

Mathematics and Visualization

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Visualization and Mathematics III

With 240 Figures, 47 in Color



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Preface

Mathematical Visualization aims at an abstract framework for fundamental objects appearing in visualization and at the application of the manifold visualization techniques to problems in geometry, topology and numerical mathematics. The articles in this volume report on new research results in this field, on the development of software and educational material and on mathematical applications.

The book grew out of the third international workshop “Visualization and Mathematics”, which was held from May 22-25, 2002 in Berlin (Germany). The workshop was funded by the DFG-Sonderforschungsbereich 288 “Differential Geometry and Quantum Physics” at Technische Universität Berlin and supported by the Zuse Institute Berlin (ZIB) and the DFG research center “Mathematics for Key Technologies” (FZT 86) in Berlin. Five keynote lectures, eight invited presentations and several contributed talks created a stimulating atmosphere with many scientific discussions.

The themes of this book cover important recent developments in the following fields:

- Geometry and Combinatorics of Meshes
- Discrete Vector Fields and Topology
- Geometric Modelling
- Image Based Visualization
- Software Environments and Applications
- Education and Communication

We hope that the research articles of this book will stimulate the readers’ own work and will further strengthen the development of the field of Mathematical Visualization.

We appreciate the thorough work of the authors and reviewers on each of the individual articles, and we thank you all. Beside the editors, the reviewers and members of the program committee were:

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Berlin, 2002
Hans-Christian Hege
Konrad Polthier

Table of Contents

Preface	V
---------------	---

Part I Geometry and Combinatorics of Meshes

Planar Conformal Mappings of Piecewise Flat Surfaces	3
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Philip L. Bowers, Monica K. Hurdal

1 An Inversive Distance Primer	6
2 Piecewise Flat Surfaces and Circle Packings	12
3 Hexagonal Refinement	15
4 Proving Convergence and Conformality	17
5 A Gallery of Quadrilaterals	23
6 Implementation: Practical Experimental, Computational, and Theoretical Issues	31

Discrete Differential-Geometry Operators for Triangulated 2-Manifolds	35
--	-----------

Mark Meyer, Mathieu Desbrun, Peter Schröder, Alan H. Barr

1 Introduction	35
2 Defining Discrete Operators	38
3 Discrete Mean Curvature Normal	40
4 Discrete Gaussian Curvature	45
5 Discrete Principal Curvatures	46
6 Results and Applications	49
7 Discrete Operators in nD	53
8 Conclusion	54

Constructing Circle Patterns Using a New Functional	59
--	-----------

Boris A. Springborn

1 Introduction	59
2 Circle Patterns	60
3 Preliminaries	61
4 The Functional	62
5 Examples	64
6 Numerical Evaluation of the Functional	66

Constructing Hamiltonian Triangle Strips on Quadrilateral Meshes 69

Gabriel Taubin

1	Introduction	70
2	Graphs and Meshes	71
3	Hamiltonian Paths	74
4	Basic Algorithm	77
5	Efficient Implementation	78
6	Diagonal Graph Structure	79
7	Transparent Vertex Caching	83
8	Implementation and Complexity	86
9	Subdivision	87
10	Borders	88
11	Conclusions and Future Work	89

Part II Discrete Vector Fields and Topology

Visualizing Forman's Discrete Vector Field 95

Thomas Lewiner, Helio Lopes, Geovan Tavares

1	Introduction	95
2	Basic Concepts	96
3	Hypergraphs and Hypertrees	99
4	Algorithm	101
5	Applications	106
6	Future Works	109

Identifying Vector Field Singularities Using a Discrete Hodge Decomposition 113

Konrad Polthier and Eike Preuß

1	Introduction and Related Work	113
2	Setup	116
3	Discrete Rotation	117
4	Discrete Divergence	120
5	Hodge Type Decomposition of Vector Fields	123
6	Decomposition Algorithm and Detecting Vector Field Singularities	127
7	Examples	129
8	Conclusions and Future Work	130

Searching for Knotted Spheres in 4-dimensional Space 135

Dennis Roseman

1	Background for Random Knots	135
2	Generating Random Collections of Objects in $\mathbb{R}^n$	137
3	Methods of Generating Random Spheres in $\mathbb{R}^4$	138
4	Random Icosahedra in $\mathbb{R}^4$	141

5	Sorting Knottings into Equivalence Classes	144
6	The Role of Visualization.....	148
7	Ordering Knots	148

3D Loop Detection and Visualization in Vector Fields 151

Thomas Wischgoll, Gerik Scheuermann

1	Introduction	151
2	Mathematical Background	152
3	Loop Detection	153
4	Results.....	158

Part III Geometric Modelling

Minkowski Geometric Algebra and the Stability of Characteristic Polynomials 163

Rida T. Farouki, Hwan Pyo Moon

1	Introduction	163
2	Minkowski Geometric Algebra	164
3	Families of Curves and Envelopes.....	168
4	Stability of Characteristic Polynomials	170
5	Complex Disk Polynomials	173
6	Hurwitz Stability of Disk Polynomials.....	175
7	Γ -Stability of Disk Polynomials	179
8	Robustness Margin of Disk Polynomials	183
9	Closure	185

Subdivision Invariant Polynomial Interpolation 189

Stefanie Hahmann, Georges-Pierre Bonneau, Alex Yvart

1	Introduction	189
2	Related Works	190
3	Subdivision Invariance	191
4	Subdivision Invariant G^1 Polynomial Triangular Interpolant	193
5	Results.....	197
6	Conclusion	198

Another Metascheme of Subdivision Surfaces 201

Heinrich Müller, Markus Rips

1	Introduction	201
2	Elementary Schemes Based on Vertex Assignment	203
3	Inversion	209
4	Selected Composed Subdivision Schemes	212
5	Subdivision Schemes of Higher Order	216
6	Concluding Remarks.....	219

Geometry of the Squared Distance Function to Curves and Surfaces 221

Helmut Pottmann, Michael Hofer

1	Introduction	221
2	Graph Surface of the Squared Distance Function to a Planar Curve	223
3	Quadratic Approximations to d^2	226
4	Squared Distance Function to a Surface and its Second Order Approximants	231
5	Squared Distance Function to a Space Curve	234
6	Application to Geometric Optimization Problems	235
7	Future Research	240

Part IV Image Based Visualization

A Multiscale Fairing Method for Textured Surfaces 245

Ulrich Clarenz, Udo Diewald, Martin Rumpf

1	Introduction	245
2	Image Processing Background	246
3	Anisotropic Geometric Diffusion	247
4	Coupling Anisotropic Texture and Surface Diffusion	250
5	Comparison and Conclusions	255

Generalized Block Iterative Methods 261

Michel Leblond, François Rousselle, Christophe Renaud

1	Introduction	261
2	Generalized Block Partitioning of a Matrix	265
3	Generalized Block Iterative Methods	267
4	Application to Radiosity	273
5	Conclusion and Perspectives	280
6	Acknowledgements	283

Fast Difference Schemes for Edge Enhancing Beltrami Flow and Subjective Surfaces 287

Ravi Malladi, Igor Ravve

1	Introduction	287
2	Implicit Scheme for Beltrami Flow	289
3	Simulation Results for Beltrami Flow	292
4	Implicit Scheme for Subjective Surfaces	294
5	Simulation Results for Completing Missing Boundaries	296
6	Closing Remarks	296

Part V Software Environments and Applications

ALICE on the Eightfold Way: Exploring Curved Spaces in an Enclosed Virtual Reality Theater	305
<i>George K. Francis, Camille M.A. Goudeseune, Henry J. Kaczmariski, Benjamin J. Schaeffer, John M. Sullivan</i>	

1 Introduction	305
2 Other Fully Enclosed Virtual Reality Theaters	307
3 Cluster Architecture	307
4 Mathematical Visualization of Three-dimensional Geometries	310

Computation and Visualisation in the NUMLAB Numerical Laboratory	317
<i>Joseph M.L. Maubach, Alexandru C. Telea</i>	

1 Introduction	317
2 The Mathematical Framework	321
3 The Software Framework	322
4 An Efficient NUMLAB Finite Element Implementation	325
5 Application Design and Use	331
6 Conclusions and Future Work	333

A Generic Programming Approach to Multiresolution Spatial Decompositions	337
<i>Vinícius Mello, Luiz Velho, Paulo Roma Cavalcanti, Cláudio T. Silva</i>	

1 Introduction	337
2 Background	338
3 Concepts	339
4 Models	350
5 Applications	351
6 Conclusion	355

Mathematical Modelling and Visualisation of Complex Three-dimensional Flows	361
<i>Alfio Quarteroni, Marzio Sala, M.L. Sawley, N. Parolini, G. Cowles</i>	

1 Introduction	361
2 Mathematical Formulation	364
3 Complexity	366
4 Visualisation Techniques	370
5 Conclusions	373

webMathematica	379
<i>Tom Wickham-Jones</i>	

1 Introduction	379
2 webMathematica Technology	380
3 Web Extension Technologies	382

XII Table of Contents

4	web <i>Mathematica</i> Services.....	383
5	Summary.....	389
6	References.....	390

Part VI Education and Communication

Films: A Communicating Tool for Mathematics 393

Michele Emmer

1	Introduction	393
2	Mathematics: a Special Case?	395
3	The Mathematics and Art Project	396
4	Mathematics and Fiction Films	401
5	Final Comments	403

The Potentials of Math Visualization and their Impact on the Curriculum 407

Beau Janzen

1	Introduction	407
2	Palpability	409
3	Context	409
4	Implementation	412
5	Impact on the Nature of the Curriculum	416
6	Impact on the Content of the Curriculum	419
7	Conclusion	420

Appendix: Color Plates..... 423

Index..... 453