Converting Forms, Tricks for Gesturalizing	1160
74.1.6	Gesturalizing a Spectrum	1162
74.1.7	Using Wallpapers to Create Rhythmical Structures	1163
74.2	Improvisation and Performance with BigBang	1163
74.2.1	Improvising by Selecting States and Modifying Transformations	1164
74.2.2	Playing Sounds with a MIDI Keyboard and Modifying Them	1164
74.2.3	Playing a MIDI Grand Piano with Leap Motion	1165
74.2.4	Playing a MIDI Grand Piano with the Ableton Push	1166
74.2.5	Improvising with 12-Tone Rows	1167

Part XVIII The Multiverse Perspective

75	Gesture Theory and String Theory	1173
76	Physical and Musical Multiverses	1175
77	Hesse's Melting Beads: A Multiverse Game with Strings and Gestures	1177
77.1	Review of Hesse's Glass Bead Game	1177
77.2	Frozen Glass Beads of Facticity	1178
77.3	The Revolution of Functors	1178
77.4	Gestures in Philosophy and Science	1180
77.5	Gesture Theory in Music	1181
77.6	A Remark on Gestural Creativity	1183
77.7	Gestures and Strings	1183
77.8	Playing the Multiversed Game in a Pre-semiotic Ontology	1184

78 Euler-Lagrange Equations for Hypergestures	1185
78.1 The Problem in Performance Theory with the Physical Nambu-Goto Lagrangian	1185
78.1.1 Complex Time and Descartes' Dualistic Ontology	1186
78.2 Lagrangian Density for Complex Time	1187
78.2.1 The Lagrangian Action for Performance	1188
78.2.2 The World-Sheet of Complex Time	1190
78.2.3 The Space for a Hand's Gestures	1192
78.2.4 The World-Sheet for a Simple Case	1192
78.2.5 The Elementary Gesture of a Pianist	1192
78.2.6 The Overarching Framework Between Note Performance and Gesture Performance	1195
78.2.7 Examples of Functional Relations Between Potential and Physical Gesture	1196
78.2.8 Calculus of Variations for the Physical Gesture	1202
78.2.9 A First Solution. World-Sheet Potentials Determine a Pianist's Gesture: Calculus of Variations and Fourier Analysis	1203
78.2.10 The Calculus with Vanishing Potential	1204
78.2.11 The Calculus with General Potential	1209
78.2.12 Solution of the Differential Equation Using 2D Fourier Series	1212
78.2.13 Parallels Between Performance Operators for Scores and for Gestures	1216
78.2.14 Complex Time and the Artistic Effort	1218
78.2.15 Opening the Aesthetic Question that Is Quantified in Lagrange Potentials	1218
78.2.16 A Musical Composition by Maria Mannone Realized Using These Ideas	1219
78.3 Global Performance Hypergestures	1223
78.3.1 The Musical Situation: An Intuitive Introduction	1223
78.4 Categorical Gestures and Global Performance Hypergestures	1224
78.4.1 Categorical Gestures: The Case of Potentials	1224
78.4.2 The Mathematics of Global Performance Hypergestures	1226
78.5 World-Sheet Hypergestures for General Skeleta	1227
78.6 A Global Variational Principle for the Lagrange Formalism	1231

Part XIX Gestures in Music and Performance Theory, and in Ethnomusicology

79 Gesture Homology for Counterpoint	1235
79.1 Summary of Mathematical Theory of Counterpoint: What It Is About and What Is Missing	1235
79.2 Hypergestural Singular Homology	1236
79.3 A Classical Example of a Topological Category from Counterpoint	1237
79.3.1 Generators of $H_1({}^G X)$ for a Groupoid ${}^G X$ Defined by a Group Action	1238
79.4 The Meaning of H_1 for Counterpoint	1240
79.5 Concluding Comments	1241
80 Modulation Theory and Lie Brackets of Vector Fields	1243
80.1 Introduction	1243
80.1.1 Short Recapitulation of the Classical Model's Structure	1244
80.2 Hypergestures Between Triadic Degrees That Are Parallel to Vector Fields	1245
80.3 Lie Brackets Generate Vector Fields That Connect Symmetry-Related Degrees	1245
80.4 Selecting Parallel Hypergestures That Are Admissible for Modulation	1247
80.5 The Other Direct Modulations	1249
81 Hypergestures for Performance Stemmata	1253
81.1 Motivation, Terminology, and Previous Results	1253
81.1.1 Performance Stemmata and Performance Gestures of Locally Compact Points	1254
81.2 Gestures with Lie Operators in Stemma Theory	1255

81.3	Connecting Stemmatic Gestures for Weights and Performance Fields	1256
81.4	Homology of Weight Parameter Stemmata	1258
81.5	A Concrete Example	1260
81.6	A Final Comment	1261
82	Composing and Analyzing with the Performing Body	1263
82.1	Gesture: A Sign or a Totality?	1264
82.2	A Gesture-Based Structural Reading in <i>Rain Tree Sketch II</i> by Toru Takemitsu	1266
82.2.1	Process I: Synergy of Mirroring and Parallel Gestures	1267
82.2.2	Process II: Towards Relaxation, Balance, and Weightfulness	1270
82.3	The Last Leg of a Bodily Journey	1274
82.3.1	<i>Sheng</i> for Piano, Smartphones, and Fixed Playback	1278
82.3.2	Cross-modality of Gestures	1279
82.3.3	Learning the Smartphone Instrument	1279
82.3.4	Kinesthetic Awareness and Modes of Listening	1281
82.4	Conclusion: Foregrounding the Performer's Body	1283
83	Gestural Analysis and Classification of a Conductor's Movements	1285
83.1	Gestures and Communication in Orchestral Conducting: A Case Study	1285
83.1.1	Problematics and Solving Methods	1286
83.1.2	Results, Consequences, Applications	1288
83.1.3	Some Remarks	1289
83.2	Hints for a Mathematical Description	1289
83.3	Data Analysis	1290
83.4	Conclusion	1292
83.5	Addendum	1293
84	Reviewing Flow, Gesture, and Spaces in Free Jazz	1295
84.1	Improvisation: Defining Time	1295
84.2	Flow, Gestures, Imaginary Time and Spaces in the Music Movie <i>Imaginary Time</i>	1295
84.2.1	The Compositional Character of the Pieces	1297
84.2.2	Large Forms	1298
84.2.3	Precision of Attacks	1298
84.2.4	Co-presence of Different Time Layers	1299
84.2.5	The Reality of Imaginary Time	1300
84.2.6	Measuring Flow	1300
84.2.7	Explicit Perception of Gestures	1300
85	Gesture and Vocalization	1301
85.1	Vocal Gesture	1301
85.2	Vocal and Manual Motion	1303
85.3	Gait	1304
85.4	Hindustani Vocal Music	1305
85.5	Notic Models and Kinetic Models	1306
85.6	The Realist Pitfall	1307
85.7	The Subjectivist Pitfall	1308
85.8	Speech Gesture	1311
86	Elements of a Future Vocal Gesture Theory	1313
86.1	Why a Theory of Vocal Gestures?	1313
86.1.1	Studying the Voice Without the Singer?	1314
86.1.2	Parts of the Phonatory System and Their Functions	1314

86.1.3	Imaginary Gestures in Real Time?	1315
86.1.4	Space of Voice Parameters Gestures	1315
86.1.5	About the Importance of Breathing and of Laryngeal Movements	1316
86.1.6	Mathematical Description of Vocal Gestures	1317
86.1.7	Gestures Thought by Singers	1321
86.2	A Powerful Tool from the Past	1322
86.2.1	Gestures in Gregorian Chant Didactics	1323
86.2.2	Concept of Rhythm and Time	1323
86.2.3	The Neumes	1327
86.3	Connecting Physiology, Gestures and Notation. Toward New Neumes?	1329
86.3.1	New Neumes	1330

Part XXIV References and Index

References	R.1
Index	R.33

Book Set Contents

Part I Introduction and Orientation

1	What Is Music About?	3
1.1	Fundamental Activities	3
1.2	Fundamental Scientific Domains	5
2	Topography	9
2.1	Layers of Reality	10
2.1.1	Physical Reality	10
2.1.2	Mental Reality	11
2.1.3	Psychological Reality	11
2.2	Molino's Communication Stream	11
2.2.1	Creator and Poietic Level	12
2.2.2	Work and Neutral Level	13
2.2.3	Listener and Esthetic Level	13
2.3	Semiosis	14
2.3.1	Expressions	15
2.3.2	Content	15
2.3.3	The Process of Signification	15
2.3.4	A Short Overview of Music Semiotics	15
2.4	The Cube of Local Topography	17
2.5	Topographical Navigation	19
3	Musical Ontology	21
3.1	Where Is Music?	21
3.2	Depth and Complexity	23
4	Models and Experiments in Musicology	27
4.1	Interior and Exterior Nature	29
4.2	What Is a Musicological Experiment?	30
4.3	Questions—Experiments of the Mind	31
4.4	New Scientific Paradigms and Collaboratories	32

Part II Navigation on Concept Spaces

5	Navigation	35
5.1	Music in the EncycloSpace.....	36
5.2	Receptive Navigation	39
5.3	Productive Navigation	39
6	Denotators	41
6.1	Universal Concept Formats	42
6.1.1	First Naïve Approach to Denotators	43
6.1.2	Interpretations and Comments	48
6.1.3	Ordering Denotators and ‘Concept Leafing’	50
6.2	Forms.....	52
6.2.1	Variable Addresses	53
6.2.2	Formal Definition	54
6.2.3	Discussion of the Form Typology	56
6.3	Denotators	57
6.3.1	Formal Definition of a Denotator	57
6.4	Anchoring Forms in Modules.....	59
6.4.1	First Examples and Comments on Modules in Music	60
6.5	Regular and Circular Forms	64
6.6	Regular Denotators	66
6.7	Circular Denotators.....	72
6.8	Ordering on Forms and Denotators	75
6.8.1	Concretizations and Applications	78
6.9	Concept Surgery and Denotator Semantics.....	83

Part III Local Theory

7	Local Compositions	89
7.1	The Objects of Local Theory.....	89
7.2	First Local Music Objects	92
7.2.1	Chords and Scales	92
7.2.2	Local Meters and Local Rhythms.....	96
7.2.3	Motives	99
7.3	Functorial Local Compositions	101
7.4	First Elements of Local Theory	103
7.5	Alterations Are Tangents	107
7.5.1	The Theorem of Mason-Mazzola	108
8	Symmetries and Morphisms	113
8.1	Symmetries in Music.....	114
8.1.1	Elementary Examples	116
8.2	Morphisms of Local Compositions	128
8.3	Categories of Local Compositions	132
8.3.1	Commenting on the Concatenation Principle.....	134
8.3.2	Embedding and Addressed Adjointness	136
8.3.3	Universal Constructions on Local Compositions	138
8.3.4	The Address Question	140
8.3.5	Categories of Commutative Local Compositions	142

9	Yoneda Perspectives	145
9.1	Morphisms Are Points	147
9.2	Yoneda's Fundamental Lemma	150
9.3	The Yoneda Philosophy	152
9.4	Understanding Fine and Other Arts	153
9.4.1	Painting and Music	153
9.4.2	The Art of Object-Oriented Programming	155
10	Paradigmatic Classification	157
10.1	Paradigmata in Musicology, Linguistics, and Mathematics	158
10.2	Transformation	162
10.3	Similarity	163
10.4	Fuzzy Concepts in the Humanities	164
11	Orbits	167
11.1	Gestalt and Symmetry Groups	167
11.2	The Framework for Local Classification	168
11.3	Orbits of Elementary Structures	168
11.3.1	Classification Techniques	169
11.3.2	The Local Classification Theorem	170
11.3.3	The Finite Case	177
11.3.4	Dimension	178
11.3.5	Chords	180
11.3.6	Empirical Harmonic Vocabularies	181
11.3.7	Self-addressed Chords	185
11.3.8	Motives	187
11.4	Enumeration Theory	190
11.4.1	Pólya and de Bruijn Theory	190
11.4.2	Big Science for Big Numbers	196
11.5	Group-Theoretical Methods in Composition and Theory	198
11.5.1	Aspects of Serialism	199
11.5.2	The American Tradition	202
11.6	Esthetic Implications of Classification	211
11.6.1	Jakobson's Poetic Function	212
11.6.2	Motivic Analysis: Schubert/Stolberg "Lied auf dem Wasser zu singen..."	214
11.6.3	Composition: Mazzola/Baudelaire "La mort des artistes"	218
11.7	Mathematical Reflections on Historicity in Music	220
11.7.1	Jean-Jacques Nattiez' Paradigmatic Theme	221
11.7.2	Groups as a Parameter of Historicity	223
12	Topological Specialization	225
12.1	What Ehrenfels Neglected	225
12.2	Topology	226
12.2.1	Metrical Comparison	228
12.2.2	Specialization Morphisms of Local Compositions	230
12.3	The Problem of Sound Classification	232
12.3.1	Topographic Determinants of Sound Descriptions	232
12.3.2	Varieties of Sounds	238
12.3.3	Semiotics of Sound Classification	240
12.4	Making the Vague Precise	241

Part IV Global Theory

13 Global Compositions	245
13.1 The Local-Global Dichotomy in Music	246
13.1.1 Musical and Mathematical Manifolds	251
13.2 What Are Global Compositions?	252
13.2.1 The Nerve of an Objective Global Composition	253
13.3 Functorial Global Compositions	256
13.4 Interpretations and the Vocabulary of Global Concepts	258
13.4.1 Iterated Interpretations	258
13.4.2 The Pitch Domain: Chains of Thirds, Ecclesiastical Modes, Triadic and Quaternary Degrees	259
13.4.3 Interpreting Time: Global Meters and Rhythms	266
13.4.4 Motivic Interpretations: Melodies and Themes	270
14 Global Perspectives	273
14.1 Musical Motivation	273
14.2 Global Morphisms	274
14.3 Local Domains	280
14.4 Nerves	281
14.5 Simplicial Weights	283
14.6 Categories of Commutative Global Compositions	285
15 Global Classification	287
15.1 Module Complexes	287
15.1.1 Global Affine Functions	288
15.1.2 Bilinear and Exterior Forms	290
15.1.3 Deviation: Compositions vs. “Molecules”	291
15.2 The Resolution of a Global Composition	292
15.2.1 Global Standard Compositions	293
15.2.2 Compositions from Module Complexes	294
15.3 Orbits of Module Complexes Are Classifying	298
15.3.1 Combinatorial Group Actions	299
15.3.2 Classifying Spaces	300
16 Classifying Interpretations	303
16.1 Characterization of Interpretable Compositions	303
16.1.1 Automorphism Groups of Interpretable Compositions	306
16.1.2 A Cohomological Criterion	307
16.2 Global Enumeration Theory	309
16.2.1 Tesselation	309
16.2.2 Mosaics	310
16.2.3 Classifying Rational Rhythms and Canons	312
16.3 Global American Set Theory	314
16.4 Interpretable “Molecules”	316
17 Esthetics and Classification	319
17.1 Understanding by Resolution: An Illustrative Example	319
17.2 Varèse’s Program and Yoneda’s Lemma	323

18	Predicates	327
18.1	What Is the Case: The Existence Problem	327
18.1.1	Merging Systematic and Historical Musicology	328
18.2	Textual and Paratextual Semiosis	329
18.2.1	Textual and Paratextual Signification	330
18.3	Textuality	331
18.3.1	The Category of Denotators	331
18.3.2	Textual Semiosis	334
18.3.3	Atomic Predicates	339
18.3.4	Logical and Geometric Motivation	345
18.4	Paratextuality	349
19	Topoi of Music	351
19.1	The Grothendieck Topology	351
19.1.1	Cohomology	354
19.1.2	Marginalia on Presheaves	356
19.2	The Topos of Music: An Overview	357
20	Visualization Principles	361
20.1	Problems	361
20.2	Folding Dimensions	363
20.2.1	$\mathbb{R}^2 \rightarrow \mathbb{R}$	363
20.2.2	$\mathbb{R}^n \rightarrow \mathbb{R}$	364
20.2.3	An Explicit Construction of μ with Special Values.	365
20.3	Folding Denotators	366
20.3.1	Folding Limits	366
20.3.2	Folding Colimits	367
20.3.3	Folding Powersets	368
20.3.4	Folding Circular Denotators	369
20.4	Compound Parametrized Objects	370
20.5	Examples	371

Part V Topologies for Rhythm and Motives

21	Metrics and Rhythmics	375
21.1	Review of Riemann and Jackendoff-Lerdahl Theories	375
21.1.1	Riemann's Weights	375
21.1.2	Jackendoff-Lerdahl: Intrinsic Versus Extrinsic Time Structures	376
21.2	Topologies of Global Meters and Associated Weights	378
21.3	Macro-events in the Time Domain	380
22	Motif Gestalts	383
22.1	Motivic Interpretation	384
22.2	Shape Types	385
22.2.1	Examples of Shape Types	386
22.3	Metrical Similarity	388
22.3.1	Examples of Distance Functions	389
22.4	Paradigmatic Groups	390
22.4.1	Examples of Paradigmatic Groups	391
22.5	Pseudo-metrics on Orbits	393
22.6	Topologies on Gestalts	394

22.6.1	The Inheritance Property	395
22.6.2	Cognitive Aspects of Inheritance	396
22.6.3	Epsilon Topologies	397
22.7	First Properties of the Epsilon Topologies	399
22.7.1	Toroidal Topologies	401
22.8	Rudolph Reti's Motivic Analysis Revisited	404
22.8.1	Review of Concepts	404
22.8.2	Reconstruction	406
22.9	Motivic Weights	408

Part VI Harmony

23	Critical Preliminaries	413
23.1	Hugo Riemann	414
23.2	Paul Hindemith	414
23.3	Heinrich Schenker and Friedrich Salzer	414
24	Harmonic Topology	417
24.1	Chord Perspectives	418
24.1.1	Euler Perspectives	418
24.1.2	12-Tempered Perspectives	422
24.1.3	Enharmonic Projection	425
24.2	Chord Topologies	427
24.2.1	Extension and Intension	427
24.2.2	Extension and Intension Topologies	429
24.2.3	Faithful Addresses	431
24.2.4	The Saturation Sheaf	434
25	Harmonic Semantics	435
25.1	Harmonic Signs—Overview	436
25.2	Degree Theory	437
25.2.1	Chains of Thirds	437
25.2.2	American Jazz Theory	439
25.2.3	Hans Straub: General Degrees in General Scales	442
25.3	Function Theory	442
25.3.1	Canonical Morphemes for European Harmony	444
25.3.2	Riemann Matrices	447
25.3.3	Chains of Thirds	447
25.3.4	Tonal Functions from Absorbing Addresses	449
26	Cadence	453
26.1	Making the Concept Precise	454
26.2	Classical Cadences Relating to 12-Tempered Intonation	454
26.2.1	Cadences in Triadic Interpretations of Diatonic Scales	455
26.2.2	Cadences in More General Interpretations	456
26.3	Cadences in Self-addressed Tonalities of Morphology	457
26.4	Self-addressed Cadences by Symmetries and Morphisms	459
26.5	Cadences for Just Intonation	460
26.5.1	Tonalities in Third-Fifth Intonation	460
26.5.2	Tonalities in Pythagorean Intonation	461

27 Modulation	463
27.1 Modeling Modulation by Particle Interaction	464
27.1.1 Models and the Anthropic Principle	464
27.1.2 Classical Motivation and Heuristics	465
27.1.3 The General Background	467
27.1.4 The Well-Tempered Case	469
27.1.5 Reconstructing the Diatonic Scale from Modulation	471
27.1.6 The Case of Just Tuning	473
27.1.7 Quantized Modulations and Modulation Domains for Selected Scales	477
27.2 Harmonic Tension	481
27.2.1 The Riemann Algebra	481
27.2.2 Weights on the Riemann Algebra	482
27.2.3 Harmonic Tensions from Classical Harmony?	484
27.2.4 Optimizing Harmonic Paths	485
28 Applications	487
28.1 First Examples	488
28.1.1 Johann Sebastian Bach: Choral from “Himmelfahrtsoratorium”	488
28.1.2 Wolfgang Amadeus Mozart: “Zauberflöte”, Choir of Priests	490
28.1.3 Claude Debussy: “Préludes”, Livre 1, No.4	492
28.2 Modulation in Beethoven’s Sonata op.106, 1 st Movement	495
28.2.1 Introduction	495
28.2.2 The Fundamental Theses of Erwin Ratz and Jürgen Uhde	497
28.2.3 Overview of the Modulation Structure	498
28.2.4 Modulation $B_b \rightsquigarrow G$ via e^{-3} in W	499
28.2.5 Modulation $G \rightsquigarrow E_b$ via U_g in W	499
28.2.6 Modulation $E_b \rightsquigarrow D/b$ from W to W^*	499
28.2.7 Modulation $D/b \rightsquigarrow B$ via $U_{d/d_{\sharp}} = U_{g_{\sharp}/a}$ within W^*	500
28.2.8 Modulation $B \rightsquigarrow B_b$ from W^* to W	500
28.2.9 Modulation $B_b \rightsquigarrow G_b$ via U_{b_b} within W	501
28.2.10 Modulation $G_b \rightsquigarrow G$ via $U_{a_b/a}$ within W	501
28.3 Rhythmical Modulation in “Synthesis”	501
28.3.1 Rhythmic Modes	502
28.3.2 Composition for Percussion Ensemble	503

Part VII Counterpoint

29 Melodic Variation by Arrows	507
29.1 Arrows and Alterations	507
29.2 The Contrapuntal Interval Concept	508
29.3 The Algebra of Intervals	509
29.3.1 The Third Torus	510
29.4 Musical Interpretation of the Interval Ring	511
29.5 Self-addressed Arrows	514
29.6 Change of Orientation	515

30	Interval Dichotomies as a Contrast	517
30.1	Dichotomies and Polarity	517
30.2	The Consonance and Dissonance Dichotomy	520
30.2.1	Fux and Riemann Consonances Are Isomorphic	521
30.2.2	Induced Polarities	523
30.2.3	Empirical Evidence for the Polarity Function	523
30.2.4	Music and the Hippocampal Gate Function	527
31	Modeling Counterpoint by Local Symmetries	531
31.1	Deformations of the Strong Dichotomies	531
31.2	Contrapuntal Symmetries Are Local	533
31.3	The Counterpoint Theorem	534
31.3.1	Some Preliminary Calculations	534
31.3.2	Two Lemmata on Cardinalities of Intersections	536
31.3.3	An Algorithm for Exhibiting the Contrapuntal Symmetries	536
31.3.4	Transfer of the Counterpoint Rules to General Representatives of Strong Dichotomies	540
31.4	The Classical Case: Consonances and Dissonances	540
31.4.1	Discussion of the Counterpoint Theorem in the Light of Reduced Strict Style	541
31.4.2	The Major Dichotomy—A Cultural Antipode?	542
31.4.3	Software for Counterpoint and Theoretical Extensions	543
Part VIII Structure Theory of Performance		
32	Local and Global Performance Transformations	547
32.1	Performance as a Reality Switch	548
32.2	Why Do We Need Infinite Performance of the Same Piece?	549
32.3	Local Structure	550
32.3.1	The Coherence of Local Performance Transformations	550
32.3.2	Differential Morphisms of Local Compositions	551
32.4	Global Structure	554
32.4.1	Modeling Performance Syntax	556
32.4.2	The Formal Setup	557
32.4.3	Performance qua Interpretation of Interpretation	560
33	Performance Fields	561
33.1	Classics: Tempo, Intonation, and Dynamics	561
33.1.1	Tempo	561
33.1.2	Intonation	563
33.1.3	Dynamics	564
33.2	Genesis of the General Formalism	565
33.2.1	The Question of Articulation	565
33.2.2	The Formalism of Performance Fields	568
33.3	What Performance Fields Signify	568
33.3.1	Th.W. Adorno, W. Benjamin, and D. Raffman	569
33.3.2	Towards Composition of Performance	571
34	Initial Sets and Initial Performances	573
34.1	Taking Off with a Shifter	573
34.2	Anchoring Onset	574
34.3	The Concert Pitch	576
34.4	Dynamical Anchors	578

34.5	Initializing Articulation	578
34.6	Hit Point Theory	580
34.6.1	Distances	580
34.6.2	Flow Interpolation	582
35	Hierarchies and Performance Scores	585
35.1	Performance Cells	585
35.2	The Category of Performance Cells	586
35.3	Hierarchies	588
35.3.1	Operations on Hierarchies	590
35.3.2	Classification Issues	591
35.3.3	Example: The Piano and Violin Hierarchies	593
35.4	Local Performance Scores	594
35.5	Global Performance Scores	598
35.5.1	Instrumental Fibers	599
Part IX Expressive Semantics		
36	Taxonomy of Expressive Performance	603
36.1	Feelings: Emotional Semantics	604
36.2	Motion: Gestural Semantics	606
36.3	Understanding: Rational Semantics	609
36.4	Cross-semantical Relations	612
37	Performance Grammars	615
37.1	Rule-Based Grammars	615
37.1.1	The KTH School	617
37.1.2	Neil P. McAngus Todd	618
37.1.3	The Zurich School	619
37.2	Remarks on Learning Grammars	619
38	Stemma Theory	621
38.1	Motivation from Practising and Rehearsing	622
38.1.1	Does Reproducibility of Performances Help Understanding?	622
38.2	Tempo Curves Are Inadequate	623
38.3	The Stemma Concept	626
38.3.1	The General Setup of Matrilineal Sexual Propagation	628
38.3.2	The Primary Mother—Taking Off	629
38.3.3	Mono- and Polygamy—Local and Global Actions	632
38.3.4	Family Life—Cross-correlations	634
39	Operator Theory	637
39.1	Why Weights?	638
39.1.1	Discrete and Continuous Weights	638
39.1.2	Weight Recombination	639
39.2	Primavista Weights	640
39.2.1	Dynamics	640
39.2.2	Agogics	643
39.2.3	Tuning and Intonation	644
39.2.4	Articulation	645
39.2.5	Ornaments	645

39.3	Analytical Weights	646
39.4	Taxonomy of Operators	648
39.4.1	Splitting Operators	649
39.4.2	Symbolic Operators	650
39.4.3	Physical Operators	651
39.4.4	Field Operators	652
39.5	Tempo Operator	653
39.6	Scalar Operator	654
39.7	The Theory of Basis—Pianola Operators	655
39.7.1	Basis Specialization.....	656
39.7.2	Pianola Specialization.....	659
39.8	Locally Linear Grammars	659

Part X RUBATO®

40	Architecture	665
40.1	The Overall Modularity	666
40.2	Frame and Modules.....	668
40.3	Postscriptum: The Rubato Composer Environment	668
41	The RUBETTE® Family	669
41.1	MetroRUBETTE®.....	669
41.2	MeloRUBETTE®.....	672
41.3	HarmoRUBETTE®	674
41.3.1	A Set of New Harmonic Analysis Rubettes on RUBATO® Composer	678
41.4	PerformanceRUBETTE®	679
41.5	PrimavistaRUBETTE®	685
42	Performance Experiments	687
42.1	A Preliminary Experiment: Robert Schumann’s “Kuriose Geschichte”	687
42.2	Full Experiment: J.S. Bach’s “Kunst der Fuge”	688
42.3	Analysis.....	688
42.3.1	Metric Analysis	688
42.3.2	Motif Analysis	692
42.3.3	Omission of Harmonic Analysis	693
42.4	Stemma Constructions	694
42.4.1	Performance Setup	694
42.4.2	Instrumental Setup	700
42.4.3	Global Discussion	701

Part XI Statistics of Analysis and Performance

43	Analysis of Analysis	707
43.1	Hierarchical Decomposition	707
43.1.1	General Motivation	707
43.1.2	Hierarchical Smoothing	708
43.1.3	Hierarchical Decomposition	710
43.2	Comparing Analyses of Bach, Schumann, and Webern.....	711

44	Differential Operators and Regression	719
44.0.1	Analytical Data	720
44.1	The Beran Operator	721
44.1.1	The Concept	721
44.1.2	The Formalism	723
44.2	The Method of Regression Analysis	726
44.2.1	The Full Model	726
44.2.2	Step Forward Selection	727
44.3	The Results of Regression Analysis	727
44.3.1	Relations Between Tempo and Analysis	727
44.3.2	Complex Relationships	729
44.3.3	Commonalities and Diversities	729
44.3.4	Overview of Statistical Results	738
45	Relating Tempo to Metric, Melodic and Harmonic Analyses in Chopin's <i>Prélude op. 28, No. 4</i>	743
45.1	Introduction	743
45.2	Data	745
45.2.1	Analytical Data	745
45.2.2	Tempo Data	745
45.3	Short Summary of the Results	747
45.4	Some Philosophical Comments	748

Part XII Inverse Performance Theory

46	Principles of Music Critique	751
46.1	Boiling Down Infinity—Is Feuilletonism Inevitable?	751
46.2	“Political Correctness” in Performance—Reviewing Gould	752
46.3	Transversal Ethnomusicology	754
47	Critical Fibers	755
47.1	The Stemma Model of Critique	755
47.2	Fibers for Locally Linear Grammars	756
47.3	Algorithmic Extraction of Performance Fields	759
47.3.1	The Infinitesimal View on Expression	759
47.3.2	Real-Time Processing of Expressive Performance	760
47.3.3	Score-Performance Matching	761
47.3.4	Performance Field Calculation	762
47.3.5	Visualization	763
47.3.6	The EspressoRUBETTE®: An Interactive Tool for Expression Extraction	764
47.4	Local Sections	766
47.4.1	Comparing Argerich and Horowitz	768

Part XIII Operationalization of Poiesis

48	Unfolding Geometry and Logic in Time	773
48.1	Performance of Logic and Geometry	774
48.2	Constructing Time from Geometry	775
48.3	Discourse and Insight	776

49	Local and Global Strategies in Composition	779
49.1	Local Paradigmatic Instances	780
49.1.1	Transformations	780
49.1.2	Variations	780
49.2	Global Poetical Syntax	781
49.2.1	Roman Jakobson's Horizontal Function	781
49.2.2	Roland Posner's Vertical Function	781
49.3	Structure and Process	782
50	The Paradigmatic Discourse on <i>presto</i>[®]	783
50.1	The <i>presto</i> [®] Functional Scheme	783
50.2	Modular Affine Transformations	786
50.3	Ornaments and Variations	786
50.4	Problems of Abstraction	789
51	Case Study I: "Synthesis" by Guerino Mazzola	791
51.1	The Overall Organization	791
51.1.1	The Material: 26 Classes of Three-Element Motives	792
51.1.2	Principles of the Four Movements and Instrumentation	793
51.2	1 st Movement: Sonata Form	794
51.3	2 nd Movement: Variations	794
51.4	3 rd Movement: Scherzo	798
51.5	4 th Movement: Fractal Syntax	799
52	Object-Oriented Programming in OpenMusic	801
52.1	Object-Oriented Language	802
52.1.1	Patches	802
52.1.2	Objects	803
52.1.3	Classes	803
52.1.4	Methods	803
52.1.5	Generic Functions	804
52.1.6	Message Passing	804
52.1.7	Inheritance	805
52.1.8	Boxes and Evaluation	805
52.1.9	Instantiation	806
52.2	Musical Object Framework	806
52.2.1	Internal Representation	806
52.2.2	Interface	808
52.3	Maquettes: Objects in Time	811
52.4	Meta-object Protocol	813
52.4.1	Reification of Temporal Boxes	815
52.5	A Musical Example	817

Part XIV String Quartet Theory

53	Historical and Theoretical Prerequisites	825
53.1	History	825
53.2	Theory of the String Quartet Following Ludwig Finscher	826
53.2.1	Four Part Texture	826
53.2.2	The Topos of Conversation Among Four Humanists	827
53.2.3	The Family of Violins	828

54	Estimation of Resolution Parameters	831
54.1	Parameter Spaces for Violins	831
54.2	Estimation	833
55	The Case of Counterpoint and Harmony	839
55.1	Counterpoint	839
55.2	Harmony	840
55.3	Effective Selection	840

Part XV Gesture Philosophy for Music

56	The Topos of Gestures	843
57	Gesture Philosophy: Phenomenology, Ontology, and Semiotics	845
57.1	A Short Recapitulation of Musical Ontology	845
57.1.1	Ontology: Where, Why, and How	845
57.1.2	Ontology: Facts, Processes, and Gestures	846
57.2	Jean-Claude Schmitt's Historiographic and Philosophical Treatise "La raison des gestes dans l'Occident médiéval"	846
57.2.1	Comments	847
57.3	Vilém Flusser's <i>Gesten: Versuch einer Phänomenologie</i>	848
57.3.1	A Short Introduction to Flusser's Essay	848
57.3.2	The Semiotic Neurosis	848
57.4	Michel Guérin's <i>philosophie des gestes</i>	850
57.4.1	The Essay's Structure	850
57.4.2	Gestural Ontology and Four Elementary Gestures	850
57.5	Flusser and Guérin: Some Consequences	851
57.6	A Program	852
57.6.1	Circularity	852
57.7	The Semiotic Gesture Concept of Adam Kendon and David McNeill	853
57.7.1	Comments	855
57.8	Juhani Pallasmaa and André Chastel: The Thinking Hand in Architecture and the Arts	855
57.9	Benveniste, Popelard, Wall	856
58	The French Presemiotic Approach	859
58.1	Maurice Merleau-Ponty	860
58.2	Francis Bacon and Gilles Deleuze	860
58.3	Jean Cavaillès and Charles Alunni	861
58.4	Gilles Châtelet	862
59	Cognitive Science	867
59.1	Embodiment	867
59.1.1	Embodiment Science	868
59.2	Neuroscience	871
59.2.1	Embodied AI	873
59.3	Anthropology	874
59.4	Dance	874
59.5	Disabled Gestures Versus Gestures Disabled: Parlan's Versus Peterson's Pianism	875
59.5.1	Performative Gestures: Disabled Jazz Pianists	876
59.5.2	Horace Parlan: Disabled Gestures	877
59.5.3	Parlan with Bass (and Drums)	877

59.5.4	Parlan with Rhythm Section	878
59.5.5	Parlan as Soloist	879
59.5.6	Parlan's Duets with Archie Shepp	880
59.5.7	Disabled Gestures	881
59.5.8	Gestures Disabled: Oscar Peterson	882
59.5.9	Conclusion	884
59.6	Aristotle, Blind Lemon Jefferson, and Vilayanur S. Ramachandran Walk into a Bar	885
59.6.1	Introduction	885
59.6.2	Division by (Almost) Zero: Many Blind Bluesmen but Few Blind Blues	885
59.6.3	Seeing Blind Blues: Gesture, Flow, Circuitry, and Amplification	886
59.6.4	Epilogue: Puns as Gestures	887
60	Models from Music	889
60.1	Wolfgang Graeser	890
60.2	Adorno, Wieland, Sessions, Clynes	890
60.2.1	Theodor Wiesengrund Adorno	891
60.2.2	Renate Wieland	893
60.2.3	Roger Sessions	894
60.2.4	Manfred Clynes	895
60.3	Johan Sundberg and Neil P. McAngus Todd	896
60.4	David Lewin and Robert S. Hatten	898
60.5	Marcelo Wanderley and Claude Cadoz, Rolf Inge Godøy and Marc Leman	901
Part XVI Mathematics of Gestures		
61	Fundamental Concepts and Associated Categories	907
61.1	Introduction	907
61.2	Towards a Musical String Theory	909
61.3	Initial Investigations: Diagrams of Curves	910
61.4	Modeling a Pianist's Hand	912
61.4.1	The Hand's Model	912
61.4.2	Transforming Abstract Note Symbols into Symbolic Gestures	912
61.4.3	From Symbolic Hand Gestures to Physical Gestures	913
61.5	The Mathematical Definition of Gestures	914
61.6	Hypergestures	915
61.6.1	Spatial Hypergestures	917
61.7	Categorically Natural Gestures	918
61.8	Connecting to Algebraic Topology: Hypergestures Generalize Homotopy	919
61.9	Gestoids	922
61.9.1	The Fundamental Group, Klumpenhouwer Networks, and Fourier Representation	924
61.10	Gabriel's Spectroids and Natural Formulas	925
61.10.1	Solutions of Representations of Natural Formulas by Local Networks	927
61.11	The Tangent Category	927
61.12	The Diamond Conjecture	929
61.13	Topos Logic for Gestures	930
61.14	The Escher Theorem for Hypergestures	931
61.14.1	Hypergestures and the Escher Theorem for Fux Counterpoint	931
61.14.2	Rebecca Lazier's Vanish: Lawvere, Escher, Schoenberg	933

62	Categories of Gestures over Topological Categories	937
62.1	Gestures over Topological Categories	939
62.1.1	The Categorical Digraph of a Topological Category	940
62.1.2	Gestures with Body in a Topological Category	940
62.1.3	Varying the Underlying Topological Category	942
62.2	From Morphisms to Gestures	942
62.2.1	Diagrams as Gestures	944
62.2.2	Gestures in Factorization Categories	944
62.2.3	Extensions from Homological Algebra Are Gestures	945
62.2.4	The Bicategory of Gestures	945
62.2.5	Entering the Diamond Space	946
62.3	Diagrams in Topological Groups for Gestures	947
62.4	Gestural Interpretation of Modulations in Beethoven's op.106/Allegro	950
62.4.1	Recapitulation of the Results from Section 28.2	951
62.4.2	The Modulation B_b -major $\rightsquigarrow$ G -major Between Measure 31 and Measure 44	952
62.4.3	Lewin's Characteristic Gestures Identified?	955
62.4.4	Modulation E_b -major $\rightsquigarrow$ D -major/ B -minor from W to W^*	957
62.4.5	The Fanfare	958
62.5	Conclusion for the Categorical Gesture Approach	960
62.6	Functorial Gestures: General Addresses	961
62.7	Yoneda's Lemma for Gestures	962
62.8	Examples from Music	963
62.8.1	Collections of Acoustical Waves	963
62.8.2	Collections of Spectral Music Data	964
62.8.3	MIDI-Type ON-OFF Transformations	964
63	Singular Homology of Hypergestures	965
63.1	An Introductory Example	965
63.2	Chain Modules for Singular Hypergestural Homology	967
63.3	The Boundary Homomorphism	968
64	Stokes' Theorem for Hypergestures	973
64.1	The Need for Stokes' Theorem for Hypergestures	973
64.2	Almost Regular Manifolds, Differential Forms, and Integration for Hypergestures	973
64.2.1	Locally Almost Regular Manifolds	974
64.2.2	Differential Forms	975
64.2.3	Integration	975
64.3	Stokes' Theorem	976
65	Local Facts, Processes, and Gestures	979
65.1	Categories of Local Compositions	979
65.2	Categories of Local Networks	980
65.3	Categories of Local Gestures	982
65.3.1	Local Gestures on Topological Categories of Points	982
65.4	Connecting Functors	984
65.5	Hypernetworks and Hypergestures	985
65.5.1	Escher Theorems	985
65.6	Singular Homology of Hypernetworks and Hypergestures	985

66 Global Categories	987
66.1 Categories of Global Compositions	987
66.1.1 Simplicial Methods	988
66.2 Classification of Global Compositions	989
66.3 Non-interpretable Global Compositions	990
66.4 Categories of Global Networks	990
66.4.1 Non-interpretable Global Networks	991
66.5 Categories of Global Gestures	993
66.6 Globalizing Topological Categories: Categorical Manifolds	993
66.7 Globalizing Skeleta	996
66.8 Functorial Global Gestures	998
67 Mathematical Models of Creativity	1001
67.1 Forewarning: Invention of Gestures in Mathematics	1001
67.1.1 Thinking Exactness, Like a Rolling Mind	1001
67.1.2 Thought as an Algebra of Gestures	1002
67.2 Method and Objects, Summarily Explained: I—Preamble	1003
67.2.1 Prelude to a Discourse of a Method: “Caminos”, “Aletheia”, Irreverence	1003
67.2.2 Our Posture	1015
67.3 Method and Objects, Summarily Explained: II—Data	1021
67.3.1 Simple Objects, Structures and Invariants in Mathematics	1021
67.3.2 Complete Frameworks, Computations and Representations	1029
67.4 Creativity in Mathematics: Gestures in Historical Contexts	1031
67.4.1 Creativity: Phenomenology, Psychology and Skills, and Life	1031
67.4.2 Determination of Mathematics as a History of Its Gestures	1036
67.4.3 Invention in the Art of Mathematics	1044
67.5 On the Mathematical Invention of Coordinations	1046
67.5.1 Emergence of Coordinations	1047
67.5.2 Arrows	1050
67.5.3 Bodies, Implicit Surfaces, Abstract Relations	1055
67.5.4 Sketches	1056
67.6 Pulsation in the Living Process of Invention Among Shapes	1058
67.6.1 Productions: Objects and Relations, Problems, Pulsation	1058
67.6.2 Creativity in the Mathematical World Seen as a Living System of Shapes, in a Categorical Framework	1060
67.7 Conclusion: Categorical Presentation of Pulsations	1064
67.8 The Hegel Group Action on a Critical Concept’s Walls	1066
67.9 Introduction	1066
67.10 The Hegel Concept Group $\mathcal{G}$	1067
67.10.1 Hegel’s Initial Thought Movement in <i>Wissenschaft der Logik</i>	1067
67.10.2 The Implicit Group Structure	1070
67.10.3 The Conceptual Box Structure	1072
67.11 The $\mathcal{G}$ Action on the Yoneda Model of Creativity	1073
67.12 The Hegel Body $\mathcal{B}$ in the Concept Architecture of Forms and Denotators	1073
67.13 The Usage of $\mathcal{G}$ for the Dynamics of Creativity	1074
67.13.1 Two Preliminary Examples	1074
67.13.2 The Challenge: Creating a Spectrum of Conceptual Extensions	1075
67.13.3 Escher’s Theorem for Beethoven’s Fanfare in the “ <i>Hammerklavier</i> ” Sonata op. 106	1075
67.13.4 The Rotation $S@N$ as a Driving Creative Force in the Incipit of Liszt’s <i>Mephisto</i> <i>Walzer</i> No.1	1076
67.14 An Experimental Composition	1078
67.15 Still More Symmetries? Future Developments	1080

Part XVII Concept Architectures and Software for Gesture Theory

68	Forms and Denotators over Topological Categories	1085
68.1	The General Topos—Theoretical Framework	1085
68.1.1	The category TopCat of Small Topological Categories	1085
68.2	Forms and Denotators	1086
68.3	Mathematics of Objects, Structures, and Concepts	1087
68.4	Galois Theory of Concepts	1087
68.4.1	Introduction	1088
68.4.2	Form Semiotics	1089
68.4.3	The Category of Form Semiotics	1092
68.4.4	Galois Correspondence of Form Semiotics	1093
69	The Rubato Composer Architecture	1095
69.1	The Software Architecture	1096
69.2	The Rubette World	1097
69.2.1	Rubettes for Counterpoint	1097
69.2.2	Rubettes for Harmony	1098
69.2.3	MetroRubettes	1099
70	The BigBang Rubette and the Ontological Dimension of Embodiment	1101
71	Facts: Denotators and Their Visualization and Sonification	1103
71.1	Some Earlier Visualizations of Denotators	1103
71.1.1	Göller’s PrimaVista Browser	1103
71.1.2	Milmeister’s ScorePlay and Select2D Rubettes	1105
71.2	An Early Score-Based Version of BigBang	1106
71.2.1	The Early BigBang Rubette’s View Configurations	1107
71.2.2	Navigating Denotators	1111
71.2.3	Sonifying Score-Based Denotators	1111
71.3	BigBangObjects and Visualization of Arbitrary <i>Mod</i> [®] Denotators	1111
71.3.1	A Look at Potential Visual Characteristics of Form Types	1112
71.3.2	From a General View Concept to BigBang Objects	1114
71.3.3	New Visual Dimensions	1115
71.4	The Sonification of BigBangObjects	1116
71.5	Examples of Forms and the Visualization of Their Denotators	1117
71.5.1	Some Set-Theoretical Structures	1117
71.5.2	Tonal and Transformational Theory	1119
71.5.3	Synthesizers and Sound Design	1121
72	Processes: BigBang’s Operation Graph	1127
72.1	Temporal BigBangObjects, Object Selection, and Layers	1128
72.1.1	Selecting None and Lewin’s Transformation Graphs	1128
72.1.2	The Temporal Existence of BigBangObjects	1129
72.1.3	BigBang Layers	1131
72.2	Operations and Transformations in BigBang	1132
72.2.1	Non-transformational Operations	1132
72.2.2	Transformations	1136
72.3	BigBang’s Process View	1138
72.3.1	Visualization of Processes	1138
72.3.2	Selecting States and Modifying Operations	1139

72.3.3	Alternative and Parallel Processes	1139
72.3.4	Structurally Modifying the Graph	1141
72.3.5	Undo/Redo	1141
73	Gestures: Gestural Interaction and Gesturalization	1143
73.1	Formalizing: From Gestures to Operations	1143
73.1.1	Modes, Gestural Operations, and the Mouse	1144
73.1.2	Affine Transformations and Multi-touch	1148
73.1.3	Dynamic Motives, Sound Synthesis, and Leap Motion	1149
73.1.4	Recording, Modifying Operations and MIDI Controllers	1151
73.2	Gesturalizing and the Real BigBang: Animated Composition History	1152
73.2.1	Gesturalizing Transformations	1152
73.2.2	Gesturalizing Other Operations	1154
73.2.3	Using Gesturalization as a Compositional Tool	1154
74	Musical Examples	1157
74.1	Some Example Compositions	1157
74.1.1	Transforming an Existing Composition	1157
74.1.2	Gesturalizing and Looping with a Simple Graph	1158
74.1.3	Drawing UPIC-like Motives and Transforming	1159
74.1.4	Drawing Time-Slices	1160
74.1.5	Converting Forms, Tricks for Gesturalizing	1160
74.1.6	Gesturalizing a Spectrum	1162
74.1.7	Using Wallpapers to Create Rhythmical Structures	1163
74.2	Improvisation and Performance with BigBang	1163
74.2.1	Improvising by Selecting States and Modifying Transformations	1164
74.2.2	Playing Sounds with a MIDI Keyboard and Modifying Them	1164
74.2.3	Playing a MIDI Grand Piano with Leap Motion	1165
74.2.4	Playing a MIDI Grand Piano with the Ableton Push	1166
74.2.5	Improvising with 12-Tone Rows	1167
Part XVIII The Multiverse Perspective		
75	Gesture Theory and String Theory	1173
76	Physical and Musical Multiverses	1175
77	Hesse's Melting Beads: A Multiverse Game with Strings and Gestures	1177
77.1	Review of Hesse's Glass Bead Game	1177
77.2	Frozen Glass Beads of Facticity	1178
77.3	The Revolution of Functors	1178
77.4	Gestures in Philosophy and Science	1180
77.5	Gesture Theory in Music	1181
77.6	A Remark on Gestural Creativity	1183
77.7	Gestures and Strings	1183
77.8	Playing the Multiversed Game in a Pre-semiotic Ontology	1184

78 Euler-Lagrange Equations for Hypergestures	1185
78.1 The Problem in Performance Theory with the Physical Nambu-Goto Lagrangian	1185
78.1.1 Complex Time and Descartes' Dualistic Ontology	1186
78.2 Lagrangian Density for Complex Time	1187
78.2.1 The Lagrangian Action for Performance	1188
78.2.2 The World-Sheet of Complex Time	1190
78.2.3 The Space for a Hand's Gestures	1192
78.2.4 The World-Sheet for a Simple Case	1192
78.2.5 The Elementary Gesture of a Pianist	1192
78.2.6 The Overarching Framework Between Note Performance and Gesture Performance	1195
78.2.7 Examples of Functional Relations Between Potential and Physical Gesture	1196
78.2.8 Calculus of Variations for the Physical Gesture	1202
78.2.9 A First Solution. World-Sheet Potentials Determine a Pianist's Gesture: Calculus of Variations and Fourier Analysis	1203
78.2.10 The Calculus with Vanishing Potential	1204
78.2.11 The Calculus with General Potential	1209
78.2.12 Solution of the Differential Equation Using 2D Fourier Series	1212
78.2.13 Parallels Between Performance Operators for Scores and for Gestures	1216
78.2.14 Complex Time and the Artistic Effort	1218
78.2.15 Opening the Aesthetic Question that Is Quantified in Lagrange Potentials	1218
78.2.16 A Musical Composition by Maria Mannone Realized Using These Ideas	1219
78.3 Global Performance Hypergestures	1223
78.3.1 The Musical Situation: An Intuitive Introduction	1223
78.4 Categorical Gestures and Global Performance Hypergestures	1224
78.4.1 Categorical Gestures: The Case of Potentials	1224
78.4.2 The Mathematics of Global Performance Hypergestures	1226
78.5 World-Sheet Hypergestures for General Skeleta	1227
78.6 A Global Variational Principle for the Lagrange Formalism	1231

Part XIX Gestures in Music and Performance Theory, and in Ethnomusicology

79 Gesture Homology for Counterpoint	1235
79.1 Summary of Mathematical Theory of Counterpoint: What It Is About and What Is Missing	1235
79.2 Hypergestural Singular Homology	1236
79.3 A Classical Example of a Topological Category from Counterpoint	1237
79.3.1 Generators of $H_1({}^G X)$ for a Groupoid ${}^G X$ Defined by a Group Action	1238
79.4 The Meaning of H_1 for Counterpoint	1240
79.5 Concluding Comments	1241
80 Modulation Theory and Lie Brackets of Vector Fields	1243
80.1 Introduction	1243
80.1.1 Short Recapitulation of the Classical Model's Structure	1244
80.2 Hypergestures Between Triadic Degrees That Are Parallel to Vector Fields	1245
80.3 Lie Brackets Generate Vector Fields That Connect Symmetry-Related Degrees	1245
80.4 Selecting Parallel Hypergestures That Are Admissible for Modulation	1247
80.5 The Other Direct Modulations	1249

81	Hypergestures for Performance Stemmata	1253
81.1	Motivation, Terminology, and Previous Results	1253
81.1.1	Performance Stemmata and Performance Gestures of Locally Compact Points	1254
81.2	Gestures with Lie Operators in Stemma Theory	1255
81.3	Connecting Stemmatic Gestures for Weights and Performance Fields	1256
81.4	Homology of Weight Parameter Stemmata	1258
81.5	A Concrete Example	1260
81.6	A Final Comment	1261
82	Composing and Analyzing with the Performing Body	1263
82.1	Gesture: A Sign or a Totality?	1264
82.2	A Gesture-Based Structural Reading in <i>Rain Tree Sketch II</i> by Toru Takemitsu	1266
82.2.1	Process I: Synergy of Mirroring and Parallel Gestures	1267
82.2.2	Process II: Towards Relaxation, Balance, and Weightfulness	1270
82.3	The Last Leg of a Bodily Journey	1274
82.3.1	<i>Sheng</i> for Piano, Smartphones, and Fixed Playback	1278
82.3.2	Cross-modality of Gestures	1279
82.3.3	Learning the Smartphone Instrument	1279
82.3.4	Kinesthetic Awareness and Modes of Listening	1281
82.4	Conclusion: Foregrounding the Performer's Body	1283
83	Gestural Analysis and Classification of a Conductor's Movements	1285
83.1	Gestures and Communication in Orchestral Conducting: A Case Study	1285
83.1.1	Problematics and Solving Methods	1286
83.1.2	Results, Consequences, Applications	1288
83.1.3	Some Remarks	1289
83.2	Hints for a Mathematical Description	1289
83.3	Data Analysis	1290
83.4	Conclusion	1292
83.5	Addendum	1293
84	Reviewing Flow, Gesture, and Spaces in Free Jazz	1295
84.1	Improvisation: Defining Time	1295
84.2	Flow, Gestures, Imaginary Time and Spaces in the Music Movie <i>Imaginary Time</i>	1295
84.2.1	The Compositional Character of the Pieces	1297
84.2.2	Large Forms	1298
84.2.3	Precision of Attacks	1298
84.2.4	Co-presence of Different Time Layers	1299
84.2.5	The Reality of Imaginary Time	1300
84.2.6	Measuring Flow	1300
84.2.7	Explicit Perception of Gestures	1300
85	Gesture and Vocalization	1301
85.1	Vocal Gesture	1301
85.2	Vocal and Manual Motion	1303
85.3	Gait	1304
85.4	Hindustani Vocal Music	1305
85.5	Notic Models and Kinetic Models	1306
85.6	The Realist Pitfall	1307
85.7	The Subjectivist Pitfall	1308
85.8	Speech Gesture	1311

86 Elements of a Future Vocal Gesture Theory	1313
86.1 Why a Theory of Vocal Gestures?	1313
86.1.1 Studying the Voice Without the Singer?	1314
86.1.2 Parts of the Phonatory System and Their Functions	1314
86.1.3 Imaginary Gestures in Real Time?	1315
86.1.4 Space of Voice Parameters Gestures	1315
86.1.5 About the Importance of Breathing and of Laryngeal Movements	1316
86.1.6 Mathematical Description of Vocal Gestures	1317
86.1.7 Gestures Thought by Singers	1321
86.2 A Powerful Tool from the Past	1322
86.2.1 Gestures in Gregorian Chant Didactics	1323
86.2.2 Concept of Rhythm and Time	1323
86.2.3 The Neumes	1327
86.3 Connecting Physiology, Gestures and Notation. Toward New Neumes?	1329
86.3.1 New Neumes	1330

Part XX Appendix: Sound

A Common Parameter Spaces	1335
A.1 Physical Spaces	1335
A.1.1 Neutral Data	1336
A.1.2 Sound Analysis and Synthesis	1338
A.2 Mathematical and Symbolic Spaces	1347
A.2.1 Onset and Duration	1347
A.2.2 Amplitude and Crescendo	1348
A.2.3 Frequency and Glissando	1349
B Auditory Physiology and Psychology	1353
B.1 Physiology: From the Auricle to Heschl's Gyrus	1353
B.1.1 Outer Ear	1354
B.1.2 Middle Ear	1354
B.1.3 Inner Ear (Cochlea)	1355
B.1.4 Cochlear Hydrodynamics: The Travelling Wave	1357
B.1.5 Active Amplification of the Traveling Wave Motion	1358
B.1.6 Neural Processing	1360
B.2 Discriminating Tones: Werner Meyer-Eppler's Valence Theory	1362
B.3 Aspects of Consonance and Dissonance	1364
B.3.1 Euler's Gradus Function	1364
B.3.2 von Helmholtz' Beat Model	1365
B.3.3 Psychometric Investigations by Plomp and Levelt	1367
B.3.4 Counterpoint	1367
B.3.5 Consonance and Dissonance: A Conceptual Field	1367

Part XXI Appendix: Mathematical Basics

C Sets, Relations, Monoids, Groups	1371
C.1 Sets	1371
C.1.1 Examples of Sets	1371

C.2	Relations	1372
C.2.1	Universal Constructions	1374
C.2.2	Graphs and Quivers	1375
C.2.3	Monoids	1376
C.3	Groups	1378
C.3.1	Homomorphisms of Groups	1378
C.3.2	Direct, Semi-direct, and Wreath Products	1380
C.3.3	Sylow Theorems on p -groups	1380
C.3.4	Classification of Groups	1381
C.3.5	General Affine Groups	1382
C.3.6	Permutation Groups	1382
D	Rings and Algebras	1385
D.1	Basic Definitions and Constructions	1385
D.1.1	Universal Constructions	1386
D.2	Prime Factorization	1389
D.3	Euclidean Algorithm	1389
D.4	Approximation of Real Numbers by Fractions	1389
D.5	Some Special Issues	1390
D.5.1	Integers, Rationals, and Real Numbers	1390
E	Modules, Linear, and Affine Transformations	1391
E.1	Modules and Linear Transformations	1391
E.1.1	Examples	1391
E.2	Module Classification	1392
E.2.1	Dimension	1392
E.2.2	Endomorphisms on Dual Numbers	1394
E.2.3	Semi-simple Modules	1394
E.2.4	Jacobson Radical and Socle	1395
E.2.5	Theorem of Krull-Remak-Schmidt	1396
E.3	Categories of Modules and Affine Transformations	1396
E.3.1	Direct Sums	1397
E.3.2	Affine Forms and Tensors	1397
E.3.3	Biaffine Maps	1399
E.3.4	Symmetries of the Affine Plane	1402
E.3.5	Symmetries on $\mathbb{Z}^2$	1402
E.3.6	Symmetries on $\mathbb{Z}^n$	1403
E.3.7	Complements on the Module of a Local Composition	1403
E.3.8	Fiber Products and Fiber Sums in Mod	1404
E.4	Complements of Commutative Algebra	1405
E.4.1	Localization	1406
E.4.2	Projective Modules	1406
E.4.3	Injective Modules	1407
E.4.4	Lie Algebras	1408
F	Algebraic Geometry	1411
F.1	Locally Ringed Spaces	1411
F.2	Spectra of Commutative Rings	1412
F.2.1	Sober Spaces	1413

F.3	Schemes and Functors	1414
F.4	Algebraic and Geometric Structures on Schemes	1415
F.4.1	The Zariski Tangent Space	1415
F.5	Grassmannians	1416
F.6	Quotients	1417
G	Categories, Topoi, and Logic	1419
G.1	Categories Instead of Sets	1419
G.1.1	Examples	1420
G.1.2	Functors	1421
G.1.3	Natural Transformations	1422
G.2	The Yoneda Lemma	1423
G.2.1	Universal Constructions: Adjoints, Limits, and Colimits	1423
G.2.2	Limit and Colimit Characterizations	1425
G.3	Topoi	1427
G.3.1	Subobject Classifiers	1428
G.3.2	Exponentiation	1429
G.3.3	Definition of Topoi	1429
G.4	Grothendieck Topologies	1430
G.4.1	Sheaves	1431
G.5	Formal Logic	1432
G.5.1	Propositional Calculus	1432
G.5.2	Predicate Logic	1435
G.5.3	A Formal Setup for Consistent Domains of Forms	1437
H	Complements on General and Algebraic Topology	1443
H.1	Topology	1443
H.1.1	General	1443
H.1.2	The Category of Topological Spaces	1444
H.1.3	Uniform Spaces	1444
H.1.4	Special Issues	1445
H.2	Algebraic Topology	1445
H.2.1	Simplicial Complexes	1445
H.2.2	Geometric Realization of a Simplicial Complex	1446
H.2.3	Contiguity	1447
H.3	Simplicial Coefficient Systems	1447
H.3.1	Cohomology	1447
I	Complements on Calculus	1449
I.1	Abstract on Calculus	1449
I.1.1	Norms and Metrics	1449
I.1.2	Completeness	1450
I.1.3	Differentiation	1451
I.2	Ordinary Differential Equations (ODEs)	1451
I.2.1	The Fundamental Theorem: Local Case	1452
I.2.2	The Fundamental Theorem: Global Case	1453
I.2.3	Flows and Differential Equations	1455
I.2.4	Vector Fields and Derivations	1455
I.3	Partial Differential Equations	1455

J	More Complements on Mathematics	1457
J.1	Directed Graphs	1457
J.1.1	The Category of Directed Graphs (Digraphs)	1457
J.1.2	Two Standard Constructions in Graph Theory	1459
J.1.3	The Topos of Digraphs	1459
J.2	Galois Theory	1460
J.3	Splines	1461
J.3.1	Some Simplex Constructions for Splines	1461
J.3.2	Definition of General Splines	1462
J.4	Topology and Topological Categories	1463
J.4.1	Topology	1463
J.4.2	Topological Categories	1464
J.5	Complex Analysis	1465
J.6	Differentiable Manifolds	1466
J.6.1	Manifolds with Boundary	1467
J.6.2	The Tangent Manifold	1467
J.7	Tensor Fields	1468
J.7.1	Alternating Tensors	1468
J.7.2	Tangent Tensors	1468
J.8	Stokes' Theorem	1469
J.9	Calculus of Variations	1471
J.10	Partial Differential Equations	1471
J.10.1	Explicit Calculation	1472
J.11	Algebraic Topology	1474
J.11.1	Homotopy Theory	1474
J.11.2	The Fundamental Group(oid)	1475
J.12	Homology	1475
J.12.1	Singular Homology	1476
J.13	Cohomology	1477

Part XXII Appendix: Complements in Physics

K	Complements on Physics	1481
K.1	Hamilton's Variational Principle	1481
K.1.1	Euler-Lagrange Equations for a Non-relativistic Particle	1482
K.2	String Theory	1482
K.3	Duality and Supersymmetry	1484
K.4	Quantum Mechanics	1485
K.4.1	Banach and Hilbert Spaces	1486
K.4.2	Geometry on Hilbert Spaces	1490
K.4.3	Axioms for Quantum Mechanics	1492
K.4.4	The Spectral Theorem	1493

Part XXIII Appendix: Tables

L	Euler's Gradus Function	1497
M	Just and Well-Tempered Tuning	1499

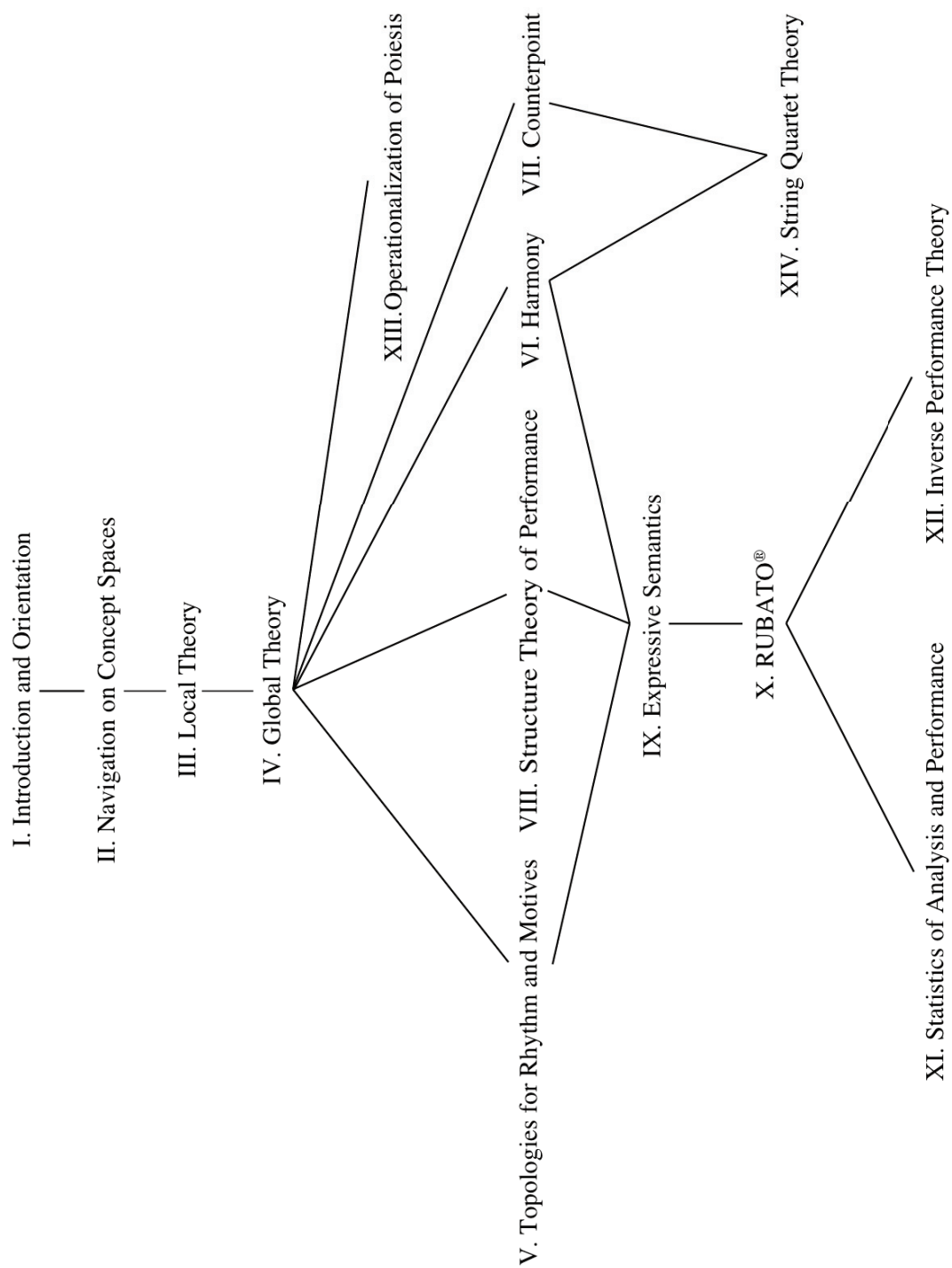
N	Chord and Third Chain Classes	1501
N.1	Chord Classes	1501
N.2	Third Chain Classes	1506
O	Two, Three, and Four Tone Motif Classes	1513
O.1	Two Tone Motifs in $OnPiMod_{12,12}$	1513
O.2	Two Tone Motifs in $OnPiMod_{5,12}$	1513
O.3	Three Tone Motifs in $OnPiMod_{12,12}$	1514
O.4	Four Tone Motifs in $OnPiMod_{12,12}$	1517
O.5	Three Tone Motifs in $OnPiMod_{5,12}$	1523
P	Well-Tempered and Just Modulation Steps	1525
P.1	12-Tempered Modulation Steps	1525
P.1.1	Scale Orbits and Number of Quantized Modulations	1525
P.1.2	Quanta and Pivots for the Modulations Between Diatonic Major Scales (No.38.1) .	1527
P.1.3	Quanta and Pivots for the Modulations Between Melodic Minor Scales (No.47.1) ..	1528
P.1.4	Quanta and Pivots for the Modulations Between Harmonic Minor Scales (No.54.1)	1530
P.1.5	Examples of 12-Tempered Modulations for All Fourth Relations	1530
P.2	2-3-5-Just Modulation Steps	1531
P.2.1	Modulation Steps Between Just Major Scales	1531
P.2.2	Modulation Steps Between Natural Minor Scales	1532
P.2.3	Modulation Steps from Natural Minor to Major Scales	1532
P.2.4	Modulation Steps from Major to Natural Minor Scales	1533
P.2.5	Modulation Steps Between Harmonic Minor Scales	1533
P.2.6	Modulation Steps Between Melodic Minor Scales	1534
P.2.7	General Modulation Behaviour for 32 Altered Scales	1535
Q	Counterpoint Steps	1537
Q.1	Contrapuntal Symmetries	1537
Q.1.1	Class No. 64	1537
Q.1.2	Class No. 68	1538
Q.1.3	Class No. 71	1539
Q.1.4	Class No. 75	1540
Q.1.5	Class No. 78	1541
Q.1.6	Class No. 82	1542
Q.2	Permitted Successors for the Major Scale	1543

Part XXIV References and Index

References	R.1
Index	R.33

Leitfaden

Leitfaden I & II



Leitfaden III

XV. Gesture Philosophy for Music	
XVI. Mathematics of Gestures	
XVII. Concept Architectures and Software for Gesture Theory	XVIII. The Multiverse Perspective
	XIX. Gestures in Music and Performance Theory, and in Ethnomusicology

XX. Appendix: Sound
XXI. Appendix: Mathematical Basics
XXII. Appendix: Complements in Physics
XXIII. Appendix: Tables
XXIV. References Bibliography Index

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