50 Years of Integer Programming 1958–2008

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Editors

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From the Early Years to the State-of-the-Art

Springer

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We dedicate this book to the pioneers of Integer Programming.

Preface

The name integer programming refers to the class of constrained optimization problems in which some or all of the variables are required to be integers. In the most widely studied and used integer programs, the objective function is linear and the constraints are linear inequalities. The field of integer programming has achieved great success in the academic and business worlds. Hundreds of papers are published every year in a variety of journals, several international conferences are held annually and software for solving integer programs, both commercial and open source, is widely available and used by thousands of organizations. The application areas include logistics and supply chains, telecommunications, finance, manufacturing and many others.

This book is dedicated to the theoretical, algorithmic and computational aspects of integer programming. While it is not a textbook, it can be read as an introduction to the field and provides a historical perspective. Graduate students, academics and practitioners, even those who have spent most of their careers in discrete optimization, will all find something useful to learn from the material in this book. Given the amount that has been accomplished, it is remarkable that the field of integer programming began only fifty years ago.

The 12th Combinatorial Optimization Workshop AUSSOIS 2008 took place in Aussois, France, 7–11 January 2008. The workshop, entitled *Fifty Years of Integer Programming*, and this book, which resulted from the workshop, were created to celebrate the 50th anniversary of integer programming. The workshop had a total of 136 participants from 14 countries ranging in experience from pioneers who founded the field to current graduate students. In addition to the formal program, the workshop provided many opportunities for informal discussions among participants as well as a chance to enjoy the spectacular Alpine setting provided by Aussois.

The book is organized into four parts. The first day of the workshop honored some of the pioneers of the field. Ralph Gomory's path-breaking paper, showing how the simplex algorithm could be generalized to provide a finite algorithm for integer programming and published in 1958, provided the justification of the anniversary celebration. The activities of the first day, led by George Nemhauser and Bill Pulleyblank, included a panel discussion with the pioneers who attended the workshop (Egon Balas, Michel Balinski, Jack Edmonds, Arthur Geoffrion, Ralph Gomory and Richard Karp) as well as three invited talks by Bill Cook, Gérard Cornuéjols and Laurence Wolsey on integer programming and combinatorial optimization from the beginnings to the state-of-the-art. The whole day is captured in two Video DVDs which come with the book (Part IV). Parts I, II, and III contain 20 papers of historical and current interest.

Part I of the book, entitled *The Early Years*, presents, in order of publication date, reprints of eleven fundamental papers published between 1954 and 1979. Ten of these papers were selected by one or more of the authors of the paper, who also wrote new introductions to the papers that explain their motivations for working on the problems addressed and their reason for selecting the paper for inclusion in this volume. The authors are Egon Balas, Michel Balinski, Alison Doig, Jack Edmonds, Arthur Geoffrion, Ralph Gomory, Alan Hoffman, Richard Karp, Joseph Kruskal, Harold Kuhn, and Ailsa Land. Each of these heavily cited papers has had a major influence on the development of the field and lasting value. The eleventh selection, which starts this section, is a groundbreaking paper by George Dantzig, Ray Fulkerson, and Selmer Johnson, with an introduction by Vašek Chvátal and William Cook. The introduction to Part I closes with a list, in chronological order, of our selection of some of the most influential papers appearing between 1954 and 1973 pertaining to the many facets of integer programming.

Part II contains papers based on the talks given by Cornuéjols, Cook, and Wolsey. The paper *Polyhedral Approaches to Mixed Integer Programming* by Michele Conforti, Gérard Cornuéjols, and Giacomo Zambelli presents tools from polyhedral theory that are used in integer programming. It applies them to the study of valid inequalities for mixed integer linear sets, such as Gomory's mixed integer cuts. The study of combinatorial optimization problems such as the traveling salesman problem has had a significant influence on integer programming. *Fifty-plus Years of Combinatorial Integer Programming* by Bill Cook discusses these connections. In solving integer programming problems by branch-and-bound methods, it is important to use relaxations that provide tight bounds. In the third paper entitled *Reformulation and Decomposition of Integer Programs*, François Vanderbeck and Laurence Wolsey survey ways to reformulate integer and mixed integer programs to obtain stronger linear programming relaxations. Together, these three papers give a remarkably broad and comprehensive survey of developments in the last fifty-plus years and their impacts on state-of-the-art theory and methodology.

Six survey talks on current hot topics in integer programming were given at the workshop by Fritz Eisenbrand, Andrea Lodi, François Margot, Franz Rendl, Jean-Philippe P. Richard, and Robert Weismantel. These talks covered topics that are actively being researched now and likely to have substantial influence in the coming decade and beyond.

Part III contains the six papers that are based on these talks. *Integer Programming and Algorithmic Geometry of Numbers* by Fritz Eisenbrand surveys some of the most important results from the interplay of integer programming and the geometry of numbers. *Nonlinear Integer Programming* by Raymond Hemmecke, Matthias Köppe, Jon Lee, and Robert Weismantel generalizes the usual integer programming model by studying integer programs with nonlinear objective functions. *Mixed Integer Programming Computation* by Andrea Lodi discusses the important ingredients involved in building a successful mixed integer solver as well as the problems that need to be solved in building the next generation of faster and more stable solvers. Symmetry is a huge obstacle encountered in solving mixed integer programs efficiently. In *Symmetry in Integer Programming*, François Margot presents several techniques that have been used successfully to overcome this difficulty. Semidefinite programming is a generalization of linear programming that provides a tighter relaxation to integer programs than linear programs. In *Semidefinite Relaxations for Integer Programming*, Franz Rendl surveys how semidefinite models and algorithms can be used effectively in solving certain combinatorial optimization problems. In the 1960s Ralph Gomory created a new tight relaxation for integer programs based on group theory. Recently the group theoretic model has been revived in the study of two-row integer programs. In *The Group-Theoretic Approach in Mixed Integer Programming*, Jean-Philippe P. Richard and Santanu S. Dey provide an overview of the mathematical foundations and recent theoretical and computational advances in the study of the group-theoretic approach.

We close with the hope that the next fifty years will be as rich as the last fifty have been in theoretical and practical accomplishments in integer programming.

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About the Cover Illustration

The four figures on the cover illustrate adding Gomory mixed integer cuts to a polyhedron of dimension 3. The x-axis is horizontal, the y-axis is vertical and the z-axis is orthogonal to the cover. The starting polyhedron P shown in Fig. 1(a) is a cone with a square base and a peak having $y = 4.25$. P contains twelve integer lattice points. Suppose we solve the linear program: maximize y, subject to $y \in P$. The unique optimum will have $y = 4.25$. However, if we add the constraint that y be integral, then there are four optima, the lattice points illustrated on the edges of P having $y = 2$.

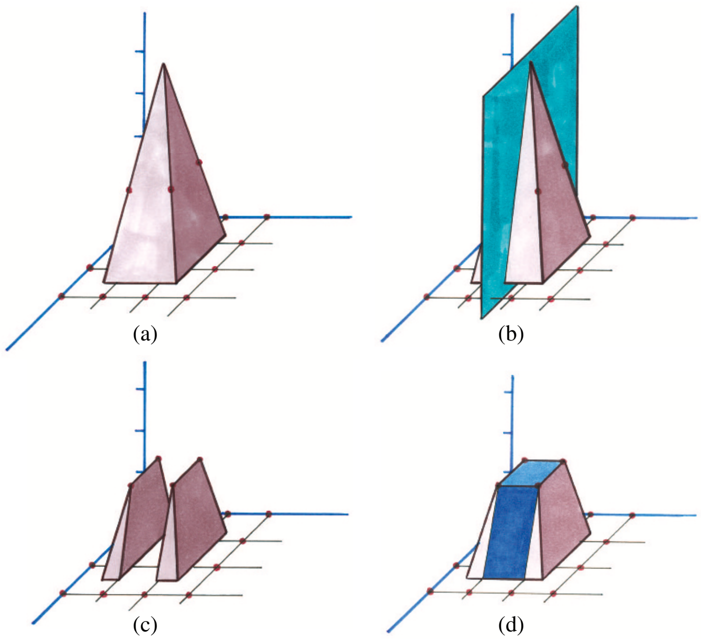

Fig. 1 The Cover Illustration.

This example is a 3-D version of a 2-D example, first shown to us by Vašek Chvátal, which Bill Cook told us that Vašek attributes to Adrian Bondy. A “standard” Chvátal-Gomory cut (CG cut) is obtained by taking a hyperplane that supports a polyhedron and which contains no lattice points in space, then moving in a direction orthogonal to the hyperplane into the polyhedron until it hits a lattice point somewhere in space (not necessarily in the polyhedron). This gives a new valid inequality for all lattice points in the polyhedron, and which cuts off part of the original polyhedron. Gomory’s fundamental result described a finite algorithm that, given any integer program, would automatically generate a finite sequence of CG cuts such that when they were added, the resulting linear program would have an integer optimum.

What cuts must be added to P to remove all points having $y > 2$? How do we generate the inequality $y \leq 2$ which must be added if the resulting linear program is going to have an integral optimum? The Bondy-Chvátal example showed that, even for dimension 2, the number of CG cuts that would have to added was unbounded, depending only on the height of the peak of the pyramid (provided that we adjust the base so that the lattice points in P having $y = 2$ continue to lie on the edges). In particular, the number of CG cuts that need to be added to solve an integer program is independent of the dimension of the polyhedron, and is not polynomial in the size of a linear system necessary to define the original polyhedron.

In 1960, Gomory described a method to generate so-called mixed integer cuts. These cuts have turned out to be very powerful in practice, both for integer and mixed integer programs. They work as follows: Take a hyperplane that intersects the polyhedron and passes through no lattice points in space. In Fig. 1(b), we chose the hyperplane $x = 1.5$. Note that it passes right through P. Consider the inequalities $x \leq 1$ and $x \geq 2$ which are obtained by shifting the hyperplane left and right respectively, until it hits a lattice point in space. We construct two new polyhedra P_1 and P_2 from P, one by adding the inequality $x \leq 1$ and one by adding $x \geq 2$. Then every lattice point in P will belong to one of P_1 and P_2.

These two polyhedra are the two wedges shown in Fig. 1(c). Note that every lattice point contained in P is in one of the two wedges.

The final step is to take the convex hull of the union of P_1 and P_2. This is the polyhedron shown in Fig. 1(d). Note that one hyperplane was used to create two subproblems. Then by maximizing y over these two subproblems, we get the solution we are seeking. Balas, Ceria and Cornuéjols describe a method called *lift-and-project* for generating a cut after a polyhedron has been split into two subpolyhedra. This is discussed in Balas’ introduction to Chapter 10.

Also, everything we have done remains valid if x and z are allowed to be continuous variables and only y is required to be integral. For this reason, these types of cuts are usually called “mixed integer cuts”.

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