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Topological Data Analysis for Scientific Visualization

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Any problem which is non-linear in character, which involves more than one coordinate system or more than one variable, or where structure is initially defined in the large, is likely to require considerations of topology and group theory for its solution.

In the solution of such problems classical analysis will frequently appear as an instrument in the small, integrated over the whole problem with the aid of topology or group theory.

Marston Morse [87]

This book is dedicated to all those who supported me along the years.

Preface

This book is adapted from my habilitation thesis manuscript, which reviewed my research work since my Ph.D. thesis defense (2008), as a postdoctoral researcher at the University of Utah (2008–2010) and a permanent CNRS researcher at Telecom ParisTech (2010–2014) and at Sorbonne Universités UPMC (2014-present).

This book presents results obtained in collaboration with several research groups (University of Utah, Lawrence Livermore National Laboratory, Lawrence Berkeley National Laboratory, Universidade de Sao Paulo, New York University, Sorbonne Universités, Clemson University, University of Leeds) as well as students whom I informally or formally advised.

This research has been partially funded by several grants, including a Fulbright fellowship (US Department of State), a Lavoisier fellowship (French Ministry for Foreign Affairs), a Digiteo grant (national funding, "Uncertain TopoVis" project 2012-063D, Principal Investigator), an ANR grant (national funding, "CrABEx" project ANR-13-CORD-0013, local investigator), a CIFRE partnership with Renault, a CIFRE partnership with Kitware, a CIFRE partnership with Total, and a BPI grant (national funding, "AVIDO" project, local investigator).

During this period, I taught regularly at the University of Utah (2008–2010), Telecom ParisTech (2011–present), Sorbonne Universités (2011–present), and since 2013 at ENSTA ParisTech and University of Versailles, where I am the head instructor for the scientific visualization course.

This book describes most of the results published over this period (Chap. 3: [118, 125], Chap. 4: [63, 125, 128], Chap. 5: [16, 17, 55, 101], Chap. 6: [21, 56–58, 74, 117, 130]). I refer the interested reader to the following publications [12, 49, 50, 52, 80, 83, 95, 96, 107, 108, 111–113, 119–124, 126, 127, 132, 133] for additional results not described in this document.

The reading of this book only requires a basic background in computer science and algorithms; most of the mathematical notions are introduced in a dedicated chapter (Chap. 2).

Paris, France — Julien Tierny
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Second, I would like to thank all my collaborators over the last 8 years. The results presented in this book would not have been possible without them. Since my Ph.D. defense, I had the opportunity to work with more than 50 coauthors, and I like to think that I learned much from each one of them. More specifically, I would like to thank some of my main collaborators (in alphabetical order), hoping the persons I forgot to mention will forgive me: Timo Bremer, Hamish Carr, Joel Daniels, Julie Delon, Tiago Etiene, Attila Gyulassy, Pavol Klacansky, Josh Levine, Gustavo Nonato, Valerio Pascucci, Joseph Salmon, Giorgio Scorzelli, Claudio Silva, Brian Summa, Jean-Marc Thiery. Along the years, some of these persons became recurrent collaborators with whom I particularly enjoyed working and interacting. Some of them even became close friends (even best men!) and I am sincerely grateful for that. Special thanks go to Valerio Pascucci, who gave me a chance back in 2008 when he hired me as a postdoc, although we had never met before. I have no doubt that my career path would have been very different if we had not worked together. Working with Valerio and his group has been a real pleasure and a source of professional and personal development. I am both glad and proud to be able to say that our collaboration lasted well beyond my stay at the University of Utah and that it still continues today. Along the last 8 years, Valerio has been a careful and inspiring mentor and I am sincerely grateful for that.

Next, I would like to thank all of the colleagues I had the chance to interact with at the University of Utah, Telecom ParisTech, and Sorbonne Universités UPMC, in particular my students, who are a daily source of motivation.

Finally, I would like to thank my friends, my family, my wife, and my daughter for their constant love and support.

Contents

Notations

$\mathbb{X}$	Topological space
$\partial\mathbb{X}$	Boundary of a topological space
$\mathbb{M}$	Manifold
$\mathbb{R}^d$	Euclidean space of dimension d
$\sigma,\ \tau$	d-simplex, face of a d-simplex
$v,\ e,\ t,\ T$	Vertex, edge, triangle, and tetrahedron
$Lk(\sigma),\ St(\sigma)$	Link and star of a simplex
$Lk_d(\sigma),\ St_d(\sigma)$	d-simplices of the link and the star of a simplex
$\mathscr{K}$	Simplicial complex
$\mathscr{T}$	Triangulation
$\mathscr{M}$	Piecewise linear manifold
β_i	i-th Betti number
χ	Euler characteristic
α_i	ith barycentric coordinates of a point p relatively to a simplex σ
$f : \mathscr{T} \to \mathbb{R}$	Piecewise linear scalar field
∇f	Gradient of a PL scalar field f
$Lk^-(\sigma),\ Lk^+(\sigma)$	Lower and upper link of σ relatively to f
$o(v)$	Memory position offset of the vertex v
$\mathscr{L}^-(i),\ \mathscr{L}^+(i)$	Sub- and sur-level set of the isovalue i relatively to f
$\mathscr{D}(f)$	Persistence diagram of f
$\mathscr{C}(f)$	Persistence curve of f
$\mathscr{R}(f)$	Reeb graph of f
$l(\mathscr{R}(f))$	Number of loops of $\mathscr{R}(f)$
$\mathscr{T}(f)$	Contour tree of f
$\mathscr{J}(f),\ \mathscr{S}(f)$	Join and split trees of f
$\mathscr{MS}(f)$	Morse-Smale complex of f