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Multiresolution Methods in Scattered Data Modelling

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Für Martina, Meira und Marian.

Preface

This application-oriented work concerns the design of efficient, robust and reliable algorithms for the numerical simulation of multiscale phenomena. To this end, various modern techniques from scattered data modelling, such as splines over triangulations and radial basis functions, are combined with customized adaptive strategies, which are developed individually in this work.

The resulting multiresolution methods include thinning algorithms, multilevel approximation schemes, and meshfree discretizations for transport equations. The utility of the proposed computational methods is supported by their wide range of applications, such as image compression, hierarchical surface visualization, and multiscale flow simulation.

Special emphasis is placed on comparisons between the various numerical algorithms developed in this work and comparable state-of-the-art methods. To this end, extensive numerical examples, mainly arising from real-world applications, are provided.

This research monograph is arranged in six chapters:

1. *Introduction*;
2. *Algorithms and Data Structures*;
3. *Radial Basis Functions*;
4. *Thinning Algorithms*;
5. *Multilevel Approximation Schemes*;
6. *Meshfree Methods for Transport Equations.*

Chapter 1 provides a preliminary discussion on basic concepts, tools and principles of multiresolution methods, scattered data modelling, multilevel methods and adaptive irregular sampling. Relevant algorithms and data structures, such as triangulation methods, heaps, and quadtrees, are then introduced in Chapter 2.

Chapter 3 is devoted to radial basis functions, which are well-established and powerful tools for multivariate interpolation and approximation from scattered data. Radial basis functions are therefore important basic tools in scattered data modelling. In fact, various computational methods, which are developed in this work, essentially rely on radial basis function methods.

In Chapter 3, basic features of scattered data interpolation by radial basis functions are first explained, before more advanced topics, such as optimal recovery, the uncertainty relation, and optimal point sampling, are addressed. Moreover, very recent results concerning the approximation order and the numerical stability of polyharmonic spline interpolation are reviewed, before least squares approximation by radial basis functions is discussed.

The extensive discussion in Chapter 4 is devoted to recent developments concerning thinning algorithms with emphasis on their application to terrain modelling and image compression. In their application to image compression, adaptive thinning algorithms are combined with least squares approximation and a customized coding scheme for scattered data. This yields a novel concept for image compression. As confirmed by various numerical examples, this image compression method often gives better or comparable compression rates to the well-established wavelet-based compression method SPIHT.

In Chapter 5, various alternative multilevel approximation methods for bivariate scattered data are discussed. Starting point of this discussion is our multilevel interpolation scheme of [72], which was the first to combine thinning algorithms with scattered data interpolation by compactly supported radial basis functions. Recent improvements of the multilevel method of [72] are discussed in Chapter 5, where a new adaptive domain decomposition scheme for multilevel approximation is proposed. The performance of the various multilevel approximation schemes is compared by using one real-world model problem concerning hierarchical surface visualization from scattered terrain data.

Chapter 6 is concerned with meshfree methods for transport equations. Meshfree methods are recent and modern discretization schemes for partial differential equations. In contrast to traditional methods, such as finite differences (FD), finite volumes (FV), and finite element methods (FEM), meshfree methods do not require sophisticated data structures and algorithms for mesh generation, which is often the most time-consuming part in mesh-based simulations. Moreover, meshfree methods are very flexible and particularly useful for modelling multiscale phenomena.

In Chapter 6, a new adaptive meshfree method of characteristics, called **AMMoC**, is developed for multiscale flow simulation. The advection scheme **AMMoC** combines an adaptive version of the well-known semi-Lagrangian method with local meshfree interpolation by radial basis functions. The good performance of the particle-based method **AMMoC** for both linear and nonlinear transport problems is shown. This is done by using one model problem concerning tracer transportation in the artic stratosphere, and the popular five-spot problem from hydrocarbon reservoir simulation. The latter is concerning two-phase flow in porous media. In this model problem, our meshfree advection method **AMMoC** is compared with two leading commercial reservoir simulators, ECLIPSE and FrontSim of Schlumberger.

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I wish to take this opportunity in order to thank those of my friends and colleagues who gave me their precious and long-lasting support during my academic career, and the non-academic detour.

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I used this freedom in order to acquire two different European research and training networks, MINGLE (Multiresolution in Geometric Modelling), and NetAGES (Network for Automated Geometry Extraction from Seismic), both of which are currently funded by the European Commission, contract no. HPRN-CT-1999-00117 (MINGLE) and IST-1999-29034 (NetAGES).

Due to my involvement in MINGLE and NetAGES, several exciting joint research initiatives with academic and industrial partners were created, which helped to foster the outcome of this work. Especially the intense and very pleasant collaboration with my MINGLE postdoc student, Laurent Demaret, on image compression, and with my NetAGES PhD student, Martin Käser, on meshfree flow simulation, is very fruitful. I wish to thank both of them explicitly for their professional attitude and everlasting drive during our entire collaboration.

But I also wish to thank my colleague Jörn Behrens for he initiated our joint work on meshfree semi-Lagrangian advection schemes. Moreover, the kind invitation from Jeremy Levesley on a research visit at the University of Leicester (UK) in February/March 2001 is gratefully appreciated. Parts of our very exciting collaboration on multilevel approximation schemes were initiated during that visit, which was supported by the UK Research Council through the EPSRC grant GR/RO7769.

Useful assistance with the implementation of the various algorithms and with the preparation of the required numerical experiments was provided by the computer science students Stefan Pöhn, Konstantinos Panagiotou, Bertolt Meier, Eugene Rudoy, Georgi Todorov, and Rumen Traykov (all at Munich University of Technology).

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I dedicate this book to my wife Martina and our children, Meira and Marian, not only in gratitude for their great understanding and patience during the uncounted hours that were spent on this book.

Munich, November 2003 *Armin Iske*

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