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# Non-Linear Feedback Neural Networks

VLSI Implementations and Applications

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*It is said that a good marriage is the closest  
thing to heaven on earth. In that light,  
I dedicate this book to the person who has  
facilitated that glimpse of paradise for me ...*

*To*

*My Wife Asra*

*With Love*

# Preface

Artificial Neural Networks (ANNs), having a highly parallel architecture, have emerged as a new paradigm for solving computationally intensive tasks using collective computation in a network of neurons. They can be considered as analog computers relying on simplified models of neurons. The essential difference between the ANN's distributed computation and a modern day digital computer's functioning is that in the case of a digital computer, the processing is often centralized at one location by means of a single processor (aptly referred to as the 'uniprocessor' architecture), whereas the collective computation model of a neural network depends upon the simultaneous working of hundreds of thousands of analog neural processors. Each of these 'neurons' have very limited computational power when considered as a separate entity. However, the real power comes with their acting in unison. The immense computational abilities of a neural network are a perfect example of the benefits of teamwork!

Recurrent neural networks, which are essentially ANNs employing feedback interconnections between the neurons, were extensively explored after the concept of Lyapunov (or 'energy') functions, as a means of understanding the complex dynamics of such networks, was introduced by Hopfield and Tank. Their architecture, called the Hopfield Network, was implementable in hardware, and although it became very popular, many limitations like convergence to infeasible solutions and the requirement of a large number of neurons and interconnection weights were revealed when attempts were made to apply it to practical applications. These drawbacks warranted the exploration of alternative neural network architectures which are amenable to hardware realizations. The Nonlinear Synapse Neural Network (NOSYNN) has been proposed as one such alternative which alleviates the problems that plagued the Hopfield Network.

This book deals with VLSI implementations and applications of the NOSYNN type of nonlinear feedback neural networks. These networks have been shown to be better performing than their Hopfield Neural Network (HNN)-based counterparts, in the sense that their convergence to the exact solution is fast and guaranteed. This improvement in the performance is due to underlying difference in the nature of feedback between the HNN and the NOSYNN. While the HNN employs linear feedback (typically implemented using resistors), the NOSYNN employs nonlinear feedback (typically implemented using voltage-mode comparators). This difference in hardware also carries over to a difference in the dynamical properties

of the two networks and makes the energy functions of the two networks vastly different. While the HNN has a quadratic form of the energy function, the NOSYNN has transcendental terms in its energy function which account for better and faster convergence characteristics.

In this text, the NOSYNN architecture has been chosen as the starting point of the exploration for better hardware implementations of ANNs. Thereafter, the content progresses in two dimensions. First, the voltage-mode NOSYNN has been reconfigured and applied to a new problem, viz. the solution of linear equations. As has been mentioned above, the NOSYNN-based neural network for solving linear equations has an associated energy function which contains transcendental terms as opposed to the quadratic terms in the energy functions for the Hopfield network and its variants. It is shown that the network has only a unique minimum in the energy landscape, and moreover, the global minimum coincides exactly with the solution point of the system of linear equations. Thereafter, it has been shown that two other important problems of mathematical programming: linear programming problem (LPP) and quadratic programming problem (QPP) could also be solved by incorporating small modifications in the voltage-mode network proposed for the solution of linear equations.

Second, a ‘mixed’-mode (MM) implementation for the NOSYNN has been discussed. Applications of the ‘mixed’-mode neural circuit in solving linear equations, LPP, QPP, graph coloring, and ranking of numbers, are explained in detail. In the so-called ‘mixed’-mode hardware realization, neuron states are represented as voltages whereas the synaptic interconnections convey information in the form of currents. It has been shown that the mixed-mode implementation of the NOSYNN leads to reduction in the overall circuit complexity, as compared to the voltage-mode realization, by eliminating the resistors employed as synaptic weights. Two different VLSI realizations of the ‘mixed’-mode networks are discussed. The first employs Differential Voltage Current Conveyors (DVCCs) to implement voltage comparators with current outputs. The second class of realizations use Operational Transconductance Amplifiers (OTAs) to provide the required voltage comparison at the inputs of the comparators.

## Why this Book?

A very pertinent question that would come to the mind of a person coming across this book, while searching for a book on feedback neural networks, is ‘What does this text has to offer which any other book does not?’ The answer to the question lies in the nature of content. While a multitude of (very good) books on neural networks are heavy on theory and the related mathematics without dwelling on the actual hardware implementations, this particular text focusses on the intricacies involved when a neural circuit is actually realized in hardware. This book does not intend to replace the already established books on the subject. Rather, it offers the readers an additional cue about how to actually port the neural circuit, leaving

behind all the mathematics, to a real-world circuit, for instance, a CMOS realization. The block diagram representations that abound in many of the existing texts are sufficient for developing an understanding about the intended working of the network. However, such a representation generally assumes ideal values and behavior of the various components, which is seldom the case in an actual real-world implementation. This has led to a severe dent on the neural network field in general, with critics saying that neural networks promise the moon but deliver nothing. Almost entirety of such statements are issued by researchers who are attracted by the field only to find it full of perfectly working mathematical models which turn into nonperforming entities in a breadboard implementation. It is the contention of the author that a significant number of such misplaced ideas about neural networks can be eliminated if interested persons are provided the required help in actual hardware design and testing of such circuits.

Although it is true that the real power of neural networks lies in massively parallel structures, containing hundreds of neurons, capable of solving combinatorial (and other) problems comprising a large number of variables; the fact should not prevent a book from being able to provide small-sized scaled versions of those huge networks, just to make a reader familiar with the actual operation of the network. The approach followed in this book is simply to start with the most simple case, understand its operation, get its maths right, test it in hardware, and then move on to somewhat bigger problems. For instance, for every linear equation solver discussed in the book, first a small circuit capable of solving just two linear equations is presented and explained. It is the belief of the author that the concept of energy function is easily grasped for networks with low neuron counts. Once the operation, maths and the hardware of the two variable linear equation solver is complete, the text moves on to slightly more complex circuits before finally dealing with the generalized version of the network.

## Prerequisites

For a good understanding of the topics covered in this book, a reader should have knowledge about electronic amplifiers, particularly the operational amplifier. A basic knowledge of circuit theory is assumed. On the mathematics front, the reader should have studied differential and integral calculus as well as mathematical optimisation.

# Acknowledgments

Writing a book is seldom a ‘one-man’ effort. This remains true for this book as well. There are many whom I would like to thank and acknowledge.

I am at loss to find words suitable enough to express my gratitude to my parents for their unfaltering love and support. I am indebted to them for the difficulties, and loneliness, that they endure while I am away from them. I also wish to thank my younger brother Sarim for his deep concern and support in my work.

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