Lecture Notes Electrical Engineering

Volume 50

Yasumichi Hasegawa

# Algebraically Approximate and Noisy Realization of Discrete-Time Systems and Digital Images

Springer

Yasumichi Hasegawa
Gifu University
1-1 Yanagido
Gifu, 501-11
Japan
E-mail: yhasega@gifu-u.ac.jp

ISBN 978-3-642-03216-5 e-ISBN 978-3-642-03217-2

DOI 10.1007/978-3-642-03217-2

Library of Congress Control Number: 2009920581



*Typeset & Coverdesign:* Scientific Publishing Services Pvt. Ltd., Chennai, India.

Printed in acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

# Preface

This monograph deals with approximation and noise cancellation of dynamical systems which include linear and nonlinear input/output relationships. It also deal with approximation and noise cancellation of two dimensional arrays. It will be of special interest to researchers, engineers and graduate students who have specialized in filtering theory and system theory and digital images. This monograph is composed of two parts. Part I and Part II will deal with approximation and noise cancellation of dynamical systems or digital images respectively. From noiseless or noisy data, reduction will be made. A method which reduces model information or noise was proposed in the reference vol. 376 in LNCIS [Hasegawa, 2008]. Using this method will allow model description to be treated as noise reduction or model reduction without having to bother, for example, with solving many partial differential equations. This monograph will propose a new and easy method which produces the same results as the method treated in the reference. As proof of its advantageous effect, this monograph provides a new law in the sense of numerical experiments. The new and easy method is executed using the algebraic calculations without solving partial differential equations. For our purpose, many actual examples of model information and noise reduction will also be provided.

Using the analysis of state space approach, the model reduction problem may have become a major theme of technology after 1966 for emphasizing efficiency in the fields of control, economy, numerical analysis, and others. Noise reduction problems in the analysis of noisy dynamical systems may have become a major theme of technology after 1974 for emphasizing efficiency in control. However, the subjects of these researches have been mainly concentrated in linear systems.

In common model reduction of linear systems in use today, a singular value decomposition of a Hankel matrix is used to find a reduced order model. However, the existence of the conditions of the reduced order model are derived without evaluation of the resultant model. In the common typical noise reduction of linear systems in use today, the order and parameters of the systems are determined by minimizing information criterion.

Algebraically approximate and noisy realization problems for input/output relations can be roughly stated as follows:

**A. The algebraically approximate realization problem.**
For any input/output map, find, using only algebraic calculations, one mathematical model such that it is similar to the input/output map and has a lower dimension than the given minimal state space of a dynamical system which has the same behavior to the input/output map.

**B. The algebraically noisy realization problem.**
For any input/output map which includes noises in output, find, using only algebraic calculations, one mathematical model which has the same input/output map.

Based on these parameters, we have been able to demonstrate that our new method for nonlinear dynamical systems, fully discussed in this monograph, is effective. It is worth remembering that the development of approximate and noisy realization has been strongly stimulated by linear system theory and is well-connected to related mathematics, such as for example, matrix theory or mathematical programming. However, such development of nonlinear dynamical systems has not occurred yet because there have been no suitable mathematical methods for nonlinear systems.

In this monograph, in relation to the approximate quantity of noiseless data as being the noisy part of noisy data, we have identified the approximate realization problem as a noisy realization problem in the sense of how to unify the treatment of these problems. Our method intensively takes a positive attitude toward using computers. We will introduce a new method called the Algebraically Constrained Least Square method, for brevity, Algebraic CLS method. This method can be unified to solve both an approximate and a noisy realization problem. In the reference [Hasegawa, 2008], a CLS method was proposed for solving approximate and noisy realization problems. The method is called the analytically Constrained Least Square method, for brevity, Analytic CLS method. The analytic CLS method demands the solution of partial differential equations. Therefore, the method is too cubersome.
The proposed method seeks to find, using only algebraic calculations, the coefficients of a linear combination without the notion of orthogonal projection, and it can be applied to both approximate and noisy realization problems, i.e., in the sense of a unified manner for both approximate and noisy realization problems, a new method will be proposed that provides effective results. As has already been mentioned, common approximate and noisy realization problems have been mainly discussed via linear systems. On the other hand, there have been few developments regarding nonlinear systems. Our recent monograph, *Realization Theory of Discrete-Time Dynamical Systems* (T. Matsuo and Y. Hasegawa, Lecture Notes in Control and Information

Science, Vol. 296, Springer, 2003), indicated that any input/output map of nonlinear dynamical systems can be characterized by the Hankel matrix or the Input/output matrix. The monograph also demonstrated that obtaining a dynamical system which describes a given input/output map is equal to determining the rank of the matrix of the input/output map and the coefficients of a linear combination of column vectors in the matrix. This new insight leads to the ability of discussing fruitful approximate and noisy realization problems.

Part II deals with approximate and noisy realization problems of digital images, especially two dimensional arrays. In the reference [Hasegawa and Suzuku, 2006], a realization problem of two dimensional arrays was established over a field. In the case of real numbers, we can easily discuss approximate and noisy realization problems of two dimensional arrays in the same manner as in Part I. Therefore, we want to omit the problems over real number fields.

Also in Part II, we discuss approximate and noisy realization problems over a finite field, i.e., the quotient field modulo of the prime number. Because we cannot introduce the norm for a finite field, our problems for two dimensional arrays can be roughly stated as follows:

**A. The algebraically approximate realization problem.**
For any two dimensional array, find, using only algebraic calculations, one mathematical model such that it is similar to the two dimensional array and has a lower dimension than the given minimal state space of a mathematical model which has the same behavior to the array.

**B. The algebraically noisy realization problem.**
For any two dimensional array which includes noises in output, find, using only algebraic calculations, one mathematical model which has the same two dimensional array.

The problems will be treated as a sort of non-linear integer programming. We will propose a new method which is suitable for our problems. The method will be called a non-linear integer programming for digital images.

We wish to acknowledge Professor Tsuyoshi Matsuo, who established the foundation for realization theory of continuous and discrete-time dynamical systems, and who taught me much regarding realization theory for discrete-time non-linear systems. He would have been an author of this monograph, but in April sixteen years ago he sadly passed away. We gratefully consider him one of the authors of this manuscript in spirit.

We also wish to thank Professor R. E. Kalman for his suggestions. He stimulated us to research these realization problems directly as well as through his works. We also thank Professor Gary B. White for making the first manuscript into a more readable and elegant one.

On the occasion of becoming 65 Yasumichi Hasegawa

# Contents