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Models for Capacity Demand Estimation in a TV Broadcast Network with Variable Bit Rate TV Channels

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Abstract. Mobile TV is growing beyond the stage of experimentation and evaluation and is (about) to become part of our daily lives. Additionally, it is being delivered through heterogeneous networks and to a variety of receiving devices, which implies different versions of one and the same video content must be transported. We propose two (approximate) analytic methods for capacity demand estimation in a (mobile) TV broadcast system. In particular, the methods estimate the required transport capacity for a bouquet of channels offered on request and in different versions (video formats or in different quality) over a multicast-enabled network, encoded in non-constant bit rate targeting constant quality. We compare a transport strategy where the different versions (of one channel) are simulcast to a scalable video encoding (SVC) transport strategy, where all resolutions (of one channel) are embedded in one flow. In addition, we validate the proposed analytic methods with simulations. A realistic mobile TV example is considered with two transported resolutions of the channels: QVGA and VGA. We demonstrate that not always capacity gain is achieved with SVC as compared to simulcast since the former comes with some penalty rate and the gain depends on the system parameters.

1 Introduction

The compression efficiency of the new codecs H.264/MPEG-4 AVC [1] with respect to the codecs MPEG-2 [2] and MPEG-4 Part 2 [3] makes that the former codecs are imposing more and more in digital (IP-based) multimedia delivery systems. An extension to the AVC codec, called SVC (Scalable Video Coding) [4], allows for scalable, multi-layer encoding. A base layer ensures some basic quality of experience (e.g., targets a small-resolution device) and is complemented by one

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or several enhancement layers improving the video quality or targeting higher-resolution display screens. We consider a broadcast network where a number of channels are subject to multicast streaming to receivers of different classes of devices (resolutions) or different quality classes. The question arises which of the two following scenarios requires the least transport capacity. The first scenario simulcasts all resolutions (encoded in H.264 AVC) of all channels that are requested by at least one user. The second scenario exploits the scalability property of H.264 SVC that the bit stream of one particular resolution embeds all lower resolutions and transmits only the highest resolution that is requested by at least one user. Since all resolutions below that particular resolution piggy-back along in the stream, no other resolutions need to be transmitted in parallel in this scenario.

In [5], we demonstrated for constant bit rate (CBR) encoded video that the answer to the question formulated above is not straightforward. It is basically a trade-off between the fact that SVC encoding comes with a certain penalty rate and the fact that simulcast sends all the requested resolutions in parallel, while SVC has to transmit only one embedded stream. On the other hand, this embedded stream in the SVC case has to contain all the layers up to the highest requested resolution regardless of whether the lower ones are requested or not, while simulcast only sends the resolutions that are requested at least by one user. Whereas in our previous work [5] we considered that the channels are CBR encoded, here we describe the bit rate fluctuation of a channel stochastically by a given distribution based on real data. The extended model takes into account the fact that the required bit rates for the resolutions of a given channel are content dependent and correlated.

The rest of the paper is structured as follows. In Sect. 2, we describe the system under study. We propose two analytic methods for capacity demand estimation in Sect. 3 and 4. Our simulation approach is presented in Sect. 5. Numerical results are discussed in Sect. 6, and finally the paper is concluded in Sect. 7.

2 System under Study

We aim at developing models to estimate the capacity demand on the aggregation link in a (mobile) TV broadcast system of digital (IP-transported) channels, where a bouquet of TV channels is offered to an audience of subscribers. Aggregation network may imply also the wireless radio interface depending on the particular network design. A bouquet of K TV channels are broadcast (multicast) to the (active) subscribers (users) on request. The bit rate of a streamed channel is not constant, since constant quality is aimed for by the operator. Moreover, a system with multi-resolution receiver devices is envisioned, requiring that the content of the streamed channels is encoded in L different resolutions, either by providing separate independent versions (simulcast scenario) or by encoding them in scalable embedded/multi-layered streams (e.g., in SVC). The bouquet of offered channels is characterised by a given channel popularity distribution from which the probability ρ_k with $k \in \{1, \ldots, K\}$ that a certain channel is requested for watching is drawn. Very often, e.g. in [6], this distribution is approximated by a Zipf law [7]:

$$\rho_k = dk^{-\alpha}\,, \tag{1}$$

where d is a normalisation constant ensuring that the sum of all the probabilities ρ_k is 1, and α is the parameter of the Zipf distribution, ranging typically between 0.5 and 1.

We denote the *capacity demand* $\mathbf{C}$ under a simulcast scenario by $\mathbf{C}_{SIM}$, and under the SVC scenario by $\mathbf{C}_{SVC}$. Remark that these are random variables, since there is randomness in a user's behaviour: a user watches television or not, he selects a TV channel (respectively streamed video content) among K available channels, he is requesting it in a given resolution. We also assume that if the multicast channels are scalably encoded, only the layers up to and including the highest requested one are multicast, since otherwise, it would result in a waste of network resources.

Once a user is tuned in to a particular channel, and in contrast to [5], we assume that the required bit rate of this channel is fluctuating. We only consider fluctuations associated with the succession of scenes of programmes transported over that (selected) channel (because we assume that fluctuations associated with frame type are smoothened by the shaper in the encoder [8]). In the general case, we assume that there are no dependencies between the bit rates of the different channels but there is a dependence between the rates of the different resolutions of one channel. Under resolution we mean the different versions under simulcast (SIM) streaming or the different layers in scalable encoding of a channel under SVC streaming. This correlation between the different resolutions accounts for the fact that if a high bit rate is produced in one scene, this is probably the case for all versions/layers of a channel transporting the same content. With this assumption, a multicast channel's resolution generates with rate $\mathbf{r}_\ell$, where ℓ stands for the resolution and $\ell \in \{1, \ldots, L\}$. Remark that for simulcast (SIM) each resolution ℓ has its own specific bit rate $\mathbf{r}_\ell$, while for SVC to subscribe to resolution ℓ all layers up to the streamed layer ℓ need to be received, i.e., in the SVC case we denote by $\mathbf{r}_\ell$ the total bit rate of all layers 1 to ℓ. The channel's versions/layers have a joint distribution $\Pi_{\{\mathbf{r}_1,\ldots,\mathbf{r}_\ell,\ldots,\mathbf{r}_L\}}(r_1, \ldots, r_\ell, \ldots, r_L)$ with L marginal distributions $\{\pi_{\mathbf{r}_\ell}\}$ corresponding to the distribution associated to resolution ℓ from which its rate is drawn. In this work, we assume that all the channels behave statistically the same, hence have the same joint distribution (and resulting marginal distributions). In the first analytical method we will consider, we will only need the vectors of the resolutions' average rates and the covariance matrices. In the second analytical approach, a recursive method, the distribution of the rates of the resolutions is approximated by a discrete histogram in order to be able to calculate the convolutions of histograms (marginal distributions) numerically approximating probability density functions.

Furthermore, we assume that in all offered resolutions the channel popularity distribution (the Zipf distribution) is the same, i.e., a user associated to a given resolution selects which channel to watch according to the popularity law given in (1) irrespective of the resolution.

2.1 Capacity Demand under Simulcast

We define the random variables $\mathbf{n}_{k,\ell}$, where $1 \leq k \leq K, 1 \leq \ell \leq L$, as the number of users watching channel k in resolution ℓ. Because with simulcast every channel is encoded in L independent versions, and a channel is streamed in version ℓ if at least one user watches that channel in resolution (version) ℓ, we can express $\mathbf{C}_{SIM}$ in the following way:

$$\mathbf{C}_{SIM} = \sum_{k=1}^{K} \sum_{\ell=1}^{L} \mathbf{r}_\ell 1_{\{\mathbf{n}_{k,\ell}>0\}} , \tag{2}$$

where $1_{\{m\}}$ is the indicator function of the event m, expressing that there is a contribution to $\mathbf{C}_{SIM}$ from a channel k if it is watched by at least one user in resolution ℓ.

2.2 Capacity Demand under SVC

With scalable video encoding, in order to watch a channel in resolution ℓ, all layers 1 until ℓ of that channel are needed for decoding, because they are all interrelated (in this paper we do not consider scalable encoding such as e.g., multiple description coding, MDC). Thus, layer 1 until ℓ of channel k need to be transported if there is at least one user watching channel k in resolution ℓ, and if there is no user watching channel k in a higher resolution than ℓ. Therefore, $\mathbf{C}_{SVC}$ can be expressed as follows:

$$\mathbf{C}_{SVC} = \sum_{k=1}^{K} \sum_{\ell=1}^{L} \mathbf{r}_\ell 1_{\{\mathbf{n}_{k,L}=0,\ldots,\mathbf{n}_{k,\ell+1}=0,\mathbf{n}_{k,\ell}>0\}} . \tag{3}$$

2.3 Channel Viewing Probabilities

We assume that a user (with his receiving device) is associated only to a given resolution. The group of N users (subscribers) is divided in L fixed sets. Within a set, all clients use a terminal capable of receiving resolution ℓ, and there are N_ℓ users of type ℓ so that the sum of all N_ℓ over all L resolutions/sets is N.

The TV channels are assumed to have independent probabilities ρ_k of being watched, which are proportional to the popularity of the channels, drawn as explained above from a Zipf distribution.

We calculate the following probability generating function for the joint probability of the channels being watched by a given set of users:

$$F(z_{k,\ell}; \forall k, \ell) = E\left[\prod_{k=1}^{K} \prod_{\ell=1}^{L} z_{k,\ell}^{\mathbf{n}_{k,\ell}}\right] , \tag{4}$$

where $\sum_{k=1}^{K} \mathbf{n}_{k,\ell} = \mathbf{n}_\ell \leq N_\ell$.

Inside a set of N_ℓ users associated to layer ℓ, a user has an activity grade a_ℓ, the probability of being active watching a channel, which is the average fraction of the time that a user of type ℓ watches television. We then have that the sample space corresponding to the vector of random variables $(\mathbf{n}_{1,1}, \mathbf{n}_{2,1}, \ldots, \mathbf{n}_{K,L})$ is constituted by all tuples $(n_{1,1}, n_{2,1}, \ldots, n_{K,L})$ for which $\sum_{k=1}^{K} n_{k,\ell} = n_\ell \leq N_\ell$ (note that the random variables are expressed in Bold typeface, unlike the values they assume). The probability corresponding to a realisation of the variable, expressed by the tuple $(n_{1,1}, n_{2,1}, \ldots, n_{K,L})$, is given by:

$$\Pr\begin{bmatrix} \mathbf{n}_{1,1} = n_{1,1},\ \mathbf{n}_{2,1} = n_{2,1},\ \ldots,\ \mathbf{n}_{K,1} = n_{K,1}, \\ \cdots \\ \mathbf{n}_{1,\ell} = n_{1,\ell},\ \mathbf{n}_{2,\ell} = n_{2,\ell},\ \ldots,\ \mathbf{n}_{K,\ell} = n_{K,\ell}, \\ \cdots \\ \mathbf{n}_{1,L} = n_{1,L}, \mathbf{n}_{2,L} = n_{2,L}, \ldots, \mathbf{n}_{K,L} = n_{K,L} \end{bmatrix}$$
$$= \prod_{\ell=1}^{L} \left[\frac{N_\ell!}{(N_\ell - n_\ell)!} (1 - a_\ell)^{N_\ell - n_\ell} a_\ell^{n_\ell} \prod_{k=1}^{K} \frac{(\rho_k)^{n_{k,\ell}}}{n_{k,\ell}!} \right], \tag{5}$$

to which corresponds the following probability generating function:

$$F(z_{k,\ell}; \forall k, \ell) = \prod_{\ell=1}^{L} \left[1 - a_\ell + a_\ell \sum_{k=1}^{K} \rho_k z_{k,\ell} \right]^{N_\ell}. \tag{6}$$

The channel viewing probabilities associated to other user models can be calculated by first deriving the probability generating function corresponding to the model.

3 Capacity Demand – A Gaussian Approximation Approach

We assume first that the aggregate capacity demand is a Gaussian variable, thus, provided its average and variance (standard deviation) are known, the distribution can be found. For practical reasons we express it in the form of the complementary cumulative distribution function (CCDF), also referred to as tail distribution function (TDF). This is the probability that the required bandwidth exceeds a certain value C_{avail}, i.e., $\Pr[\{\mathbf{C}_{SIM}, \mathbf{C}_{SVC}\} > C_{avail}]$.

3.1 Average of the Aggregate Flow

The average of the aggregate flow $\mathbf{C}$ from multicast channels in L resolutions is the sum of the averages of the contributions of all the resolutions in both simulcast (SIM) or scalable encoding (SVC) scenarios.

Simulcast. In the simulcast case, the mean of the aggregate traffic $\mathbf{C}_{SIM}$ is:

$$E[\mathbf{C}_{SIM}] = \sum_{\ell=1}^{L} \sum_{k=1}^{K} E\left[\mathbf{r}_\ell 1_{\{\mathbf{n}_{k,\ell} > 0\}}\right] = \sum_{\ell=1}^{L} \sum_{k=1}^{K} E[\mathbf{r}_\ell]\, E\left[1_{\{\mathbf{n}_{k,\ell} > 0\}}\right]$$
$$= \sum_{\ell=1}^{L} E[\mathbf{r}_\ell] \sum_{k=1}^{K} \Pr[\mathbf{n}_{k,\ell} > 0], \tag{7}$$

where $E[\mathbf{r}_\ell]$ is the average rate of resolution ℓ. Here we have used the property that the average of an indicator function is the probability of its event. $\Pr[\mathbf{n}_{k,\ell} > 0]$ expresses the probability that a given channel k is requested in the given resolution ℓ by at least one user (hence the channel is multicast streamed).

SVC. In the SVC case, the mean of the aggregate traffic is:

$$\begin{aligned} E[\mathbf{C}_{SVC}] &= \sum_{\ell=1}^{L}\sum_{k=1}^{K} E\left[\mathbf{r}_\ell 1_{\{\mathbf{n}_{k,L}=0,\ldots,\mathbf{n}_{k,\ell+1}=0,\mathbf{n}_{k,\ell}>0\}}\right] \\ &= \sum_{\ell=1}^{L}\sum_{k=1}^{K} E[\mathbf{r}_\ell]\, E\left[1_{\{\mathbf{n}_{k,L}=0,\ldots,\mathbf{n}_{k,\ell+1}=0,\mathbf{n}_{k,\ell}>0\}}\right] \\ &= \sum_{\ell=1}^{L} E[\mathbf{r}_\ell] \sum_{k=1}^{K} \Pr[\mathbf{n}_{k,L}=0,\ldots,\mathbf{n}_{k,\ell+1}=0,\mathbf{n}_{k,\ell}>0]\,, \end{aligned} \tag{8}$$

where $E[\mathbf{r}_\ell]$ stands for the average contribution of a scalable flow of up to and including layer ℓ.

3.2 Variance of the Aggregate Flow

In order to obtain the variance, first the second-order moments of the aggregate traffic under the SIM and SVC scenarios (respectively of $\mathbf{C}_{SIM}$ and $\mathbf{C}_{SVC}$) are derived below. With the index k for the probabilities that a channel k is watched by a given number of users $\mathbf{n}_{k,\ell}$, we make distinction between the channels, as there is correlation between the rates of the versions/layers of one and the same channel, but there is no correlation between those of different channels.

Simulcast. The second-order moment of $\mathbf{C}_{SIM}$ is expressed as follows:

$$\begin{aligned} E\left[\mathbf{C}_{SIM}^2\right] &= \sum_{k_1=1}^{K}\sum_{k_2=1}^{K}\sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} E\left[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2} 1_{\{\mathbf{n}_{k_1,\ell_1}>0,\mathbf{n}_{k_2,\ell_2}>0\}}\right] \\ &= \sum_{k=1}^{K}\sum_{\ell=1}^{L} E\left[{\mathbf{r}_\ell}^2\right] E\left[1_{\{\mathbf{n}_{k,\ell>0}\}}\right] + \sum_{k=1}^{K}\sum_{\ell_1=1}^{L}\sum_{\substack{\ell_2=1\\ \ell_2\neq\ell_1}}^{L} E[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2}]\, E\left[1_{\{\mathbf{n}_{k,\ell_1}>0,\mathbf{n}_{k,\ell_2}>0\}}\right] \\ &\quad + \sum_{k_1=1}^{K}\sum_{\substack{k_2=1\\ k_2\neq k_1}}^{K}\sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} E[\mathbf{r}_{\ell_1}]\, E[\mathbf{r}_{\ell_2}]\, E\left[1_{\{\mathbf{n}_{k_1,\ell_1}>0,\mathbf{n}_{k_2,\ell_2}>0\}}\right] \\ &= \sum_{k=1}^{K}\sum_{\ell=1}^{L} E\left[{\mathbf{r}_\ell}^2\right] \Pr[\mathbf{n}_{k,\ell>0}] + \sum_{k=1}^{K}\sum_{\ell_1=1}^{L}\sum_{\substack{\ell_2=1\\ \ell_2\neq\ell_1}}^{L} E[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2}] \Pr[\mathbf{n}_{k,\ell_1}>0,\mathbf{n}_{k,\ell_2}>0] \\ &\quad + \sum_{k_1=1}^{K}\sum_{\substack{k_2=1\\ k_2\neq k_1}}^{K}\sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} E[\mathbf{r}_{\ell_1}]\, E[\mathbf{r}_{\ell_2}] \Pr[\mathbf{n}_{k_1,\ell_1}>0,\mathbf{n}_{k_2,\ell_2}>0]\,. \end{aligned} \tag{9}$$

In (9) we accounted for the fact that the probabilities of the bit rates are independent from the probabilities of requesting a channel, and also for the fact that the rates of the different resolutions of one and the same channel are correlated. The following relation holds for the covariance: $\mathrm{Cov}[\mathbf{r}_{\ell_1}, \mathbf{r}_{\ell_2}] = E[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2}] - E[\mathbf{r}_{\ell_1}]\, E[\mathbf{r}_{\ell_2}]$. The terms $E[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2}]$ in (9) are thus directly related to the covariance terms $\mathrm{Cov}[\mathbf{r}_{\ell_1}, \mathbf{r}_{\ell_2}]$ which compose a covariance matrix, the main diagonal of which is formed by the variance terms $\mathrm{Var}[\mathbf{r}_\ell]$ related to $E\left[{\mathbf{r}_\ell}^2\right]$.

The square of the average of $\mathbf{C}_{SIM}$ can be also expressed as three types of terms corresponding to the three types of terms in (9). The variance of the aggregate traffic $\mathbf{C}_{SIM}$ is $\mathrm{Var}[\mathbf{C}_{SIM}] = E\left[\mathbf{C}_{SIM}^2\right] - E[\mathbf{C}_{SIM}]^2$. Under simulcast, the variance of $\mathbf{C}_{SIM}$ has the following form:

$$\begin{aligned}
\mathrm{Var}[\mathbf{C}_{SIM}] = &\sum_{k=1}^{K}\sum_{\ell=1}^{L} \Pr[\mathbf{n}_{k,\ell} > 0]\Big(E\left[\mathbf{r}_\ell^2\right] - E[\mathbf{r}_\ell]^2 \Pr[\mathbf{n}_{k,\ell} > 0]\Big) \\
&+ \sum_{k=1}^{K}\sum_{\ell_1=1}^{L}\sum_{\substack{\ell_2=1\\ \ell_2\neq\ell_1}}^{L} \Big(E[\mathbf{r}_{\ell_1}\mathbf{r}_{\ell_2}]\Pr[\mathbf{n}_{k,\ell_1} > 0, \mathbf{n}_{k,\ell_2} > 0] \\
&\qquad - E[\mathbf{r}_{\ell_1}]\, E[\mathbf{r}_{\ell_2}]\Pr[\mathbf{n}_{k,\ell_1} > 0]\Pr[\mathbf{n}_{k,\ell_2} > 0]\Big) \\
&+ \sum_{k_1=1}^{K}\sum_{\substack{k_2=1\\ k_2\neq k_1}}^{K}\sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} E[\mathbf{r}_{\ell_1}]\, E[\mathbf{r}_{\ell_2}] \Big(\Pr[\mathbf{n}_{k_1,\ell_1} > 0, \mathbf{n}_{k_2,\ell_2} > 0] \\
&\qquad - \Pr[\mathbf{n}_{k_1,\ell_1} > 0]\Pr[\mathbf{n}_{k_2,\ell_2} > 0]\Big)\,.
\end{aligned} \tag{10}$$

If we denote by $\mathbf{R}_{k\ell}$ the variable $\mathbf{r}_\ell 1_{\{\mathbf{n}_{k,\ell}>0\}}$, we can rewrite more concisely and meaningfully the equation above. The terms in the first double sum in the equation above correspond to the sum of the variances of the contribution of resolution ℓ of channel k to the aggregate flow, $\mathrm{Var}[\mathbf{R}_{k\ell}]$. Similarly, the second type of terms correspond to the covariance between two resolutions of a channel, and the third type of terms correspond to the contribution of the covariance between channels. The equation above can be replaced by the more elegant one given below:

$$\begin{aligned}
\mathrm{Var}[\mathbf{C}_{SIM}] = &\sum_{k=1}^{K}\sum_{\ell=1}^{L}\mathrm{Var}[\mathbf{R}_{k\ell}] + \sum_{k=1}^{K}\sum_{\ell_1=1}^{L}\sum_{\substack{\ell_2=1\\ \ell_2\neq\ell_1}}^{L}\mathrm{Cov}[\mathbf{R}_{k\ell_1}, \mathbf{R}_{k\ell_2}] \\
&+ \sum_{k_1=1}^{K}\sum_{\substack{k_2=1\\ k_2\neq k_1}}^{K}\sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L}\mathrm{Cov}[\mathbf{R}_{k_1\ell_1}, \mathbf{R}_{k_2\ell_2}]\,.
\end{aligned} \tag{11}$$

SVC. The formula for the variance of $\mathbf{C}_{SVC}$ is the same as (10) but with the following substitutions:

$$\begin{aligned}
&\Pr[\mathbf{n}_{k,\ell} > 0] \quad \text{replaced by} \\
&\qquad \Pr[\mathbf{n}_{k,L} = 0, \ldots, \mathbf{n}_{k,\ell+1} = 0, \mathbf{n}_{k,\ell} > 0]\,; && (12)\\
&\Pr[\mathbf{n}_{k,\ell_1} > 0, \mathbf{n}_{k,\ell_2} > 0]\,(\text{where}\,\ell_1 \neq \ell_2)\,\text{replaced by}\,0; && (13)\\
&\Pr[\mathbf{n}_{k_1,\ell_1} > 0, \mathbf{n}_{k_2,\ell_2} > 0]\,(\text{where}\,k_1 \neq k_2)\,\text{replaced by} \\
&\qquad \Pr[\mathbf{n}_{k_1,L} = 0, \ldots, \mathbf{n}_{k_1,\ell_1+1} = 0, \mathbf{n}_{k_1,\ell_1} > 0, \\
&\qquad\quad \mathbf{n}_{k_2,L} = 0, \ldots, \mathbf{n}_{k_2,\ell_2+1} = 0, \mathbf{n}_{k_2,\ell_2} > 0]\,. && (14)
\end{aligned}$$

3.3 Calculation of the Probabilities

In Sect. 2.3, the channel viewing probabilities were defined. If the probability generating function $F(z_{k,\ell}; \forall k, \ell)$ associated to a certain user behaviour model is known, then the probability that some of the $\mathbf{n}_{k,\ell}$ are equal to 0 is obtained as follows: those $z_{k,\ell}$ corresponding to $\mathbf{n}_{k,\ell} = 0$ are set to 0, the rest of the arguments $z_{k,\ell}$ are set to 1. Let $\mathbf{n}_{k,\ell} = 0$ be the event A; let $\mathbf{n}_{k,L} = 0, \ldots, \mathbf{n}_{k,\ell+1} = 0$ be the event B and $\mathbf{n}_{k,L} = 0, \ldots, \mathbf{n}_{k,\ell+1} = 0, \mathbf{n}_{k,\ell} = 0$ be the event C. Additionally, we put a subscript 1 or respectively 2 if ℓ (and k) has a subscript 1 or respectively 2. The following relations are taken into account to derive the probabilities appearing in equations (7)–(14):

$$\begin{aligned}
&\Pr[\bar{A}] = 1 - \Pr[A]\,, \\
&\Pr[\bar{A}_1 \cap \bar{A}_2] = 1 - \Pr[A_1] - \Pr[A_2] + \Pr[A_1 \cap A_2]\,, \\
&\Pr[\bar{A} \cap B] = \Pr[B] - \Pr[C]\,, \\
&\Pr[\bar{A}_1 \cap \bar{A}_2 \cap B_1 \cap B_2] \\
&= \Pr[B_1 \cap B_2] - \Pr[C_1 \cap B_2] - \Pr[B_1 \cap C_2] + \Pr[C_1 \cap C_2]\,. && (15)
\end{aligned}$$

4 Capacity Demand – A Recursive Analytical Approach

Enumerating every possible outcome of the random variable of the capacity demand and calculating its associated probability would lead to a combinatorial explosion. Therefore, we use a recursive approach in this section (an extension of the "divide and conquer approach" in [5]) in order to calculate the CCDF (TDF) of the capacity demand. Since the bit rate of a resolution of a channel is no longer deterministic as in [5], but described by a probability density function (pdf), we will need to approximate this pdf by a discretely binned histogram, which has as consequence that this approach is, in contrast to [5], no longer exact. We define for every $\ell \in \{1, \ldots, L\}$ the random variables $\mathbf{w}_\ell$ and $\mathbf{c}_\ell$, where $\mathbf{w}_\ell$ represents the number of users that watch a channel in resolution ℓ; in the simulcast case $\mathbf{c}_\ell$ denotes the number of channels that need to be provided in version ℓ, while in the SVC case $\mathbf{c}_\ell$ denotes the number of channels for which layers 1 until ℓ need to be transported, but no layers higher than ℓ. Let $\mathcal{C}$, respectively $\mathcal{W}$, denote the set of all possible values the vector of random variables $(\mathbf{c}_1, \ldots, \mathbf{c}_L)$, respectively $(\mathbf{w}_1, \ldots, \mathbf{w}_L)$, can take. In [5], an exact recursive method was presented to calculate the probabilities $\Pr[(\mathbf{c}_1, \ldots, \mathbf{c}_L) = (c_1, \ldots, c_L) | (\mathbf{w}_1, \ldots, \mathbf{w}_L) = (w_1, \ldots, w_L)]$ for all $(c_1, \ldots, c_L) \in \mathcal{C}$ and all $(w_1, \ldots, w_L) \in \mathcal{W}$.

Then by the theorem on total probability, we obtain for all $(c_1, \ldots, c_L) \in \mathcal{C}$:

$$\Pr[(\mathbf{c}_1, \ldots, \mathbf{c}_L) = (c_1, \ldots, c_L)] = \sum_{(w_1,\ldots,w_L)\in\mathcal{W}} \Pr[(\mathbf{c}_1, \ldots, \mathbf{c}_L) = (c_1, \ldots, c_L) | (\mathbf{w}_1, \ldots, \mathbf{w}_L) = (w_1, \ldots, w_L)] \cdot \Pr[(\mathbf{w}_1, \ldots, \mathbf{w}_L) = (w_1, \ldots, w_L)] \,. \tag{16}$$

Note that $\mathcal{W}$ and $\Pr[(\mathbf{w}_1, \ldots, \mathbf{w}_L) = (w_1, \ldots, w_L)]$ depend on the considered user model.

With all $(c_1, \ldots, c_L) \in \mathcal{C}$, there corresponds a probability distribution of the bandwidth required for providing the channels in the desired resolutions:

$$\{\pi_{\mathbf{C}(c_1,\ldots,c_L)}\} = (\bigotimes_{i=1}^{c_1}\{\pi_{\mathbf{r}_1}\}) \otimes (\bigotimes_{i=1}^{c_2}\{\pi_{\mathbf{r}_2}\}) \otimes \ldots \otimes (\bigotimes_{i=1}^{c_L}\{\pi_{\mathbf{r}_L}\}) \,, \tag{17}$$

where $\otimes$ denotes the convolution operation. If we then weigh these distributions with the probabilities calculated in (16), the overall probability distribution of the required capacity is obtained:

$$\{\pi_{\mathbf{C}}\} = \sum_{(c_1,\ldots,c_L)\in\mathcal{C}} \{\pi_{\mathbf{C}(c_1,\ldots,c_L)}\} \Pr[(\mathbf{c}_1, \ldots, \mathbf{c}_L) = (c_1, \ldots, c_L)] \,. \tag{18}$$

Note that convolution of two probability distributions containing both n outcomes gives a new probability distribution with n^2 outcomes (less in case some combinations of outcomes result in the same outcomes). This means that the probability distributions $\{\pi_{\mathbf{C}(c_1,\ldots,c_L)}\}$ in (17) have approximately $n^{c_1+\ldots+c_L}$ outcomes, assuming each $\{\pi_{\mathbf{r}_\ell}\}$ contains n outcomes. So, it is clearly not realistic to calculate the exact distribution $\{\pi_{\mathbf{C}}\}$ as described above. Therefore, in practice we do not work in (17) with the exact probability distributions $\{\pi_{\mathbf{r}_\ell}\}$, but rather with histograms derived for them. For the numerical examples presented further on in this paper, we first calculated the maximal possible capacity value $maxC$ corresponding to the considered scenario. We then divided the capacity range $[0, maxC]$ in bins of a certain width. The width of the bins, which we denote by *unit*, was chosen in such a way that $[0, maxC]$ is divided in a predefined number of intervals. We then constructed histograms for the $\{\pi_{\mathbf{r}_\ell}\}$'s by dividing their range in intervals of type $[m \cdot unit, (m + 1) \cdot unit[$, $m = 0, 1, \ldots$ After applying first formula (17) on these histograms, and then formula (18), we end up with a histogram $\{\pi^*_{\mathbf{C}}\}$ for $\{\pi_{\mathbf{C}}\}$. Of course, the use of histograms will cause the resulting $\{\pi^*_{\mathbf{C}}\}$ to be an approximation of the exact $\{\pi_{\mathbf{C}}\}$. The smaller *unit* is chosen, the better the approximation of $\{\pi_{\mathbf{C}}\}$ will be, since the histogram of $\{\pi^*_{\mathbf{C}}\}$ will consist of more bins.

Notice that in this section we did not take into account the correlation between the bit rates associated with the different resolutions of the same channel. In the SVC case it does not matter that all layers are correlated as all layers associated with the highest resolution that is demanded need to be transported. Only the fluctuations of the totality of all transported layers matter. For the simulcast transport mode, the correlation between bit rates of various resolutions does matter, and as such this recursive method neglects it. Due to the nature of the recursive method, it is very hard to take this correlation into account.

5 Simulation Approaches

We wrote an event-driven C-based simulator, which can simulate both simulcast and SVC streaming transport modes. This simulator generates a number of realisations of the variable *capacity demand* $\mathbf{C}$ ($\mathbf{C}_{SIM}$ under simulcast and $\mathbf{C}_{SVC}$ under SVC). For every realisation, a user is either tuned into a given channel k and resolution ℓ or is inactive (governed by the user model and activity grade). According to the selected transport mode (simulcast (SIM) or SVC), the activity grade of the users, and the popularity distribution of the multimedia content (i.e., the Zipf parameter α), we measure the TDF of the variable $\mathbf{C}$, over a sufficiently large number of realisations, depending on the required accuracy. The random number generating function interpreting the activity grade of users follows a uniform distribution.

A multivariate Gaussian variable for the rates of a channel's resolutions is generated at every simulation step, drawn from a distribution defined by the input average vector and covariance matrix. In this paper, we refer to this simulator as *Gaussian simulator*.

We constructed another (C-based) simulator too. The difference with the previous one is that the channel's resolutions rates are drawn randomly from a list of the average rates of real movies. We refer here to this simulator as *histogram simulator*. Were this list of movies' average rates long enough (sorted and possibly with assigned probabilities to every rate realisation or in a histogram form), the rates' probability distribution would have been reasonably well approximated.

6 Results

In this section we will estimate the capacity demand in a realistic mobile TV example by the four approaches described above (two analytical and two simulation ones). In this way we will validate the different approaches and will point out their advantages and their drawbacks.

6.1 Mobile TV Example Settings

For a mobile TV case, we take as guidelines the mobile TV example proposed in the DVB standardisation body [9]. All K channels are encoded in the same number of resolutions L, every resolution corresponding to a certain spatial/temporal format or quality level which is the same (or varies slightly) for all the channels from the bouquet of channels. In the mobile TV example, a channel is encoded in two spatial resolutions (with possible temporal scaling too) with a QVGA base resolution (version, layer) with bit rate in the range 250–400 kbps and a VGA resolution of 750–1500 kbps. In a simulcast scenario, two versions of a channel are encoded and transmitted, while in an SVC scenario, the VGA resolution is encoded from which either a QVGA resolution is streamed or if the VGA resolution is requested, the spatial enhancement layer is transmitted too.

We start from experimental data for 20 movies encoded in two resolutions (QVGA and VGA), both single-layered in AVC mode and multi-layered in SVC mode (encoded in baseline profile and in SVC layered mode). The two resolutions are: base resolution QVGA 320x240@15Hz for some of the movies or 12.5Hz for the other, and its spatial and temporarily scaled resolution VGA 640x480@30Hz for some of the movies or respectively 25Hz for the other. The rates are taken such that delivering a resolution in either of the transport modes (simulcast or SVC) provides the same video quality (Peak Signal-to-Noise Ratio, PSNR). Moreover, there are two encoding experiments and hence sets of input data, we refer to as SET1 and SET2. SET1 targets a low picture quality (i.e., a PSNR of approximately 32 dB), while SET2 targets a higher quality (approximately a PSNR of 34 dB) for both resolutions. Normally, the base layer of an SVC encoded video is encoded in AVC; however in the experimental data we use, the QVGA resolution in SVC has some 8% higher bit rate (for the same quality) as compared to the lowest resolution of simulcast. An SVC VGA resolution of the channel has some 21% higher bit rate than the corresponding AVC VGA version, which we refer to as "penalty rate of SVC". Note that the values specified above (8% and 21%) are the averages over all the movies. The penalty values of each movie differ.

The vectors of the resolutions' averages and covariance matrices for the two experimental sets and the two H.264 coding modes (AVC and SVC) are given in Table 1.

We set the network parameters as follows. Two cases are considered: with $K = 20$ and $K = 50$ channels. The case with a bouquet of 20 channels is representative of the currently deployed mobile TV trials and commercial deployments (as e.g., in [10]); the case of 50 channels is representative of enhanced services

Table 1. Vectors of the average bit rates and covariance matrices

Data SET1	AVC (simulcast)		SVC	
	QVGA	VGA	QVGA	VGA
Average	259	934	277	1110
Covariance matrix	14423	49721	16274	65208
	49721	176543	65208	261328
Data SET2	**AVC (simulcast)**		**SVC**	
	QVGA	VGA	QVGA	VGA
Average	436	1559	474	1899
Covariance matrix	45924	154047	54040	216475
	154047	528443	216475	867215

in future. In general, the known mobile (broadcast) TV technologies, to name but a few – DVB-H/SH (Digital Video Broadcasting-Handheld/Satellite services to Handhelds), MBMS (Multimedia Broadcast Multicast Service) (e.g., TDtv), MediaFLO, T/S-DMB (Terrestrial/Satellite-Digital Multimedia Broadcasting), etc., have the potential to deliver approximately up to 50 channels. For the parameter of the Zipf distribution, we choose $\alpha = 0.6$ as in our previous work [5]. We set the number of users N_ℓ to 100 for every of the two resolutions (QVGA and VGA) and their activity grade a_ℓ to 0.6.

6.2 Validation of the Proposed Methods

The TDFs of $\mathbf{C}_{SIM}$ and $\mathbf{C}_{SVC}$ by the two analytical and two simulation methods are displayed in Figure 1 for data SET1 and in Figure 2 for data SET2. In both figures, the upper graphs display the capacity demand TDF by the analytical Gaussian approximation approach and the TDFs obtained by the two simulation approaches (with the Gaussian simulator and the histogram simulator); in the bottom graphs, a comparison is made of the two analytical approaches (Gaussian and recursive).

Estimation of Capacity Demand. A network (broadcast) operator wants to dimension its network for a given probability of unavailability (the probability that the required network resources exceed the available bandwidth), i.e., $\Pr[\{\mathbf{C}_{SIM}, \mathbf{C}_{SVC}\} > C_{avail}]$. We call this probability $P_{unavail}$. In Table 2 we summarise the required bandwidth as estimated by the four approaches for $P_{unavail} = 10^{-4}$. For the case with 20 channels, both for SET1 and SET2, an SVC transport strategy outperforms the simulcast one: under SET1, the required capacity demand with simulcast is 31.1 Mbps while with SVC it is 29.7 Mbps; under SET2, the required capacity demand with simulcast is approximately 52.5 Mbps while with SVC it is 51.7 Mbps (these approximate figures are taken from

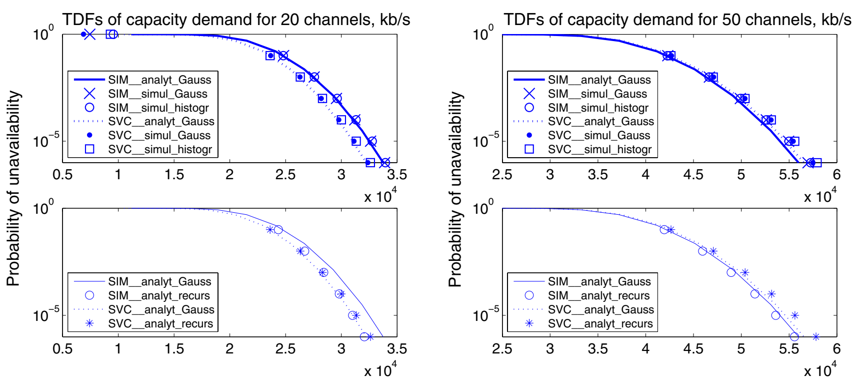

Fig. 1. Comparison of TDFs of capacity demand $\mathbf{C}_{SIM}$ and $\mathbf{C}_{SVC}$ for data SET1 by the four methods: upper graphs – by analytical Gaussian approach and the two simulation approaches; bottom graph – comparison of the two analytical approaches

Table 2. Numerical comparison of the capacity demand by the four methods at $P_{unavail} = 10^{-4}$ for the cases of Figure 1 and Figure 2

Data SET1				
Capacity demand [kbps]	$K = 20$		$K = 50$	
	simulcast	SVC	simulcast	SVC
Analytical Gaussian	31 129	29 709	51 965	52 472
Analytical recursive	29 820	30 004	51 428	53 172
Relative error	4.20%	0.99%	1.03%	1.33%
Gaussian simulator	31 206	29 778	52 617	53 111
Relative error	0.25%	0.23%	1.25%	1.22%
Histogram simulator	31 316	30 009	52 609	53 144
Relative error	0.60%	1.01%	1.24%	1.28%

Data SET2				
Capacity demand [kbps]	$K = 20$		$K = 50$	
	simulcast	SVC	simulcast	SVC
Analytical Gaussian	52 548	51 661	87 361	90 687
Analytical recursive	50 189	52 308	86 322	92 062
Relative error	4.49%	1.25%	1.19%	1.52%
Gaussian simulator	52 753	51 833	88 571	91 869
Relative error	0.39%	0.33%	1.38%	1.30%
Histogram simulator	52 883	52 309	88 531	92 051
Relative error	0.64%	1.25%	1.34%	1.50%

the analytical Gaussian approach, which accords very well with the two simulation approaches and a little less with the recursive approach, at least in the simulcast case). However, in the case of 50 channels with the same network settings, simulcast is more efficient than SVC: under SET1, the required capacity demand with simulcast is approximately 52.0 Mbps while with SVC it is 52.5 Mbps; under SET2, the required capacity demand with simulcast is approximately 87.4 Mbps while with SVC it is 90.7 Mbps.

This shows that it is not straightforward whether an SVC transport scheme will be beneficial for saving on network resources, but that this depends on the network environment (parameters). SVC becomes the more efficient transport mode when all resolutions of all channels are actively requested (for which the prerequisites are high activity grade of users, small user population, small bouquet of channels). If the number of resolutions in the broadcast system increases (and the bit rate penalty under SVC encoding remains small), the SVC mode is expected to become more effective compared to simulcasting all resolutions.

Comparison of the Four Methods. The four methods to estimate the capacity demand on an aggregation link transporting K multicast channels in L

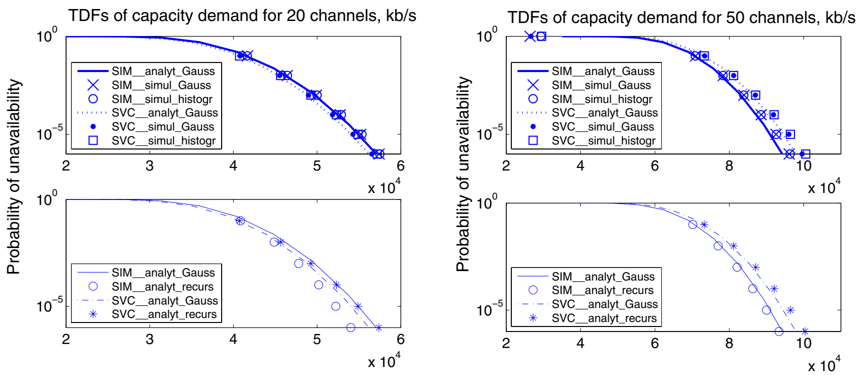

Fig. 2. Comparison of TDFs of capacity demand $\mathbf{C}_{SIM}$ and $\mathbf{C}_{SVC}$ for data SET2 by the four methods: upper graphs – by analytical Gaussian approach and the two simulation approaches; bottom graphs – comparison of the two analytical approaches

different resolutions, are consistent in the resulting TDFs of required bandwidth. However, the recursive method deviates most from the three other methods in the calculation of $\mathbf{C}_{SIM}$. The reason for this is a drawback of the approach, not allowing to account for the correlation between a channel's resolutions rates. This has no impact on the results for $\mathbf{C}_{SVC}$ since its central moments do not depend on the off-diagonal elements of the covariance matrix. This explains why the TDFs for $\mathbf{C}_{SIM}$ are qualitatively different by the two analytical methods in the lower left graphs in Figure 1 and in Figure 2. The recursive method calculates the distribution of the number of channels to be provided exactly but it needs to approximate the joint distribution of the bit rate of both resolutions by two marginal histograms. This binning and ignoring the correlations introduces an error.

The TDFs by the two simulation approaches are in a very good correspondence. Hence, it is reasonable to assume that the joint probability distribution of the bit rates of the resolutions can be approximated by a multivariate Gaussian one (under certain conditions). They show also a good match with calculated TDFs by the analytical Gaussian approach for the case of 20 channels (see the upper left graphs in Figure 1 and in Figure 2). Unfortunately, for the case of 50 channels, the analytical Gaussian method is not very exact (relative error at $P_{unavail} = 10^{-4}$ up to 1.5%). Normally however, for large network scenarios (and if the ratio N/K is large), the Gaussian analytical approach shows a good correspondence with results by the other approaches. Moreover, this method yields results in the fastest way as compared to the three other methods, it allows to calculate other user models (e.g., the ones in [5]), and it is robust to calculate large network scenarios. The drawback of the simulation methods is that the higher the required accuracy, the longer the simulation run must be.

7 Conclusion

We presented and compared four approaches to estimate the capacity demand on the aggregation link in a broadcast network with K streamed channels in L different resolutions. A realistic mobile TV scenario was calculated and simulated with two data sets (corresponding to different video quality), and the required bandwidth was predicted both in a simulcast and in an SVC transport mode. We demonstrate that not always SVC outperforms simulcast in terms of resource efficiency and that SVC becomes more beneficial with larger number of channel resolutions, smaller bit rate penalty, a more active audience of subscribers, and a small bouquet of channels.

The Gaussian approximation analytical method is the fastest one but results start to deviate with a small N/K ratio. The recursive analytical method does not account for the correlation between the bit rates of the different resolutions (which is a drawback in a simulcast scenario) and also its accuracy depends on the coarseness of the binning process. The simulation methods can be considered as most accurate of the presented approaches but their accuracy depends on the length of the simulation runs.

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