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Treewidth

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Preface

This book deals with problems related to the treewidth parameter of graphs. Tree-decompositions and path-decompositions of graphs of bounded tree- or pathwidth have been found applicable to a wide range of difficult problems. The study of the treewidth parameter has led to a theory with many deep results and challenging open problems.

The book starts with an introduction containing a short overview of the contents of the various chapters. The second chapter offers a smooth and gentle introduction to the most fundamental properties of treewidth, pathwidth and triangulations of graphs. The next three chapters deal with problems that are solved using structural properties of graphs with bounded treewidth. Chapter 6 deals with the approximation of treewidth for graphs in general. Chapter 7 up through 12 illustrate various methods for approximations and computations of treewidth and pathwidth for various classes of graphs. Finally, in Chapter 13, 14 and 15 we describe linear time algorithms for the computation of treewidth and pathwidth of treewidth-bounded graphs and pathwidth-bounded graphs.

The research for this book, which is partly a revised version of my PhD thesis, was primarily done in the period from June 1989 to June 1993 when I was at the department of computer science of Utrecht University (The Netherlands). I would like to thank the people of the computer science department at Utrecht for creating a pleasant and stimulating working atmosphere. Especially, I thank Hans Bodlaender for introducing me to this area of research. I also thank Jan van Leeuwen. This book has benefitted very much from his comments and suggestions. I am grateful to Dieter Kratsch of the department of mathematics and computer science of the Friedrich-Schiller university at Jena (Germany) for drawing my attention to perfect graphs. I thank him and Haiko Müller for inviting me to Jena and for the many discussions on special graph classes we had during that time.

I prepared the final version of this book at the mathematics and computing science department at Eindhoven University. I thank the people of the 'Club' who have read and commented on various parts of the book. Especially I thank Anne Kaldewaij for giving me the opportunity and the stimulation to work on this book from June 1993 to June 1994. Finally, I thank my wife Annelies Jacobs. She took part in everything except in typing the manuscript.

June 1994 Ton Kloks

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[168] Véga, W. F. de la and Y. Manoussakis, Grids in random graphs, *Random Structures and Algorithms* **5**, (1994), pp. 329–336.

[169] Wagner, K., Über eine Eigenschaft der ebenen Komplexe, *Math. Ann.* **114**, (1937), pp. 570–590.

[170] Wagner, K., Monotonic coverings of finite sets, *Journal of Information Processing and Cybernetics*, EIK, **20**, (1984), pp. 633–639.

[171] Wald, J. A. and C. J. Colbourn, Steiner trees, partial 2-trees and minimum IFI networks, *Networks* **13**, (1983), pp. 159–167.

[172] Walter, J. R., Representations of chordal graphs as subtrees of a tree, *J. Graph Theory* **2**, (1978), pp. 265–267.

[173] Whitesides, S. H., An Algorithm for finding clique cut-sets, *Information Processing Letters* **12**, (1981), pp. 31–32.

[174] Yannakakis, M., Computing the minimum fill-in is NP-complete, *SIAM J. Alg. Disc. Meth.* **2**, (1981), pp. 77–79.