Teachers and Children Playing with Factorization: Putting Prime Slaughter to the Test

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Abstract. This study presents results from the evaluation of Prime Slaughter, a computer game aimed at supporting learning of factorization and prime numbers. The game was tested and re-conceptualized during a whole-day participatory workshop, involving two classes of pupils and their math teacher. As a result, it was possible to see that social play elicits fundamental questions about the nature of abstract concepts, in our case the operations involved in factorization and the relationship between natural numbers and primes, supporting sense making and reflections through verbal articulation. Moreover, new insights were gathered, in relation to enrich the game, taking inspiration from emergent meaning regarding the different forms of play allowed and the need to better support multi-player interaction.

Keywords: Playful learning, factorization, social interaction.

1 Introduction

Prime numbers and factorization are challenging and fundamental topics for mathematics and computer sciences, moreover, they are regarded as confusing and boring by pupils that have to understand and work with them for the first time. In order to facilitate learning of these topics Prime Slaughter was designed, a computer game aimed at representing factorization and primality in a playful and intuitive way. The scenario envisioned for Prime Slaughter (PS for short), was as a mediating tool for learning, to support sense making, so that a teacher could use the game as grounding for theoretical concepts and for their students to practice.

A one-day evaluation was conducted involving two classes and their math teacher, in which the participants had to play with Prime Slaughter and afterwards design low-fidelity prototypes for the game, with different materials, so to gain inspirations about how the game could be improved.

The design and gameplay of our current prototype is explained in section 2, together with related work. Section 3 presents our empirical study, focusing on methods, and the test setup. Section 4 discusses the results and user-feedback we obtained from the test, and section 5 concludes the paper.

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2 Prime Slaughter

The development of Prime Slaughter aimed at creating a mediating tool for learning, which could allow students approaching the concepts of prime numbers and factorization, to gain a multimodal grounding for the knowledge to be learnt.

Theoretically the creation of PS is based upon play, intended as a resource to foster conceptual thinking. Play mediated through tangible artefacts allows individuals to move towards an imaginary world, in which they can reflect upon their actions in play and their implications for the imaginary world (Vygotsky 1978). In this way, artefacts become tangible symbols eliciting what if questions, providing a grounding for exploration of meaning and conceptual thinking (Sutton-Smith 1987).

In the scenario we envisioned, Prime Slaughter is supposed to be used as a learning tool, to support communication between teachers and learners in a form of collaborative sense making. Hence, in our scenario, teachers may introduce factorization and prime numbers to their class and use the game to demonstrate meanings and dynamics, by playing, in a situation of apprenticeship in thinking (Rogoff 1990). In this way, the teachers can use Prime Slaughter to support their students to go beyond what they already know, supporting theoretical explanations with a multimodal and interactive representation of the concept. By multimodal representation, we intend a rich illustration of theoretical concepts, combining multiple sensory modalities (van Leeuwen 2005), specifically through audio-visual feedback in the game, and also social interaction and verbal communication among the players. Prime Slaughter is intended to provide an interactive representation of concepts, but it is not supposed to be self-explanatory, the use scenario is conceived so to rely upon teachers' experience to make it effective. When fully completed, teachers may also use the different levels of PS to perform distal arrangement (Rogoff 1990), defined as a segmentation of tasks, through which teachers adapt the content of learning and related activities to the level of mastery expressed by the students. In this sense, the teachers could assign initial or more advanced levels to the students, asking them to pay attention to different elements of the game, according to their individual needs.

Prime Slaughter is also supposed to be used independently by students as part of their homework, so that after being introduced to factorization, they can practice by themselves in order to develop their theoretical understanding through play. In this sense, it became a priority in the project to accommodate individual needs regarding play, which is also intended as a state of mind, in which learners can act in an internally motivated and exploratory way, in which they should be given the possibility to choose their course of actions (Sutton-Smith 1987, Apter 2007). Hence based upon empirical data gathered in a previous study (Marchetti and Petersson Brooks 2011), according to which different individuals tend to select among two different forms of play: a competitive and an exploratory one. It was observed that some children enjoyed play in a military form of play, in which the goal is to fight against an enemy or another player, eventually challenging and teasing each other through their play. On the contrary, other children may prefer to explore how can they modify and affect the game environment, without having to fight or to win (Marchetti and Petersson Brooks 2012).

2.1 The Prototype

The current prototype of Prime Slaughter has only one level, composed of 6 rooms, as visible in Fig. 1. In the game numbers to factorize are mapped into enemies, i.e. monsters, while division is mapped into slicing with a sword. To maintain the coherence with the source domain, swords are also numbers, but only prime numbers can be swords. In the game various clues are offered to the player, to make more sense of the source domain, for example the size of the monster-numbers depends on the number of factors. A 12-monster is therefore larger than a 6-monster, and prime monsters (e.g. 3-monsters) are the smallest. Since 1 is not a prime, 1-monsters are special and they vanish as soon as they are created. At the beginning the player only has a 2-sword, so only even monsters can be sliced in 2 halves, however, prime monsters can be captured and converted into swords, so the player can collect swords and engage larger and larger monsters.

Two different modalities of play are supported in Prime Slaughter: a classical adventure gameplay, in which a hero kills the jelly monsters, and an exploratory play style, in which players prune number-trees and collect number-fruits.

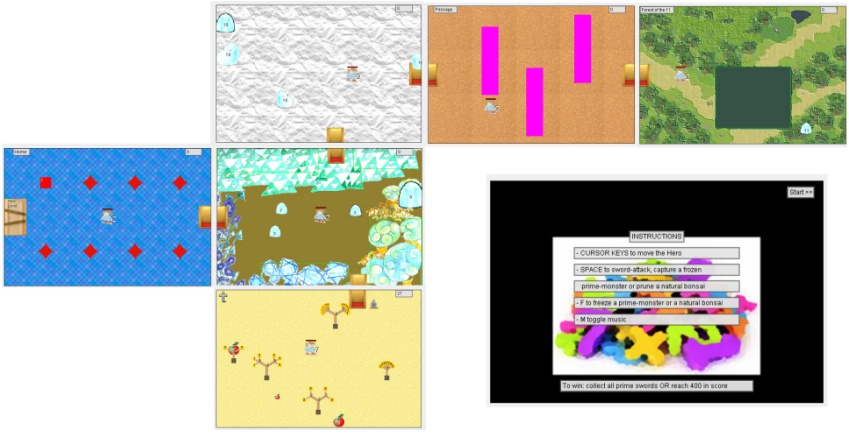

Fig. 1. Map of Prime Slaughter's first prototype. The first screen presented to the player shows instructions, then the player's character starts in the left-most room and explores all rooms. The lowest room is special: monsters cannot enter it, instead.

The number-trees are only present in the Bonsai Graveyard room (Fig. 1, lower room of the level), and since no monsters are present in the room, players can enter and leave the Graveyard to switch modality of play. Finally, testing whether a number is divisor of another is transposed into exploration of the rooms of the game, sword collection and slicing of number-monsters: in fact slicing a 3-monster with a 2-sword causes damage to the player, hence learning about divisibility provides an advantage in our game.

2.2 Related Work

In our previous study (Valente and Marchetti 2012) we surveyed math-related games, especially free online games, and found that they can be clustered in 3 groups: augmented exercises (the large majority), exploratory/manipulative environments, and very few genuine math games. We could not find any games about factorization, except computer-augmented exercises; however, if we consider fractions (a domain that makes use of factorization, for instance when adding 2 fractions) we have the IXL[1] website with augmented exercises, and the Refraction game[2], a rare and very good example of playful learning.

Considering other media, not only games as is our case, the novel *White Light* (Rucker 2001) offers an example of transposition of complex mathematical concepts into narrative. *White Light* is about Cantor's Continuum Problem, a complex mathematical problem related to the cardinality of infinite sets, and the baffling properties of infinity are a recurring theme in the book. The main character is a math teacher, who is somehow transported on a distant planet where he stays at a hotel with infinite rooms (in specific the hotel has $\aleph_0$ rooms, i.e. one room for every natural number), the Hilbert Hotel. The hotel is full, so the mathematician suggests moving every guest in the next room, in this way they will all still have a room, but the first room will be empty for him. There are many more examples of difficult and abstract mathematical concepts that become concrete everyday problems in the novel, for instance the main character wants to meet Cantor, who happens to live on the distant planet. However, Cantor's front door has a lock that can only be opened by typing all digits of pi, so the math teacher uses one of the classical algorithms to generate the digits of pi, and types the first in 1 second, the second digit in half a second, the third in a quarter of a second, and so forth. Luckily for the teacher, in the distant planet physics allows for infinite acceleration, so he can practically compose all of pi in 2 seconds and finally meet Cantor. Mathematically the trick works because of the convergence of the power series

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 2$$

which in turn is related to Zeno's paradox about Achilles and the Tortoise (as discussed also in Hofstadter 1999), the origins of calculus and Newtonian physics.

For us Rucker is very inspiring by the way he turns abstract puzzles into practical problems, and also because of the surreal and goliardic way he mixes serious topics with ordinary and often squalid situations. Making fun of (or at least having fun with) serious, tough concepts is quite close to our idea of modeling games after abstract math concepts, as it introduces a rebellious element in learning experience, linked to freedom and learners' ownership over their own learning process (Apter 2007).

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[1] See `http://www.ixl.com/math/grade-2` at the *fraction* section.

[2] Playable at `http://www.centerforgamescience.org/site/games/refraction`

3 Evaluation and Method

A whole-day participatory workshop was set up in our university, with students from a local school in Bramming (Denmark), and their math teacher. The target group for the study is primary school and junior high school students, who are facing the topic of factorization and prime numbers for the first time, in the Danish school system it corresponds to kids 12-16 years old. All the students knew about prime numbers and factorization prior to the study (the youngest ones were introduced to the concept just the day before by their math teacher). The workshop itself was structured so that in the morning the children could test the game, alone or in pairs, and in the afternoon they could engage in a prototyping workshop in small groups.

The goal of the study was to evaluate if Prime Slaughter could support learning of factorization in a meaningful way, allowing the learners to reflect upon the represented concepts and be able to grasp their meaning. The study should also evaluate if the teachers could see learning potential in Prime Slaughter, also from the perspective of using it as a tool to demonstrate abstract concepts through the game.

During the morning the students were welcomed and introduced to Prime Slaughter by one of the authors, who also teaches math at the university, with their own teacher attending the session and assisting them in case of need. Then the students could test the game in a computer room, some of them preferred to sit alone while others were happy to share a computer with a classmate. During the session, the researchers conducted ethnographic observations and situated interviews (Pink 2006), observing the students' actions without interfering, unless some difficulty may emerge, the students called for assistance, or a particular event was spotted. The students were also filmed (with a mobile video camera in the morning and with fixed cameras in the afternoon participatory design workshop), to document their interaction and the discussions that the game elicited. An interview was also conducted with the math teacher, so that his perspective could be considered in the evaluation of the game.

In the afternoon the students were then invited to create new low-fidelity prototypes for a new version of the game, they were asked to split into groups of more or less four participants. A participatory design workshop, intended as a cooperative inquiry in which the pupils acted as design partners (Druin 2002), was run in one of the university's classroom. Each group was supposed to occupy a large table and was assigned a *design kit*, including colored cardboard, a few colored markers, a set of post-its, scissors, glue, and an envelope with screenshot prints from the different rooms of the PS game. The prints were supposed to be used as probes in the design phase (Preece et al. 2011), so that they could remember the different rooms and features of the game look like, their functionalities and their experience with them. Moreover, they could use the probes in designing their prototypes, in case they wanted. The workshop was also filmed and close-up shots of the artefacts created by each group were taken. The researchers also took field notes during the observation, so to register interesting occurrences and compare them later to the video material.

Data analysis, conducted after the workshop, involved the video material, field notes and artefacts. Visual ethnographic methods (Pink 2006) and the critical accident

technique (Gremler 2004) were used as a framework to analyze video material and the artefacts created by the students. In this particular case, the critical incident technique was combined with visual ethnography (Pink 2006), so that the analysis focused on identifying meaningful incidents, during ethnographic observations, supported by situated interviews, so that the participants were interviewed while they played and while they were making prototypes, and while something interesting seemed to happen. The goal of the study was also to balance the design process of the game, which was initially decided by the authors, giving more ownership to potential users in this initial test.

Specific incidents were identified, in order to gather data about: interaction, in relation to specific features and their functionalities, signs of engagement or frustration, indicating the participants enjoyed playing our game, learning, in particular focusing on think aloud reasoning and performance in the game. These results are being used in redesigning a new prototype, which will be soon reevaluated with the same group; the results from the workshop are discussed in details in the next section.

4 Results and Discussion

In the whole-day workshop several insights were gathered, regarding how Prime Slaughter affects:

1. Learning and understanding of factorization, in relation to social play.
2. Playability and design suggestions for the creation of the next prototype.

These aspects are discussed in details in the following subsections.

4.1 Learning and Social Play

Interestingly the aspects of learning and social interaction through the game emerged as strictly interconnected. The game itself, in particular in the goal-directed activity of collecting prime-number swords in order to win, succeeded in eliciting critical questions regarding primality. Observing the students play, we noticed that individual players were sometimes talking also to other players, but in most cases called the researchers (their teacher was not present, and tested the game separately). Emerging questions opened up the possibilities to see how students relate to concept of prime numbers, what did they grasped or not from the introduction they received, such questions included: “What is the next prime number we need for our collection?”, “How can we determine the next prime number?”, “How can we define a prime number?”. It was also noticed that doubts about the identification of the next prime numbers emerged especially for two digits numbers. Generally it seems that students expect odd numbers to be prime, as they have absorbed the notion that all the even numbers are divisible by 2, but confusion may emerge with odd numbers such as 9 and 15.

In the interview with the teacher, he expressed positive remarks on the game, stating that the students were very excited both before and after testing it, and suggesting that it may “support learning of a subject that is considered boring and hard to grasp”.

These findings suggest that Prime Slaughter can also be used by the teacher as a teaching aid, to gain a clearer understanding of what the students have understood and what they find difficult, in order to support their individual needs. Considering the teacher's remarks and the students' questions, the game fits well the scenario of apprenticeship in thinking we originally had in mind, in which adults act as facilitators, supporting learners in acquiring new knowledge (Rogoff 1990).

Fig. 2. Pairs of girls and boys sharing the keyboard, one moves the hero while the other one kills and freezes monsters

The PS prototype was designed with individual players in mind, but many students played in pairs, and were able to find strategies to enjoy the game sharing one computer with a single keyboard. Some started taking turns, with one player actively playing while the other advised or commented; in other cases they shared the controls, with a player steering the character and the other killing and freezing the monsters (Fig. 2). They also expressed to each other their emotional state, talking, laughing and pointing at the screen to indicate a playmate that an action was required. Moreover, participants playing in pairs were able to support each other, discussing their questions together, hence they called for assistance only when in serious troubles. On the other hand, individual players were able to play and get in flow: some appeared very focused and quiet, others instead expressed lively their emotional state, verbally and non-verbally, for instance a girl who was eager to freeze a prime monster to gain a new sword was sitting in a tense attitude leaning towards the screen and shouted several times, but not too loud: "Freeze, freeze, freeze!" or "Bang, bang, bang!". These results suggest that PS provides adequate support for both individuals and pairs, but that social play can be a vital element in terms of user experience and learning. However, these results also suggest that the players are smartly adapting to the given interface, but that there might be other possibilities to explore, in order to support better social play and learning; for example provide a single-machine multiplayer mode and use classic Nintendo-style controllers.

4.2 Design Suggestions

Regarding possible improvements for the game, several girls suggested to introduce a distinctively feminine character, so that before starting the game, the players could choose if playing as male or female, similarly to Role Play Games. This request seems to introduce an identity aspect, which might relate to the reflective level of aesthetic experience (Norman 2004), dealing with identity and perception of self.

Moreover, several participants proposed that future levels would be shaped as mazes, with a more articulated layout and closed doors that could be opened with hidden keys (Fig. 4). These features already exist in classic games like the Legend of Zelda, which was a source of inspiration in first place, and may enrich the gameplay with new dynamics. Moreover, it could be interesting to explore how to map them into the factorization theme. For instance, the concept of a key relates both to doors and treasure chests, and to cryptography, where keys are often related to prime numbers and factorization plays a central role.

Fig. 3. Prototype of a maze level with doors and hidden keys, Lego and paper

The current PS prototype has a trap room called the "passage" (Fig. 1): the first time the player traverses it, the room appears empty, but at the next visit the room is full of 11-monsters. This trap room was highly appreciated by the students, for different reasons: some said that it was fun because of the challenge of killing so many monsters at once, on the other hand it is the only room in PS that resembles a maze, an interesting feature in itself for the test participants.

Many participants preferred the rooms with the monsters: a girl said it is fun to kill the monsters, and rarely played with the Bonsai Graveyard room (the special room where number-trees grow). However, most participants liked to shift from one room to the other and play both modalities. In some cases they realized they could strategically use the different areas to ease the collection of swords. For instance obtaining a 19 sword is quite difficult in our game. In PS monsters, trees and fruits range from 1 to 20, so 19 is the largest prime in the game. Moreover, monsters from 10 to 20 can only be found in the room called "plain" (Fig. 1), and in every room monsters spawn randomly. If the plain room does not contain any 19-monster, the player would have to kill all monsters to trigger the spawning of new (random) monsters, and

eventually repeat this process until a 19-monster is eventually available, then freeze it. Hence, some of the players realizing the need a 19-sword to complete their collections, may go to the bonsai-tree room and prune trees until they get a 19 apple; in some cases this solution was suggested by the researchers. In the next prototype, it could be interesting to explore how to suggest this possibility through the game, so that it becomes an intuitive way of play.

Other improvements suggested by the children were a prize, such as a trophy, so as to allow players to gain an acknowledgement of their success and also more challenging monsters in the transition from one level to the next. Finally, several students completed the game, then decided to start playing again from the beginning; some even played the game multiple times during the test, which we interpret as a sign of appreciation. The observations, interviews and suggestions obtained in this study are the basis for the design of our next prototypes.

5 Conclusions and Future Work

The Prime Slaughter game was designed and implemented to enable primary and early secondary school students to play with prime numbers and factorization. The game design was informed by the results obtained in a 1 year field study exploring and assessing techniques for transposing dynamic and complex domain-specific knowledge into games. Empirical results from the field study suggested implementing different forms of play in the game: a competitive form of play, which was mapped into the 2D adventure game, and a designerly-creative play, which was mapped into a puzzle game (Valente and Marchetti 2011).

PS was later evaluated with a math teacher and two classes from a school from Bramming, Denmark. The participants, 12 to 16 years old, tested the game in pairs or individually and afterwards created low-fidelity prototypes with paper and Lego bricks, to give us suggestions for improvement. The players took advantage of the 2 modalities: they either tended to play with their preferred one (and the monsters rooms were the most popular), or interchanged between the two modalities, for fun or to collect swords easier. Interestingly while playing the participants asked themselves, their mates and the researchers, critical questions about primality, which prime to use to slice a particular monster, and how many primes are there between 1 and 20.

The main contribution of this work is to show how a computer game can effectively turn an abstract concept into a virtual artefact, that learners can then probe in a playful way, as well as study empirically, to explore alternative hypothesis circa its workings and verify them through experiments. This allows the learners to move from cognitive to more experiential reflection (Vygotsky 1978). The results from the test and participatory session are used as new requirements for the next PS prototype, and to reflect upon the value of social-mediated interaction in the classroom. Guidelines are also emerging from this study, that allow for a systematic analysis and design of knowledge transposition games for playful learning (Marchetti, Valente and Jensen 2013).

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