

Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

445

A.J.M. van Gasteren

On the Shape of Mathematical Arguments

Foreword by Edsger W. Dijkstra



Springer-Verlag

Berlin Heidelberg New York London

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Author

Antonetta J. M. van Gasteren
University of Groningen, The Netherlands
and
University of Utrecht, The Netherlands

Correspondence address:

Edelweislaan 20
5582 BW Waalre, The Netherlands

CR Subject Classification (1987): F.3.1, D.2.4

ISBN 3-540-52849-0 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-52849-0 Springer-Verlag New York Berlin Heidelberg

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Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
2145/3140-543210 – Printed on acid-free paper

Foreword

While current curricula extensively teach existing mathematics, they pay scant attention to the doing of mathematics, i.e., to the question of how to design and to present solutions. If any attention to these issues is paid at all, they are treated separately: design of solutions, i.e., “problem solving” or “mathematical invention”, is viewed as a psychological issue, as a matter of mathematical intuition, while presentation is viewed as a matter of personal style or as an issue of education. Most mathematicians consider psychology and pedagogy as sciences too soft to be respectable, and consequently the subject of how to do mathematics has almost been tabooed.

The great merit of A.J.M. van Gasteren’s work is to have broken this taboo. She has done so, firstly, by recognizing that, in the case of mathematical arguments, the traditional separation of content and form is untenable, and, secondly, by showing how, via notation and formula manipulation, design and presentation emerge as two sides of the same coin. She has broken the taboo because, in this united setting, the issues involved are purely technical: for instance, the question of whether a certain notational convention —of equal relevance for the derivation as for the presentation of a solution!— is geared to our manipulative needs is a technical question that has nothing to do with intuition or taste.

That formalization should and can aid the mathematician rather than add to his burden is well-known. This book reveals a wider vision, viz., that the more formal the solutions we are heading for, the better we shall be able to teach the art and science of their design. In doing so, this work represents a crucial first step towards mathematical methodology in the truest sense of the word, and for that reason I greatly welcome its publication in Springer’s Lecture Notes Series: the more people enjoy this book and are inspired by it, the better.

Edsger W. Dijkstra

Preface

For me, exploring the presentation and design of mathematical proofs has been a very exciting activity. To see how virtually every proof and program can be made much clearer and crisper, and to develop techniques and skills for doing so, is such a rewarding experience that my hope is to share it with the readers of this monograph. To appreciate the text, the reader needs some maturity in mathematics, but no specialized mathematical knowledge is required to follow the example arguments in the first part.

Of all those who made my explorations and the writing of this text possible, I want to mention first and foremost Edsger W. Dijkstra and W.H.J. Feijen, who educated me and who shared and guided my interests. The Eindhoven Tuesday Afternoon Club, in varying formations, has always been an indispensable forum for discussion and critique. In their capacity as members of my dissertation committee, R.C. Backhouse, P.L. Cijsouw, and F.E.J. Kruseman Aretz carefully studied, and commented on, this text.

For many years, BP International Limited provided financial support for my research under their much appreciated Venture Research Scheme; especially D.W. Braben, head of BP's Venture Research Unit, has been an ever interested and stimulating ally. V.H. Backhouse and D.B.M. Klaassen took care of the typesetting and typing of this text, accepting and implementing all my well-considered but non-standard typographic wishes. H. Paas of the University of Groningen provided them with technical assistance. M.C. Dijkstra-Debets and D.B.M. Klaassen, finally, deserve mention for their moral and other support.

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