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Analysis and Design of Univariate Subdivision Schemes

With 76 Figures

Springer

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Preface

'Subdivision' is a way of representing smooth shapes in a computer. A curve or surface (both of which contain an infinite number of points) is described in terms of two objects. One object is a sequence of vertices, which we visualise as a polygon, for curves, or a network of vertices, which we visualise by drawing the edges or faces of the network, for surfaces.

The other object is a set of rules for making denser sequences or networks. When applied repeatedly, the denser and denser sequences are claimed to converge to a limit, which is the curve or surface that we want to represent.

This book focusses on curves, because the theory for that is complete enough that a book claiming that our understanding is complete is exactly what is needed to stimulate research proving that claim wrong. Also because there are already a number of good books on subdivision surfaces.

The way in which the limit curve relates to the polygon, and a lot of interesting properties of the limit curve, depend on the set of rules, and this book is about how one can deduce those properties from the set of rules, and how one can then use that understanding to construct rules which give the properties that one wants.

This book therefore has four main parts. First are a set of 'Prependices' which are potted descriptions of little bits of mathematics which turn out to be useful background. These can be skipped at first reading, or by those who know the material anyway.

Then a chapter introducing the concepts for the reader who has not encountered subdivision curves and surfaces before; and how the rules are described in ways that we can apply mathematical arguments to.

Third a set of chapters dealing with the ways that we can analyse properties of the limit curves in terms of the rules, followed by a shorter set of chapters suggesting how we can work the analyses in reverse, to design rules which will give the schemes that result from them desired properties.

Where a chapter introduces techniques worth trying out, it also includes a few exercises to help the reader check that they have indeed been learned.

Finally come some ideas about efficient implementation and some appendices tidying up material best kept out of the way of the main flow of ideas, including some topics open for research, a short account of the development of the subject so far, and a bibliography of research papers.

I would like to thank the following people for their help and forbearance during the writing of this book: first, of course, my wife for her patience when there were other things she would have liked me to be doing, but also David Levin for his patient explanation of z-transforms to me, Adi Levin for showing me how the joint spectral radius worked and Carl de Boor for showing me how to say what I meant about eigenanalysis in terms of invariant subspaces and for the idea of Theorem 4.

Then there are the colleagues who have read early proofs, the referees whose comments have triggered significant improvements, and the staff at Springer who have tolerated my views as to how the book should look.

Thank you all.

July 2010, Ely

Table of Contents

Part V. Implementation

Part VI. Appendices