Explaining Time-Table-Edge-Finding Propagation for the Cumulative Resource Constraint

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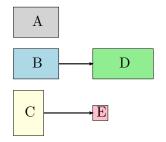
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Abstract

Cumulative resource constraints can model scarce resources in scheduling problems or a dimension in packing and cutting problems. In order to efficiently solve such problems with a constraint programming solver, it is important to have strong and fast propagators for cumulative resource constraints. One such propagator is the recently developed time-table-edge-finding propagator, which considers the current resource profile during the edge-finding propagation. Recently, lazy clause generation solvers, *i.e.*, constraint programming solvers incorporating nogood learning, have proved to be excellent at solving scheduling and cutting problems. For such solvers, concise and accurate explanations of the reasons for propagation are essential for strong nogood learning. In this paper, we develop the first explaining version of time-table-edge-finding propagation and show preliminary results on resource-constrained project scheduling problems from various standard benchmark suites. On the standard benchmark suite PSPLib, we were able to close one open instance and to improve the lower bound of about 60% of the remaining open instances. Moreover, 6 of those instances were closed.

1. Introduction

A cumulative resource constraint models the relationship between a scarce resource and activities requiring some part of the resource capacity for their execution. Resources can be workers, processors, water, electricity, or, even, a dimension in a packing and cutting problem. Due to its relevance in many industrial scheduling and placement problems, it is important to have strong and fast propagation techniques in constraint programming (CP) solvers that detect inconsistencies early and remove many invalid values from the domains of the variables involved. Moreover, when using CP solvers that incorporate "fine-grained" nogood learning it is also important that each inconsistency and each



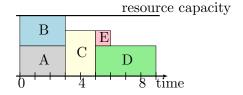


Figure 1: Five activities with precedence relations.

Figure 2: A possible schedule of the activities.

value removal from a domain is explained in such a way that the full strength of nogood learning is exploited.

In this paper, we consider renewable resources, i.e., resources with a constant resource capacity over time, and non-preemptive activities, i.e., whose execution cannot be interrupted, with fixed processing times and resource usages. In this work, we develop explanations for the time-table-edge-finding (TTEF) propagator [34] for use in lazy clause generation (LCG) solvers [22, 9].

Example 1.1. Consider a simple cumulative resource scheduling problem. There are 5 activities A, B, C, D, and E to be executed before time period 10. The activities have processing times 3, 3, 2, 4, and 1, respectively, with each activity requiring 2, 2, 3, 2, and 1 units of resource, respectively. There is a resource capacity of 4. Assume further that there are precedence constraints: activity B must finish before activity D begins, written $B \ll D$, and similarly $C \ll E$. Figure 1 shows the five activities and precedence relations, while Fig. 2 shows a possible schedule, where the start times are: 0, 0, 3, 5, and 5 respectively.

In CP solvers, a cumulative resource constraint can be modelled by a decomposition or, more successfully, by the global constraint cumulative [2]. Since the introduction of this global constraint, a great deal of research has investigated stronger and faster propagation techniques. These include time-table [2], (extended) edge-finding [21, 33], not-first/not-last [21, 25], and energetic-reasoning propagation [4, 6]. Time-table propagation is usually superior for highly disjunctive problems, i.e., in which only some activities can run concurrently, while (extended) edge-finding, not-first/not-last, and energetic reasoning are more appropriate for highly cumulative problems, i.e., in which many activities can run concurrently.[4] The reader is referred to [6] for a detailed comparison of these techniques.

Vilim [34] recently developed TTEF propagation which combines the time-table and (extended) edge-finding propagation in order to perform stronger propagation while having a low runtime overhead. Vilim [34] shows that on a range of highly disjunctive open resource-constrained project scheduling problems from the well-established benchmark

library PSPLib,¹ TTEF propagation can generate lower bounds on the project deadline (*makespan*) that are superior to those found by previous methods. He uses a CP solver without nogood learning. This result, and the success of LCG on such problems, motivated us to study whether an explaining version of this propagation yields an improvement in performance for LCG solvers.

In general, nogood learning is a resolution step that infers redundant constraints, called *nogoods*, given an inconsistent solution state. These nogoods are permanently or temporarily added to the initial constraint system in order to reduce the search space and/or to guide the search. Moreover, they can be used to short circuit propagation. How this resolution step is performed is dependent on the underlying system.

LCG solvers employ a "fine-grained" nogood learning system that mimics the learning of modern Boolean satisfiability (SAT) solvers (see e.g. [20]). In order to create a strong nogood, it is necessary that each inconsistency and value removal is explained concisely and in the most general way possible. For LCG solvers, we have previously developed explanations for time-table and (extended) edge-finding propagation [27]. Moreover, for time-table propagation we have also considered the case when processing times, resource usages, and resource capacity are variable [24]. Explanations for the time-table propagator were successfully applied on resource-constraint project scheduling problems [27, 29] and carpet cutting [28] where in both cases the state-of-the-art of exact solution methods were substantially improved. The explanations defined here are similar to the step-wise ones for the (extended) edge-finding propagation in [27], but there we do not consider the resource profile and are more complex. Moreover, the proposed explanations for edge-finding propagation in [27] has never been implemented.

Explanations for the propagation of the cumulative constraint have also been proposed for the PaLM [14, 13] and SCIP [1, 7, 12] frameworks. In the PaLM framework, explanations are only considered for time-table propagation, while the SCIP framework additionally provides explanations for energetic reasoning propagation and a restricted version of edge-finding propagation. Neither framework consider bounds widening in order to generalise these explanations as we do in this paper. Other related works include [32], which presents explanations for different propagation techniques for problems only involving disjunctive resources, *i.e.*, cumulative resources with unary resource capacity, and generalised nogoods [15]. A detailed comparison of explanations for the propagation of cumulative resource constraints in LCG solvers can be found in [24].

In this paper we develop explanations for the TTEF cumulative propagator in LCG solvers. The explaining TTEF propagation is then compared with the explaining time-table propagation from [27] in the LCG solver on RCPSP using the reengineered LCG solver [9] which was also used for the experiments presented in [27].

2. Cumulative Resource Scheduling

In cumulative resource scheduling, a set of (non-preemptive) activities V and one cumulative resource with a (constant) resource capacity R is given where an activity i is

¹See http://129.187.106.231/psplib/.

specified by its start time S_i , its processing time p_i , its resource usage r_i , and its energy $e_i := p_i \cdot r_i$. In this paper we assume each S_i is an integer variable and all others are assumed to be integer constants. Further, we define est_i (ect_i) and lst_i (lct_i) as the earliest and latest start (completion) time of i.

In this setting, the cumulative resource scheduling problem is defined as a constraint satisfaction problem that is characterised by the set of activities \mathcal{V} and a cumulative resource with resource capacity R. The goal is to find a solution that assigns values from the domain to the start time variables S_i $(i \in \mathcal{V})$, so that the following conditions are satisfied.

$$est_i \leq S_i \leq lst_i, \qquad \forall i \in \mathcal{V}$$

$$\sum_{i \in \mathcal{V}: \tau \in [S_i, S_i + p_i)} r_i \leq R \qquad \forall \tau$$

where τ ranges over the time periods considered. Note that this problem is NP-hard [5]. We shall tackle problems including cumulative resource scheduling using CP with nogood learning. In a CP solver, each variable $S_i, i \in \mathcal{V}$ has an initial domain of possible values $D^0(S_i)$ which is initially $[est_i, lst_i]$. The solver maintains a current domain D for all variables. CP search interleaves propagation with search. The constraints are represented by propagators that, given the current domain D, creates a new smaller domain D' by eliminating infeasible values. The current lower and upper bound of the domain $D(S_i)$ are denoted by $lb(S_i)$ and $ub(S_i)$, respectively. For more details on CP see e.g. [23].

For a learning solver we also represent the domain of each variable S_i using Boolean variables $[S_i \leq v]$, $est_i \leq v < lst_i$. These are used to track the reasons for propagation and generate nogoods. For more details see [22]. We use the notation $[v \leq S_i]$, $est_i < v \leq lst_i$ as shorthand for $\neg [S_i \leq v - 1]$, and treat $[v \leq S_i]$, $v \leq est_i$ and $[S_i \leq v]$, $v \geq lst_i$ as synonyms for true. Propagators in a learning solver must explain each reduction in the domain by building a clausal explanation using these Boolean variables.

Optimisation problems are typically solved in CP via branch and bound. Given an objective obj which is to be minimised, when a solution is found with objective value o, a new constraint obj < o is posted to enforce that we only look for better solutions in the subsequent search.

3. TTEF Propagation

In this section we develop explanations for TTEF propagation. For a more detailed description about TTEF propagation the reader is referred to [34].

TTEF propagation splits the treatment of activities into a fixed and free part. The former results from the activities' compulsory part whereas the latter is the remainder. The fixed part of an activity i is characterised by the length of its compulsory part $p_i^{TT} := \max(0, ect_i - lst_i)$ and its fixed energy $e_i^{TT} := r_i \cdot p_i^{TT}$. The free part has a processing time $p_i^{EF} := p_i - p_i^{TT}$ and a free energy of $e_i^{EF} := e_i - e_i^{EF}$. Let \mathcal{V}^{EF} be the set of

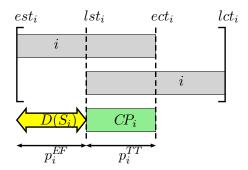


Figure 3: A diagram illustrating an activity i when started at est_i or lst_i , and its possible range of start times, as well as the compulsory part CP_i , and the fixed and free parts of the processing time.

activities with a non-empty free part $\{i \in \mathcal{V} \mid p_i^{EF} > 0\}$. An illustration of this is shown in Figure 3.

TTEF propagation reasons about the energy available from the resource and energy required for the execution of activities in specific time windows. The start and end times of these windows are determined by the earliest start and the latest completion times of activities $i \in \mathcal{V}^{EF}$. These time windows [begin, end) are characterised by the so-called $task\ intervals\ \mathcal{V}^{EF}(a,b) := \{i \in \mathcal{V}^{EF} \mid est_a \leq est_i \land lct_i \leq lct_b\}$ where $a,b \in \mathcal{V}^{EF}$, $begin := est_a$, and $end := lct_b$.

It is not only the free energy of activities in the task interval $\mathcal{V}^{EF}(a,b)$ that is considered, but also the energy resulting from the compulsory parts in the time window $[est_b, lct_b)$. This energy is defined by $ttEn(a,b) := ttAfter[est_a] - ttAfter[lct_b]$ where $ttAfter[\tau] := \sum_{t \geq \tau} \sum_{i \in \mathcal{V}: lst_i \leq t < ect_i} r_i$. Furthermore, we also consider activities $i \in \mathcal{V} \setminus \mathcal{V}^{EF}(a,b)$ in which a portion of their

Furthermore, we also consider activities $i \in \mathcal{V} \setminus \mathcal{V}^{EF}(a,b)$ in which a portion of their free part must be run within the time window as described in [34]. Let lst_i^{EF} be the latest start time of the free part of an activity, i.e., $lct_i - p_i^{EF}$. Then activity i's free part consumes at least $r_i \cdot (lct_b - lst_i^{EF})$ energy units in $[est_a, lct_b)$ if $est_a \leq est_i$ and $lst_i^{EF} < lct_b$. We define the energy contributed by such activities by $rsEn(a,b) := \sum_{i \in \mathcal{V} \setminus \mathcal{V}^{EF}(a,b): est_a \leq est_i} max(0, lct_b - lst_i^{EF})$. In summary, TTEF propagation considers three ways in which an activity i can

In summary, $\overline{\text{T}}\text{TEF}$ propagation considers three ways in which an activity i can contribute to energy consumption within a time window determined by a task interval $\mathcal{V}^{EF}(a,b)$. First, the free parts that must fully be executed in the time window; second, the compulsory parts that must lies in the time window; and third, some free parts that must partially be run in the time window. Thus, the considered length of an activity i is

$$p_i(a,b) := \begin{cases} p_i & i \in \mathcal{V}^{EF}(a,b) \\ \max(0,lct_b - lst_i) & i \notin \mathcal{V}^{EF}(a,b) \land est_a \leq est_i \\ \max(0,\min(lct_b,ect_i) - \max(est_a,lst_i)) & others \end{cases}$$

The considered energy consumption is $e_i(a,b) := r_i \cdot p_i(a,b)$ in the time window.

3.1. Explanation for the TTEF Consistency Check

The consistency check is one part of TTEF propagation that checks whether there is a resource overload in any task interval.

Proposition 3.1 (Consistency Check). The cumulative resource scheduling problem is inconsistent if

$$R \cdot (lct_b - est_a) - energy(a, b) < 0 \to \bot$$
 (1)

where $energy(a,b) := \sum_{i \in \mathcal{V}^{EF}(a,b)} e_i^{EF} + ttEn(a,b) + rsEn(a,b)$.

This check can be done in $\mathcal{O}(l^2+n)$ runtime, where $l=|\mathcal{V}^{EF}|$, if the resource profile is given. The corresponding algorithm is shown in Alg. 1 in App. A.

A naïve explanation for a resource overload in the time window $[est_a, lct_b)$ only considers the current bounds on activities' start times S_i .

$$\bigwedge_{i \in \mathcal{V}: p_i(a,b) > 0} \llbracket est_i \leq S_i \rrbracket \land \llbracket S_i \leq lst_i \rrbracket \rightarrow \bot$$

However, we can easily generalise this explanation by only ensuring that at least $p_i(a, b)$ time units are executed in the time window. This results in the following explanation.

$$\bigwedge_{i \in \mathcal{V}: p_i(a,b) > 0} \llbracket est_a + p_i(a,b) - p_i \le S_i \rrbracket \land \llbracket S_i \le lct_b - p_i(a,b) \rrbracket \to \bot$$

Note that this explanation expresses a resource overload with respect to energetic reasoning propagation which is more general than TTEF.

Let $\Delta := energy(a,b) - R \cdot (lct_b - est_a) - 1$. If $\Delta > 0$ then the resource overload has extra energy. We can use this extra energy to further generalise the explanation, by reducing the energy required to appear in the time window by up to Δ . For example, if $r_i \leq \Delta$ then the lower and upper bound on S_i can simultaneously be decreased and increased by a total amount in $\{1, 2, ..., \min(\lfloor \Delta/r_i \rfloor, p_i(a,b))\}$ units without resolving the overload. If $r_i \cdot p_i(a,b) \leq \Delta$ then we can remove activity i completely from the explanation. In a greedy manner, we try to maximally widen the bounds of activities i where $p_i(a,b) > 0$, first considering activities with non-empty free parts. If Δ_i denotes the time units of the widening then it holds $p_i(a,b) \geq \Delta_i \geq 0$ and $\sum_{i \in \mathcal{V}: p_i(a,b) > 0} \Delta_i \cdot r_i \leq \Delta$ and we create the following explanation.

$$\bigwedge_{i \in \mathcal{V}: p_i(a,b) - \Delta_i > 0} \llbracket est_a + p_i(a,b) - p_i - \Delta_i \leq S_i \rrbracket \wedge \llbracket S_i \leq lct_b - p_i(a,b) + \Delta_i \rrbracket \rightarrow \bot$$

The last generalisation mechanism can be performed in different ways, e.g. we could widen the bounds of activities that were involved in many recent conflicts. Further study is required to identify which are the most appropriate.

3.2. Explanation for the TTEF Start Times Propagation

Propagation on the lower and upper bounds of the start time variables S_i are symmetric; Consequently we only present the case for the lower bounds' propagation. To prune the lower bound of an activity u, TTEF bounds propagation tentatively starts the activity u at its earliest start time est_u and then checks whether that causes a resource overload in any time window $[est_a, lct_b)$ ($\{a, b\} \subseteq \mathcal{V}^{EF}$). Thus, bounds propagation and its explanation are very similar to that of the consistency check.

The work of [34] considers four positions of u relative to the time window: right ($est_a \leq est_u < lct_b < ect_u$), inside ($est_a \leq est_u < ect_u \leq lct_b$), through ($est_u < est_a \wedge lct_b < ect_u$), and left ($est_u < est_a < ect_u \leq lct_b$). The first two of these positions correspond to edge-finding propagation and the last two to extended edge-finding propagation. We first consider only the right and inside positions, i.e., $est_a \leq est_u$. Note that a could be u. Then,

$$R \cdot (lct_b - est_a) - energy(a, b, u) < 0 \rightarrow \left\lceil \frac{rest(a, b, u)}{r_u} \right\rceil \le S_u$$
 (2)

where $energy(a, b, u) := energy(a, b) - e_u(a, b) + r_u \cdot (\min(lct_b, ect_u) - est_u)$ and

$$rest(a,b,u) := energy(a,b,u) - (R - r_u) \cdot (lct_b - est_a) - r_u \cdot (\min(lct_b,ect_u) - est_u) .$$

The first two terms in the sum of energy(a,b,u) gives the energy consumption of all considered activities except u, whereas the last term is the required energy of u if it is scheduled at est_u in the time window $[est_a, lct_b)$. The propagation, including explanation generation, can be performed in $\mathcal{O}(l^2+k\cdot n)$ runtime, where $l=|\mathcal{V}^{EF}|$ and k the number of bounds' updates, if the resource profile is given. Moreover, TTEF propation does not necessarily consider each $u \in \mathcal{V}^{EF}$, but those only that maximise $\min(e_u^{EF}, r_u \cdot (lct_b - est_a)) - r_u \cdot \max(0, lct_b - lst_u^{EF})$ and satisfy $est_a \leq est_u$. The corresponding algorithm is shown in Alg. 2 in App. A.

A naïve explanation for a lower bound update from est_u to $newLB := \lceil rest(a, b, u)/r_u \rceil$ with respect to the time window $\lceil est_a, lct_b \rceil$ additionally includes the previous and new lower bound on the left and right hand side of the implication, respectively, in comparison to the naïve explanation for a resource overload.

$$\llbracket est_u \leq S_u \rrbracket \wedge \bigwedge_{i \in \mathcal{V} \setminus \{u\}: p_i(a,b) > 0} \llbracket est_i \leq S_i \rrbracket \wedge \llbracket S_i \leq lst_i \rrbracket \to \llbracket newLB \leq S_u \rrbracket$$

As we discussed in the case of resource overload, we perform a similar generalisation for the activities in $V \setminus \{u\}$, and for u we decrease the lower bound on the left hand side as much as possible so that the same propagation holds when u is executed at that

suite	sub-suites	#inst	#act	p_{i}	$\#\mathrm{res}$	notes	
AT [3]	st27/st51/st103	48 each	25/49/101	1-12	6 each		
PSPLib [16]	J30 [17]/J60/J90	480 each	30/60/90	1-10	4 each		
	Ј120	600	30	1 - 10	4		
BL [4]	BL20/BL25	20 each	20/25	1-6	3 each		
Pack [8]		55	15–33	1-19	2-5		
KSD15_D [18]		480	15	1-250	4	based of	on J30
Pack_d [18]		55	15–33	1-1138	2-5	based	on
						Pack	

Table 1: Specifications of the benchmark suites.

decreased lower bound.

$$[[est_a + lct_b - newLB + 1 - p_u \le S_u]] \land$$

$$\bigwedge_{i \in \mathcal{V} \setminus \{u\}: p_i(a,b) > 0} [[est_a + p_i(a,b) - p_i \le S_i]] \land [[S_i \le lct_b - p_i(a,b)]]$$

$$\rightarrow [[newLB \le S_u]] \quad (3)$$

Again this more general explanation expresses the energetic reasoning propagation and the bounds of activities in $\{i \in \mathcal{V} \setminus \{u\} \mid p_i(a,b) > 0\}$ can further be generalised in the same way as for a resource overload. But here the available energy units Δ for widening the bounds is $rest(a,b,u) - r_u \cdot (newLB - 1) + 1$. Hence, $0 \le \Delta < r_u$ indicate that the explanation only can further be generalised a little bit. We perform this generalisation as for the overload case.

4. Experiments on Resource-constrained Project Scheduling Problems

We carried out extensive experiments on RCPSP instances comparing our solution approach using both time-table and/or TTEF propagation. We compare the obtained results on the lower bounds of the makespan with the best known so far. Detailed results are available at http://www.cs.mu.oz.au/~pjs/rcpsp.

We used six benchmark suites for which an overview is given in Tab. 1 where #inst, #act, p_i , and #res are the number of instances, number of activities, range of processing times, and number of resources, respectively. The first two suites are highly disjunctive, while the remainder are highly cumulative.

The experiments were run on a X86-64 architecture running GNU/Linux and a Intel(R) Core(TM) i7 CPU processor at 2.8GHz. The code was written in Mercury [30] using the G12 Constraint Programming Platform [31].

We model an instance as in [27] using global cumulative constraints cumulative and difference logic constraints $(S_i + p_i \leq S_j)$, resp. In addition, between two activities i, j in disjunction, *i.e.*, two activities which cannot concurrently run without overloading some

resource, the two half-reified constraints [10] $b \to S_i + p_i \le S_j$ and $\neg b \to S_j + p_j \le S_i$ are posted where b is a Boolean variable.

We run cumulative constraint propagation using different phases:

- (a) time-table consistency check in $\mathcal{O}(n + p \log p)$ runtime,
- (b) TTEF consistency check in $\mathcal{O}(l^2+n)$ runtime as defined in Section 3.1,
- (c) time-table bounds' propagation in $\mathcal{O}(l \cdot p + k \cdot \min(R, n))$ runtime, and
- (d) TTEF bounds' propagation in $\mathcal{O}(l^2 + k \cdot n)$ runtime as defined in Section 3.2 where k, l, n, p are the numbers of bounds' updates, unfixed activities, all activities, and height changes in the resource profile, resp.

Note that in our setup phase (d) TTEF bounds' propagation does not take into account the bounds' changes of the phase (c) time-table bounds' propagation. For the experiments, we consider three settings of the cumulative propagator: tt executes phases (a) and (c), ttef(c) (a-c), and ttef (a-d). Note that phases (c) and (d) are not run if either phase (a) or (b) detects inconsistency.

4.1. Upper Bound Computation

For solving RCPSP we use the same branch-and-bound algorithm as we used in [27], but here we limit ourselves to the search heuristic HOTRESTART which was the most robust one in our previous studies [26, 27]. It executes an adapted search of [4] using serial scheduling generation for the first 500 choice points and, then, continues with an activity based search (a variant of VSIDS [20]) on the Boolean variables representing a lower part $x \leq v$ and upper part v < x of the variable x's domain where x is either a start time or the makespan variable and v a value of x's initial domain. Moreover, it is interleaved with a geometric restart policy [35] on the number of node failures for which the restart base and factor are 250 failures and 2.0, respectively. The search was halted after 10 minutes.

The results are given in Tab. 2 and 3. For each benchmark suite, the number of solved instances (#svd) is given. The column $\mathrm{cmpr}(a)$ shows the results on the instances solved by all methods, where a is the number of such instances. The left entry in that column is the average runtime on these instances in seconds, and the right entry is the average number of failures during search. The entries in column $\mathrm{all}(a)$ have the same meaning, but here all instances are considered where a is the total number of instances. For unsolved instances, the number of failures after 10 minutes is used.

Table 2 shows the results on the highly disjunctive RCPSPs. As expected, the stronger propagation (ttef(c), ttef) reduces the search space overall in comparision to tt, but the average runtime is higher by a factor of about 5%–70% and 50%–100% for ttef(c) and ttef. Interestingly, ttef(c) and ttef solved respectively 1 and 2 more instances on J60 and closed the instance j120_1_1 on J120 which has an optimal makespan 105. This makespan corresponds to the best known upper bound. However, the stronger propagation does not generally pay off for a CP solver with nogood learning.

Table 2: UB results on highly disjunctive RCPSPs.

			ј30			1		J 60		
	#svd	cmpr	(480)	all(480)	#svd	cmpr	(429)	all(480)
tt	480	0.12	1074	0.12	1074	430	1.82	5798	64.25	93164
ttef(c)	480	0.20	1103	0.20	1103	431	2.00	4860	64.39	80845
ttef	480	0.23	991	0.23	991	432	3.04	5191	64.87	62534
			л90					Ј120		
	#svd	cmpr	(400)	all(480)	#svd	cmpr	(280)	all(600)
tt	400	5.03	9229	104.09	132234	283	9.71	15022	322.35	398941
ttef(c)	400	6.93	9512	105.69	104297	282	13.47	16958	324.73	297562
ttef	400	8.10	8830	106.66	72402	283	14.97	13490	324.66	186597
			AT							
	#svd	cmpr	(129)	all(144)					
tt	132	8.90	19997	66.22	87226					
ttef(c)	130	9.36	16466	69.41	72056					
ttef	129	13.55	17239	74.60	63554					

Table 3: UB results on highly cumulative RCPSPs. BL \parallel PACK

								1 11011		
	#svd	cmp	r(40)	all(40)		#svd	cmp	$\operatorname{or}(16)$	all((55)
tt	40	0.16	2568	0.16	2568	16	77.65	245441	447.69	699615
ttef(c)	40	0.02	370	0.02	370	39	37.22	122038	186.79	292101
ttef	40	0.02	269	0.02	269	39	44.44	105751	188.23	257747
		K	SD15_	.D				Pack_	D	
	#svd	cmpr	(480)	all(480)	#svd	cmp	$\operatorname{or}(37)$	all((55)
tt	480	0.01	26	0.01	26	37	32.72	42503	218.26	184293
ttef(c)	480	0.01	26	0.01	26	37	23.96	32916	212.37	170301
ttef	480	0.01	26	0.01	26	37	36.93	37004	221.11	157015

Table 4: LB results on AT, PACK, and PACK_D

	A.	Γ	Pac	CK	Pack_d			
			0/4/12					
ttef	7/2/3	+44	1/4/11	+101	2/5/10	+618		

Table 5: LB results on J60, J90, and J120

			J60				$_{\rm J90}$				J120								
		+1	+2	+3	+1	+2	+3	+4	+5	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10
1 min	ttef(c)	4	1	-	12	1	-	-	-	27	8	4	-	-	-	2	-	-	_
1 111111	ttef	7	5	-	25	14	3	1	-	90	20	10	5	2	-	-	2	-	-
10 :	ttef(c)	21	2	-	25	7	-	-			16	4	4	2	-	-	1	1	_
10 mins	ttef	13	6	3	35	17	6	3	1	116	39	9	9	4	1	-	-	1	1

Table 3 presents the results on highly cumulative RCPSPs which clearly shows the benefit of TTEF propagation, especially on BL for which ttef(c) and ttef reduce the search space and the average runtime by a factor of 8, and PACK for which they solved 23 instances more than tt. On PACK_D, ttef(c) is about 50% faster on average than tt while ttef is slightly slower on average than tt. No conclusion can be drawn on KSD15_D because the instances are easy for LCG solvers.

4.2. Lower Bound Computation

The lower bound computation tries to solve RCPSPs in a destructive way by converging to the optimal makespan from below, i.e., it repeatedly proves that there exists no solution for current makespan considered and continues with an incremented makespan by 1. If a solution found then it is the optimal one. For these experiments we use the search heuristic HotStart as we did in [26, 27]. This heuristic is HotRestart (as decribed earlier) but no restart. We used the same parameters as for HotRestart. For the starting makespan, we choose the best known lower bounds on J60, J90, and J120 recorded in the PSPLib at http://l29.187.106.231/psplib/ and [34] at http://vilim.eu/petr/cpaior2011-results.txt. On the other suites, the search starts from makespan 1. Due to the tighter makespan, it is expected that the TtEf propagation will perform better than for upper bound computation on the highly disjunctive instances. The search was cut off at 10 minutes as in [26, 27].

Table 4 shows the results on AT, PACK, and PACK_D restricted to the instances that none of the methods could solve using the upper bound computation, that are 12, 16, and 18 for AT, PACK, and PACK_D, respectively. An entry a/b/c for method x means that x achieved respectively a-times, b-times and c-times a worse, the same and a better lower bound than tt. The entry +d is the sum of lower bounds' differences of method x to tt. On PACK and PACK_D, ttef(c) and ttef clearly perform better than tt. On the highly disjunctive instances in AT, ttef(c) and tt are almost balanced whereas tt could generate better lower bounds on more instances as ttef. The lower bounds' differences on AT are dominated by the instance st103_4 for which ttef(c) and ttef retrieved a lower

bound improvement of 54 and 53 time periods with respect to tt.

The more interesting results are presented in Tab. 5 because the best lower bounds are known for all the remaining open instances (48, 77, 307 in J60, J90, J120).² An entry in a column +d shows the number of instances for that the corresponding method could improve the lower bound by d time periods. On these instances, we run at first the experiments with a runtime limit of one minute as it was done in the experiments for TTEF propagation in [34] but he used a CP solver without nogood learning. tt could not improve any lower bound because its corresponding results are already recorded in the PSPLib. ttef(c) and ttef improved the lower bounds of 59 and 183 instances, respectively, which is about 13.7% and 42.4% of the open instances. Although, the experiments in [34] were run on a slower machine³ the results confirm the importance of nogood learning. For the experiments with 10 minutes runtime, we excluded tt due to time constraints and expected inferior results to ttef(c) and ttef. With the extended runtime, ttef(c) and ttef could improved the lower bounds of more instance, namely 151 and 264 instances, respectively, which is about 35.0% and 61.1%. Moreover, 3, 1, and 1 of the remaining open instances on J60, J90, and J120, respectively, could be solved optimally. See App. B for the listing of the closed instances and the new lower bounds.

5. Conclusion and Outlook

We present explanations for the recently developed TTEF propagation of the global cumulative constraint for lazy clause generation solvers. These explanations express an energetic reasoning propagation which is a stronger propagation than the TTEF one.

Our implementation of this propagator was compared to time-table propagation in lazy clause generation solvers on six benchmark suites. The preliminary results confirms the importance of energy-based reasoning on highly disjunctive RCPSPs for CP solvers with nogood learning.

Moreover, our approach with TTEF propagation was able to close one instance. It also improves the best known lower bounds for 264 of the remaining 432 remaining open instances on RCPSPs from the PSPLib.

In the future, we want to integrate the extended edge-finding propagation into TTEF propagation as it was originally proposed in [34], to perform experiments on cutting and packing problems, and to study different variations of explanations for TTEF propagation. Furthermore, we want to look at a more efficient implementation of the TTEF propagation as well as an implementation of energetic reasoning.

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²Note that the PSPLib still lists the instances j60_25_5, j90_26_5, j120_8_3, j120_48_5, and j120_35_5 as open, but we closed the first four ones in [27] and [19] closed the last one.

³Intel(R) Core(TM)2 Duo CPU T9400 on 2.53GHz

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A. TTEF propagation algorithms

Algorithm 1 shows the TTEF consistency check used. The outer loop (lines 2–16) iterates over all distinctive possible end times for the time windows while the inner loop (lines 7–16) iterates over all possible start times. In line 11 (12), it checks whether a must fully (partially) be executed in the current time window and further ones checked in the same inner loop. If so it adds the required free energy units e_a^{EF} of a to E. In line 13, it calculates the still available energy units in the time window [begin, end) taking the energy units from the resource profile ttEn(a,b) into account. If this results in a resource overload then a corresponding explanation is generated (line 15) and the algorithm fails; otherwise, the algorithm succeeds.

Algorithm 1: TTEF consistency check.

```
Input: X an array of activities sorted in non-decreasing order of the earliest start time.
   Input: Y an array of activities sorted in non-decreasing order of the latest completion time.
  end = \infty:
  for y := n down to 1 do
       b:=Y[y];
       if lct_b = end then continue;
4
       end := lct_b;
       E := 0;
       for x := n down to 1 do
            a := X[x];
            if end \leq est_a then continue;
            begin := est_a;
10
            if lct_a \leq end then E := E + e_a^{EF};
11
            if lst_a^{EF} < end then E := E + r_a \cdot (end - lst_a^{EF});
12
            avail := R \cdot (end - begin) - E - ttEn(a, b);
13
            if avail < 0 then
14
                explainOverload(begin, end);
15
                return false;
16
17 return true;
```

Algorithm 2 shows the lower bounds propagation algorithm. As for Alg. 1 the outer loop (lines 3–24) and inner loop (lines 7–24) iterate over the end and start times of the time windows [begin, end), but require more book keeping. In line 6, it initialises E. u, and enReqU where: E records the required energy units by the considered activities that must fully or partially be run in the time window; and u stores the activity that maximises $\min(e_u^{EF}, r_u \cdot (end - begin)) - r_u \cdot \max(0, end - lst_u^{EF})$ and that value is saved in enReqU. If a must be fully or partially be executed in the time window then the corresponding energy units are added to E in lines 11 and 14, resp. The desired activity for pruning is computed in lines 13, 15, and 16, whereas the available energy units are calculated in line 17. In the case that there is not sufficient energy available then the condition of line 18 holds and the algorithm determines the first possible start time for u (lines 19, 20). If that is larger than the recorded earliest start time in est'_u then the algorithm generates the explanation (line 22) and postpones the update (line 23) after finishing with the outer loop (line 25).

Algorithm 2: TTEF lower bounds propagator on the start times.

```
Input: X an array of activities sorted in non-decreasing order of the earliest start time.
 Input: Y an array of activities sorted in non-decreasing order of the latest completion time.

1 for i \in \mathcal{V}^{EF} do est'_i := est_i;
 2 end := \infty; k := 0;
   for y := n down to 1 do
        b := Y[y];
        if lct_b = end then continue;
        end := lct_b; E := 0; u := -\infty; enReqU := 0;
        for x := n down to 1 do
            a := X[x];
             if end \leq est_a then continue;
             begin := est_a;
10
             if lct_a \leq end then E := E + e_a^{EF};
11
             else
12
                 enIn := r_a \cdot \max(0, end - lst_a^{EF});
13
                 E := E + enIn;
14
                 enReqA := min(e_a^{EF}, r_a \cdot (end - est_a)) - enIn;
                 \mathbf{if}\ enReqA > enReqU\ \mathbf{then}\ u := a;\ enReqU := enReqA;
             avail := R \cdot (end - begin) - E - ttEn(a, b);
             if enReqU > 0 and avail - enReqU < 0 then
18
                 rest := E - avail - r_a \cdot \max(0, end - lst_a);
                 lbU := begin + \lceil rest/r_u \rceil;
20
                 if est'_u < lbU then
21
                      expl := explainUpdate(begin, end, u, est'_u, lbU);
22
                      Update[++k] := (u, lbU, expl);
23
                      est'_u := lbU;
24
25 for z := 1 to k do updateLB(Update[z]);
```

Table 6: New lower bounds on J60.

inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB
9_1	85	9_5	81	9_6	106	9_7	103	9_8	95	9_10	89	13_1	105
13_{-2}	103	13_3	84	13_4	98	13_7	82	13_8	115	13_9	96	13_10	113
25_{-2}	96	25_4	106	25_6	106	29_1	97	29_6	144	29_7	115	29_8	97
$41_{-}3$	90	41_10	106	45_1	90			'		'			

Table 7: New lower bounds on J90.

inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB
5_3	84	5_5	109	5_7	106	5_8	97	5_9	114	5_10	95	9_2	122
9_3	98	9_{-4}	120	9_5	127	9_6	113	9_7	103	9_8	111	9_9	106
$9_{-}10$	105	13_{-2}	119	13_3	105	13_5	109	13_7	116	13_8	113	13_9	117
13_10	114	21_{-7}	106	21_8	108	25_1	117	25_{-2}	122	25_3	113	25_4	128
25 - 5	110	25-6	113	25_8	131	25_9	98	25_10	119	29_1	126	29_2	122
29_{-4}	139	29-6	117	29_{-7}	160	29_8	146	29_9	120	30_9	92	37_{-2}	114
$41_{-}1$	129	41_{-2}	154	41_3	149	41_4	142	41_5	116	41_6	124	41_{-7}	145
41_{-8}	148	$41_{-}9$	110	41_10	144	45_1	143	45_2	138	45_3	144	45_4	126
45 - 6	163	$45_{-}7$	129	45_8	150	45_9	145	45_10	156	46_9	86		

B. Closed Instances and New Lower Bounds on PSPLib

From the open instances, we closed the instances 9.3 (100), 9.9 (99), 25.10 (108) on J60, 5.6 (86) on J90, and 1.1 (105), 8.6 (85) on J120 where the number in brackets shows the optimal makespan. We computed new lower bounds on the remaining open instances from the PSPLib. Tables 6–8 list these new lower bounds where the column "inst" shows the name of the instance and the column "LB" the corresponding new lower bound.

C. Best Lower and Upper Bounds Retrieved

For a later comparison, Tables 9–11 show the best lower and upper bounds for AT, PACK, and PACK_D retrieved by one of the methods tt, ttef(c), and ttef. The column "inst" shows the instance name and the column "LB/UB" the corresponding lower and upper bound. If these bounds are equal then only one number is given.

Table 8: New lower bounds on J120.

inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB	inst	LB
6_1	134	6_2	127	6_5	117	6_6	141	6_8	141	6_9	150	6_10	158
$7_{-}1$	99	$7_{-}3$	98	7_{-4}	106	7_6	116	$7_{-}7$	114	7_{-8}	93	7_9	87
$7_{-}10$	112	8_2	102	8_5	100	8_9	90	8_10	92	9_{-4}	85	11_1	157
11_{-2}	147	11_3	189	11_4	178	11_5	194	11_6	192	11_{-7}	149	11_8	153
11_10	164	12_{-1}	126	12_2	112	12_{-4}	122	12_5	155	12-6	116	13_1	124
$13_{-}3$	116	13_{-4}	109	13_6	96	13_9	83	14_2	91	$14_{-}5$	94	14_7	90
16_{-1}	181	16_{-3}	221	16_4	191	16_6	195	16_8	183	$17_{-}5$	124	17_6	134
18_8	102	$18_{-}9$	89	18_10	97	26_{-1}	155	26_2	159	26 - 3	158	26_4	161
$26_{-}5$	139	$26_{-}6$	171	26_{-7}	147	26_8	168	26_9	161	$26_{-}10$	178	27_1	107
$27_{-}2$	110	$27_{-}3$	142	27_{-4}	105	$27_{-}5$	106	27_6	133	$27_{-}7$	119	27_8	136
$27_{-}9$	121	$27_{-}10$	111	28_1	106	31_1	181	31_2	176	$31_{-}3$	160	31_4	195
$31_{-}5$	187	$31_{-}6$	182	31_7	191	31_8	176	31_9	176	$31_{-}10$	202	32_1	144
32_{-2}	123	$32_{-}5$	133	32_6	122	32_8	132	33_1	105	33_{-2}	107	33_3	102
33_{-4}	107	33_8	107	33_9	109	34_1	76	34_2	103	$34_{-}3$	99	34_5	102
36_{-1}	201	$36_{-}3$	218	36_5	213	$36_{-}7$	196	36_9	203	37_{-2}	141	37_5	195
37 - 8	169	$37_{-}9$	138	38_1	105	38_2	119	38_4	138	38-6	119	38_7	103
38 - 10	137	39_{-2}	105	40_1	80	42_{-1}	107	46_1	172	46_{-2}	187	46_3	163
$46_{-}5$	136	$46_{-}7$	158	46_9	157	46_10	175	47_{-1}	130	$47_{-}3$	119	47_{-4}	120
$47_{-}5$	126	$47_{-}6$	128	$47_{-}7$	114	47_8	124	47_10	128	48_{-4}	123	51_1	186
51_{-2}	200	51_{-3}	193	51_4	197	51_6	193	51_{-7}	185	51_{-8}	186	51_9	190
$51_{-}10$	201	52_{-1}	161	52_2	169	52_3	126	52_{-4}	157	$52_{-}5$	158	52_6	183
$52_{-}7$	142	52_{-8}	148	52_9	142	52_10	131	53_1	138	53_{-2}	109	53_4	138
$53_{-}5$	109	53_6	101	53_8	135	53_10	124	54_1	102	$54_{-}5$	107	54_6	104
54 - 8	100	$54_{-}9$	105	57_1	173	57_{-2}	151	57_3	176	$57_{-}5$	170	57_6	176
$57_{-}7$	156	$57_{-}9$	157	58_2	122	58_3	117	58_4	138	$58_{-}5$	116	58_6	135
$58_{-}7$	143	58_8	126	58_9	126	59_5	104	59_6	112	59 - 8	107	59_9	117
59 - 10	128	60 - 3	88	60_7	91								

Table 9: Lower and upper bounds for AT. $\,$

inst	LB/UB	inst	LB/UB	inst	LB/UB	inst	LB/UB	inst	LB/UB
27_{-1}	41	27_{-2}	53	27_3	68	27_4	112/114	27_5	56
$27_{-}6$	73	$27_{-}7$	54	27 - 8	95	$27_{-}9$	38	$27_{-}10$	45
$27_{-}11$	57	$27_{-}12$	73	$27_{-}13$	38	$27_{-}14$	55	$27_{-}15$	46
$27_{-}16$	75	$27_{-}17$	55	$27_{-}18$	55	$27_{-}19$	79	$27_{-}20$	152
$27_{-}21$	92	$27_{-}22$	86	27 - 23	82	$27_{-}24$	106	27 - 25	51
$27_{-}26$	53	$27_{-}27$	58	$27_{-}28$	95	$27_{-}29$	51	27 - 30	76
27 - 31	75	$27_{-}32$	82	27 - 33	66	$27_{-}34$	61	27 - 35	115
27 - 36	146	27 - 37	78	27 - 38	100	$27_{-}39$	119	27_40	130
27_41	60	$27_{-}42$	53	27 - 43	75	27_44	88	27 - 45	49
27 - 46	65	27_47	75	27 - 48	80	51_{-1}	98	51_{-2}	96
$51_{-}3$	133	51_{-4}	161/219	$51_{-}5$	97	51_6	126	$51_{-}7$	120
51-8	194	$51_{-}9$	74	51 - 10	73	51_11	99	51 - 12	116/137
$51_{-}13$	84	$51_{-}14$	86	$51_{-}15$	86	$51_{-}16$	132	$51_{-}17$	84
$51_{-}18$	99	$51_{-}19$	170	$51_{-}20$	274	$51_{-}21$	145	$51_{-}22$	168
$51_{-}23$	183	$51_{-}24$	228	$51_{-}25$	95	51_26	89	$51_{-}27$	113
$51_{-}28$	164	51_29	98	51_30	105	51_31	130	$51_{-}32$	139
$51_{-}33$	116	$51_{-}34$	115	$51_{-}35$	173	51_36	300	$51_{-}37$	162
$51_{-}38$	177	$51_{-}39$	189	$51_{-}40$	218	$51_{-}41$	102	$51_{-}42$	108
51-43	121	$51_{-}44$	174	51-45	122	51-46	125	$51_{-}47$	151
$51_{-}48$	167	103_1	158	103_{-2}	182	103_3	216/259	103-4	280/445
103 - 5	191	103_6	207/209				207/294		139
103_10	119		160/169		213/302		127	103_14	152
$103_{-}15$	157/168		167/179		209	103_18	232	103_19	301
103_20	475	103_21	276	103_22	295	103_23	368	103_24	449
103 - 25	177	103_26	183	$103_{-}27$		103_28	295	$103_{-}29$	225
103_30	231	103_31	227	103_32	281	103_33	220	103_34	264
103_35	341	103_36	575	103_37	327	103_38	376	103_39	389
103_40	451	103_41	191	103_42	187	103_43	260	103_44	375
103-45	216	103_46	251	103-47	262	103_48	300		

Table 10: Lower and upper bounds for PACK.

inst	LB/UB	inst	LB/UB	inst	LB/UB	inst	LB/UB	inst	LB/UB	inst	LB/UB
001	23	002	32	003	29	004	43/44	005	42	006	47
007	41	008	44	009	57/72	010	38	011	44	012	45
013	36	014	45	015	43	016	63	017	62	018	60
019	59	020	62	021	51	022	59	023	51	024	56
025	69/70	026	54	027	55	028	64	029	43	030	20
031	70	032	80	033	78	034	73	035	73/77	036	100/106
037	116/138	038	86	039	99/111	040	87/91	041	27	042	29
043	105	044	103	045	86/87	046	110/128	047	103/107	048	76/77
049	29	050	94/109	051	29	052	85	053	97/113	054	92/100
055	91/97			'		Į.		ı		'	

Table 11: Lower and upper bounds for PACK_D.

inst	LB/UB								
001	612	002	745/747	003	624/625	004	1381	005	983
006	1119	007	1082	008	1274	009	1593/1951	010	1216
011	940	012	1234/1241	013	829	014	1565	015	1198
016	1783/1813	017	1641/1651	018	1462/1480	019	1526/1542	020	1661
021	1606	022	1787	023	1092	024	1625	025	2061/2147
026	926	027	1789/1793	028	1897/1962	029	1233	030	597
031	1949	032	2943	033	3390	034	2371	035	2305
036	2175/2191	037	3325/3614	038	2180	039	2730/2734	040	3024
041	679	042	838	043	2439	044	3050	045	2712
046	3243/3277	047	2740/2745	048	2446	049	675	050	2687/2716
051	838	052	2253	053	2521	054	2750	055	2628