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Richard Moot Christian Retoré

The Logic of Categorial Grammars

A Deductive Account of Natural Language Syntax and Semantics

Springer

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Voor Tessa
À Antoine, Mathilde et Garance

Pour le moment, la linguistique générale m'apparaît comme un système de géométrie. On aboutit à des théorèmes qu'il faut démontrer.

de Saussure (1911)
in Godel (1957)

I think it is correct to say that the difference between the structural linguist and the formal logician is one of stress and degree rather than of kind.

Bar-Hillel (1954)

The correct way to use the insights and techniques of logic is in formulating a general theory of linguistic structure.

Chomsky (1955)

La logique contemporaine est robuste et élégante. Mais elle est aussi maigre. Les logiciens contemporains sont tout à fait conscients de ce fait, et ils s'efforcent d'engraisser un peu la bête. Aristote et sa théorie des catégories auront peut-être toujours quelques conseils à offrir.

Barnes (2005)

Preface

Audience

This book is intended for students (from the third year) in computer science, formal linguistics, and mathematical logic as well as for colleagues interested in categorial grammars and their logical foundations. Though the logical viewpoint of categorial grammars is well-established, until now there has been no textbook available to students and researchers who want to discover all the main results for categorial grammars. It is the goal of this book to fill this gap.

The book has few formal prerequisites; nevertheless, notions of mathematical logic will help.

In addition, some familiarity with proof theory and the typed λ-calculus will help, and for readers who feel they need additional background we recommend (Girard et al, 1988; Girard, 2011), and (Girard, 1995, 2011) for more details on linear logic.

Occasionally, we will assume the reader knows what context-free grammar is, and is aware of their elementary properties such as Chomsky and Greibach normal forms. If some background on formal language theory is needed, we recommend (Hopcroft and Ullman, 1979; Harrison, 1978)

Finally, for some background in linguistics, we recommend (Akmajian et al, 2001).

Nevertheless we have made every effort to make this book as self-contained as possible, and none of this additional background knowledge is strictly mandatory.

The exercises at the end of every chapter, sometimes with hints, should help the reader assimilate the new notions and check that they are well understood.

Contents

These lecture notes present categorial grammars as deductive systems, in the approach called parsing-as-deduction, and we include detailed proofs of their main properties. We provide background and motivation for all results, modern proofs of many of the classical results for categorial grammars, together with our own results, in particular on proof nets.

Chapter 1: Our book starts with AB grammars (Ajdukiewicz-Bar-Hillel) also called classical categorial grammars, basic Lambek grammars... We establish their correspondence with context-free grammars, and briefly present their learnability properties.

Chapter 2: Lambek's syntactic calculus is at the center of this book, and despite all the years it is still a pleasure to read Lambek's original article (Lambek, 1958), written with an elegance which is rarely encountered. In this chapter, we present the correspondence between Lambek grammars and context-free grammars following Pentus, (Pentus, 1993b), and study the structure of the parse-structures/deductions: normalization, sub formula property, decidability, interpolation.

Chapter 3: Here, we turn our attention to one of the important applications of categorial grammar: how to automatically turn a syntactic analysis, that is a deduction, into a lambda term, which combined with lexical lambda terms, produces a logical formula corresponding to the meaning of the analyzed sentence. As we will see, the answer to this question is very simple and builds upon a fundamental result from logic and lambda-calculus: the Curry–Howard isomorphism. We also discuss several applications of this correspondence in the context of Montague semantics.

Chapter 4: The non-associative Lambek calculus NL is a variant of the Lambek calculus where the objects of study are *trees* of formulas instead of lists of formulas. We also discuss Kripke models and polynomial parsing algorithms for NL.

Chapter 5: The multimodal Lambek calculus extends the non-associative Lambek calculus by introducing controlled versions of the structural rules of associativity and commutativity to the logic. Some illustrative examples show how multimodality allows us to move beyond context-free languages.

Chapter 6: In this chapter, we present proof nets for the Lambek calculus, graphs that better describe a deduction than the usual trees, since no two proof nets correspond to the same linguistic analysis. They also enable different parsing techniques, and they can even be used to measure the complexity of human processing.

Chapter 7: In the final chapter, we combine ideas from Chaps. 5 and 6 to give proof nets for the multimodal Lambek calculus, emphasizing their usefulness for parsing categorial grammars. We also discuss Grail, an implementation of multimodal grammars based on proof nets.

How to Read This Book

As with most books, the simplest way to read this one is from start to finish. However, to help the reader who is eager to get to one of the later chapters as soon as possible or the teacher who has only time for a limited amount of course material, Fig. 0.1 gives a list of chapter dependencies.

The "main track" is listed on the left of the figure: and it represents the minimal dependencies required to continue to the next chapter. Having read the core sections,

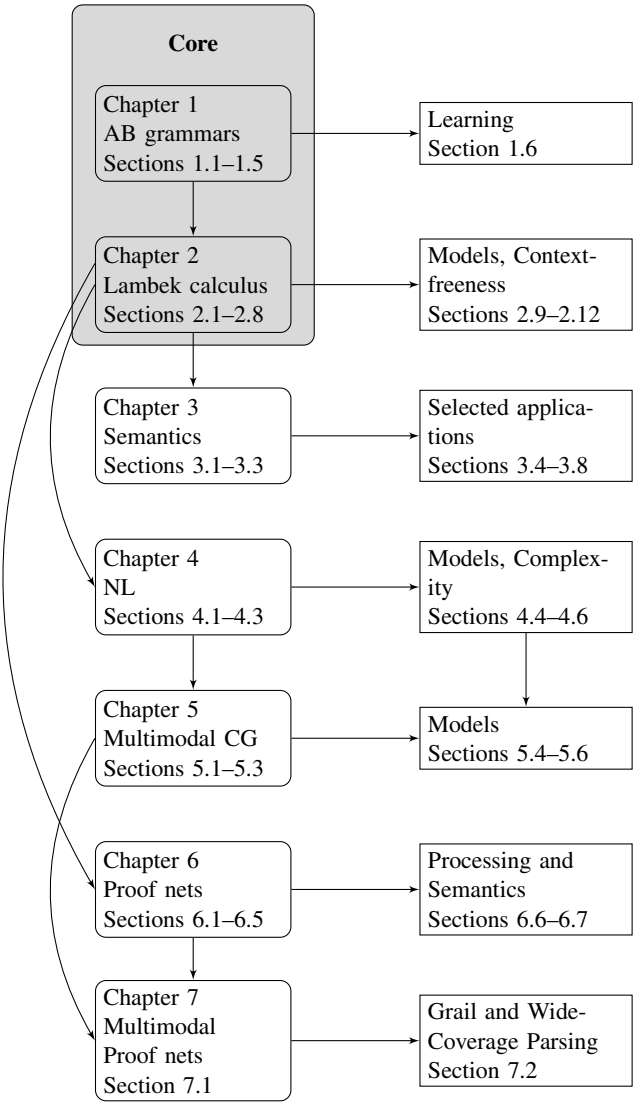

Fig. 0.1. Chapter dependencies

in the gray-shaded box, the reader can continue reading in a number of ways: either continue onward to the chapter on Montague Semantics or jump directly to the non-associative Lambek calculus or to the proof net chapter. It should be noted that this main track is rather restricted and that many fundamental results are found in the "optional" track on the right of the figure — which includes important topics such as

model theory, Pentus's context-freeness result for the Lambek calculus and formal learning theory, but which are sections that the reader can skip without it affecting his comprehension of later chapters. The goal of Fig. 0.1 is to allow the eclectic teacher, researcher, or student pick his most efficient path through the material in this book to the topics which interest him most.

Further Reading

For further general references on the logical view of categorial grammars we recommend two chapters of the Handbook of *Logic and Language* (van Benthem and ter Meulen, 1997), namely (Buszkowski, 1997; Moortgat, 1997), and the revised and extended chapter (Moortgat, 2011) in (van Benthem and ter Meulen, 2011) as well as the article on type-logical grammar on the Stanford Encyclopedia of Philosphy website (Moortgat, 2010).

Other recommended recent references, which are in many ways complementary to the current textbook, are Glyn Morrill's textbook (Morrill, 2011), which contains a wealth of material on linguistic applications and on proof nets and processing (as discussed briefly in our Sect. 6.6) and Richard Oehrle's introduction to multimodal categorial grammar (Oehrle, 2011).

Finally, let us say that our logical view of categorial grammar is not the only one, and regarding the combinatorial view of categorial grammars we recommend (Steedman, 1997).

Recent Advances in Categorial Grammars

Though this book discusses only the classical results, categorial grammars in the tradition of the Lambek calculus are still an active area of research. As an indication of some recent developments, we mention only the Lambek–Grishin calculus (Moortgat, 2009; Bernardi and Moortgat, 2010) and the Discontinous Lambek calculus (Morrill, 2011; Morrill et al, 2011). The background provided by the current book should be more than sufficient to allow the interested reader to follow these recent developments.

In the mid-1990s, the introduction of the minimalist program moved mainstream syntax closer to categorial grammars. Working with a formalization of minimalist grammars introduced by Stabler (1997), Lecomte, Retoré and some of their students, Amblard, Anoun, have been able to make this correspondence precise by introducing Categorial Minimalist Grammars, within the parsing-as-deduction paradigm, thus enriching minimalist grammars with a neat computational semantics (Lecomte and Retoré, 1999; Lecomte, 2011).

Let us also mention a different syntactic formalism, the *abstract categorial grammars* of de Groote (2001). They rely on the representation of trees and strings in the simply typed lambda calculus (as opposed to using the structure of the antecedent to represent trees and strings as we do here). Thus, it is closer to the interface between categorial syntax and semantics discussed in our chapter on Montague semantics than to usual categorial grammar with parsing-as-deduction. Abstract categorial

grammars define interesting classes of formal languages, with hierarchies that more or less match the ones of formal language theory, and there is a natural correspondence with semantics.

History of the Book

A first version of these notes was written by Christian Retoré, for a lecture on "The Logic of Categorial Grammars" at ESSLLI 2000. The section on categorial grammar acquisition was added for an ACL 2001 tutorial.

Thereafter it was used and improved on various occasions including a lecture at ESSLLI 2003, a crash course at EALING 2003 and 2008, Master lectures in Bordeaux from 2002 to 2011, and in Verona in 2006 and 2010.

Richard Moot extended these early notes, in part for an ESSLLI 2004 course, by adding chapters on the non-associative Lambek calculus, the multimodal Lambek calculus and multimodal proof nets as well as by adding supplementary material and exercises to the other chapters.

Thereafter we produced joint revisions, additions and updates of the book with an independent chapter on Montague semantics.

Acknowledgments

Introductions are always difficult to write and we benefited from the solid advice of Eco (1987) to guide us.

We would like to thank the editorial team at Springer for their support and encouragement during the long gestation period of this book — read Chap. 23 of (Kahneman, 2011) for some thoughtful comments on estimated times until the completion of a book! — and the anonymous Springer referees for their insightful and thorough comments.

We would like to thank our employers, CNRS and Université de Bordeaux gathered in our LaBRI lab, as well as the projects supported by LaBRI, INRIA and the Conseil Régional d'Aquitaine *Grammaire du Français* and *Itipy*. Further financial support was provided by the ANR projects *Prélude* and *Loci*. The current collaboration started with the creation of our research group SIGNES (LABRI-CNRS and INRIA), and special thanks go to their respective directors Serge Dulucq and Claude Puech for their support in creating the team.

Special thanks also go to our proofreaders Ivano Ciardelli and Noémie-Fleur Sandillon-Rezer, who spotted many typos and other mistakes and suggested several improvements and clarifications.

We would also like to thank the following colleagues, course participants and readers for their comments on the lectures and on earlier drafts of the current lecture notes: Anne Abeillé, Vito Michele Abrusci, Maxime Amblard, Houda Anoun, Nicholas Asher, Christian Bassac, Denis Béchet, Nicolas Belgolo, Claire Beyssade, Philippe Blache, Roberto Bonato, Pierre Bourreau, Joan Busquets, Claudia Casadio, Pierre Castéran, Lionel Clément, Francis Corblin, Bruno Courcelle, Dick Crouch, Laurence Danlos, Denis Delfitto, Alexandre Dikovsky, Luca Ducchesi, Gaetano Fiorin, Marie-Renée Fleury, Annie Foret, Christophe Fouqueré,

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Last but not least, we have a special thought for one of our commentators who brought his enthusiasm to every ESSLLI: Paul Gochet, who passed away on June 21, 2011.

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