Effects of Correlation and Degree of Balance in Random Synaptic Inputs on the Output of the Hodgkin-Huxley Model

David Brown Jianfeng Feng

Laboratory of Computational Neuroscience, The Babraham Institute
Cambridge CB2 4AT, UK

Abstract. We examine the effects of degree of balance between inhibitory and excitatory random synaptic inputs, and of positive correlation between the inputs on the mean and variability of the output of the classical Hodgkin-Huxley (HH) model for squid giant axon, using computer simulation. The mean interspike interval (ISI) and the coefficient of variation of ISI change little as the degree of balance changes, unlike the leaky integrate-and-fire model, frequently used in stochastic network modelling as an approximation to more biophysically based models. Low correlations (up to about 0.1) between 100 excitatory inputs each firing at 100 Hz reduce the mean(ISI) to below a third of its value when the inputs are independent, and CV by a factor of 5 from a near-Poisson range to one associated with regular firing.

1 Introduction

In vivo cell recordings demonstrate that many neurones fire irregularly. For example, the coefficient of variation of interspike intervals of neurones in the visual cortex of the monkey is greater than 0.5[23]. A comparison between *in vitro* and *in vivo* experiments supports the assertion that the irregularity results from random synaptic input from other neurones, both inhibitory and excitatory [11]. How the output variability relates to the characteristics of random synaptic input is a major theme in computational neuroscience [13, 24]. A better understanding of the origins of apparent randomness in neuronal firing will help us to clarify general principles underlying neuronal circuitry [17, 18, 20–22], and to assess whether rate or timing coding is the fundamental mode of information transmission [9, 12, 21].

Many neurons fire irregularly when driven weakly by random synaptic input, and quite regularly when driven very hard. In between these extremes, neurons vary in their response to random input. In previous work [2, 5], we have considered how the degree of balance and positive correlation between synaptic inputs affect the mean level and variability of firing of the leaky integrate-and-fire (I&F) model, with and without reversal potentials. CV falls in the physiological range (between 0.5 and 1) for a range of values of r, the ratio of the frequencies of inhibitory to excitatory input, the range becoming wider when reversal potentials are included. Here we demonstrate that the behaviour of the classical Hodgkin-Huxley model of squid giant axon is very different in that it can fire quite irregularly over the complete range of degrees of balance between inhibition and excitation, unlike the leaky I&F model, which generally fires regularly when inhibitory inputs are absent.

We also explore how values of the mean and coefficient of variation of ISI (the latter abbreviated to CV in the remainder of the paper) change as the simultaneous synaptic inputs become positively correlated. [30] obtained average correlation coefficients between observed spike counts in simultaneously recorded neurons in the middle temporal visual area of 0.12. We therefore consider as an example the case of p=100 excitatory inputs each of 100 Hz with a maximum correlation of 0.1, and $q = rp$ inhibitory inputs, for r between 0 and 1; and we demonstrate that as the pairwise correlation between these inputs increases to 0.1, the mean ISI falls to approximately a third of its values under independence, and CV falls to about one fifth of its value under independence. In other words, the neuron fires much faster and much more regularly.

2 Models

2.1 Neuronal and input model

The HH model is

$$CdV = I_{syn} + g_K n^4 (V - V_K)dt + g_{N_a} m^3 h (V - V_{N_a})dt + g_L (V - V_L)dt \tag{1}$$

where I_{syn} is the synaptic current, as described next. The model parameters and remaining equations are as in [10].

The model neurons were subjected to input from p excitatory synapses each following a Poisson process of rate λ_E, and q inhibitory synapses each with Poisson rate λ_I. For both model simulations,the effect of an EPSP/IPSP is an instantaneous perturbation of membrane potential of magnitude $a = 0.5$mV.

To examine the effect of departures from exact balance in the case of independent inputs, one simulation was performed for each combination of p and r taken from the following: p = 25, 50, 75, 100, 150, 200 with $\lambda_E = \lambda_I$= 100 Hz, and $r = q/p$ = 0, 0.1, 0.2 ... 1.0.

2.2 Correlated inputs and re-expression of their effects as a Wiener Process

First, we use martingale decomposition to approximate the Poisson synaptic input in a form more convenient for computer simulation. The decompositions of the excitatory and inhibitory components of the synaptic input,

$$I_{syn} = dE_i(t) + dI_i(t)$$

are

$$dE_i(t) \sim \lambda_E dt + \sqrt{\lambda_E} dB_i^E(t)$$

and

$$dI_i(t) \sim \lambda_I dt + \sqrt{\lambda_I} dB_i^I(t)$$

where $B_i^E(t)$ and $B_i^I(t)$ are standard Brownian motions. Thus synaptic input can be approximated by

$$I_{syn} = a\sum_{i=1}^{p} \lambda_E dt - b\sum_{i=1}^{q} \lambda_I dt + a\sqrt{\lambda_E}\sum_{i=1}^{p} dB_i^E(t) - b\sqrt{\lambda_I}\sum_{i=1}^{q} dB_i^I(t) \tag{2}$$

Since the sum of Brownian motions also forms a Brownian motion we can rewrite the above equation

$$\begin{aligned} I_{syn} &= (ap\lambda_E - bq\lambda_I)dt \\ &\quad + \sqrt{a^2\lambda_E \sum_{i=1}^{p}\sum_{j=1}^{p} c^E(i,j) + b^2\lambda_I \sum_{i=1}^{q}\sum_{j=1}^{q} c^I(i,j)}\, dB(t) \\ &= (ap\lambda_E - bq\lambda_I)dt \\ &\quad + \sqrt{a^2 p\lambda_E + b^2 q\lambda_I + a^2\lambda_E \sum_{i\neq j}^{p} c^E(i,j) + b^2\lambda_I \sum_{i\neq j}^{q} c^I(i,j)}\, dB(t) \end{aligned} \tag{3}$$

These approximations were used in the simulations of correlated inputs. The approximation accuracy has been shown to be adequate for the present purposes in results to be reported elsewhere [3]. On the other hand, direct simulation of the Poisson inputs was used in the results in Figure 1, dealing with independent inputs. The results for c=0 in Figure 2 can therefore be directly compared with those for p=100 in Figure 1, to assess the accuracy of the approximation; the results are quite comparable.

3 Results

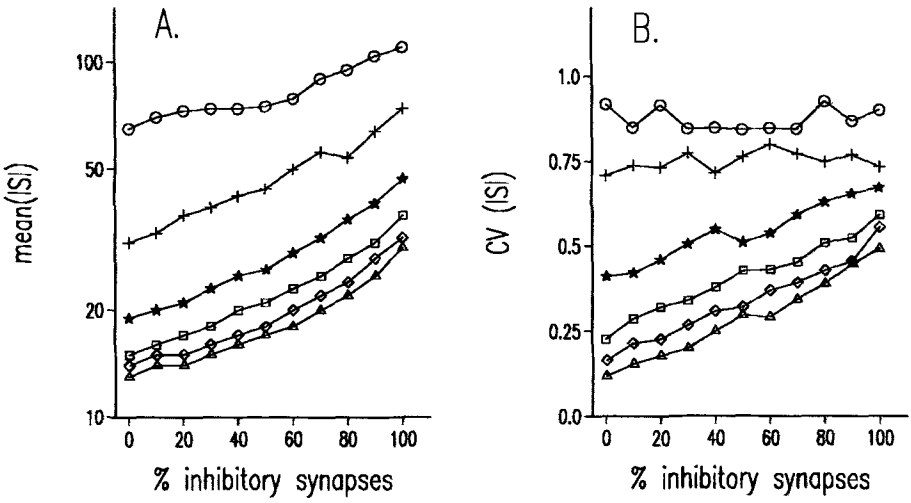

Fig. 1. Response of a HH neuron subject to p excitatory Poisson inputs each of 100Hz, and $q = rp$ inhibitory inputs with EPSP size = IPSP size = 0.5mv. A.mean(ISI) vs r for each value of p . B. CV(ISI) vs r for each value of p. Key: from top to bottom of each frame: p=75,100,150,200,250,300

3.1 Effects of degree of balance

Very few or no spikes were obtained at the lowest numbers of excitatory neurones, $p = 25, 50$, for all values of r, so only the results for

higher values of p – for which reliable statistics could be obtained – are reported here. Mean ISI varies between 14 and 110 ms (see Fig (1)), which are within the physiological range. For $p = 75$ and 100, CV(ISI) is approximately independent of r taking values of about 0.8 and 0.7 respectively (Figure 1A). For higher values of p, CV(ISI) is positively correlated with r, taking values as low as 0.1 for $p = 300$ and $r = 0$. This appears to occur because the drive to the neuron is so high that the refractory period becomes a significant portion of the mean interspike interval. Once an effective refractory period of 12 ms is subtracted from each interspike interval, CV(ISI) is approximately unity, the expectation for completely Poissonian output.

The behaviour of I&F neurons in response to stochastic synaptic input has been described elsewhere ([2, 4–6]). For low levels of input (viz $p = 20$), significant output only occurs for low levels of inhibitory input, $r = 0.0 - 0.2$. Mean (ISI) takes a very wide range of values as r is varied from the order of 6-15 ms when r=0.1 to 1 second when $0.7 < r < 0.9$ for p taking higher values i.e. between 50 and 100 (results not shown here). By contrast, mean (ISI) for the HH model shows a much weaker correlation with r of lower slope; i.e. inhibitory input has a much greater impact on the firing rate of the I&F neuron than the HH neuron. For $50 \leq p \leq 100$, CV of the I&F neuron is in the near Poisson range for $r > 0.5$ approximately , falling substantially to near 0.25, as $r \longrightarrow 0$. For a wide range of values of r, CV thus takes values more typical of regular firing. For the HH neuron, on the other hand, CV is independent of r for $p = 75$ and 100; and for higher values of p, CV only falls as a result of the neuron's refractory period.

3.2 Positive correlation between inputs

For all the simulations reported here, the correlations between the excitatory inputs was the same as that between the inhibitory inputs, so $c_E = c_I = c$. Moving from independence to very slight positive correlation ($c = 0.01$), has the effect of reducing CV substantially (see Fig(2)) from about 0.7 to about 0.5, approximately independently of the value of r. As correlation is increased further, CV falls further so that, for $0.07 < c < 0.10$, CV is close to 0.25, which is the range consistent with quite regular firing. There are

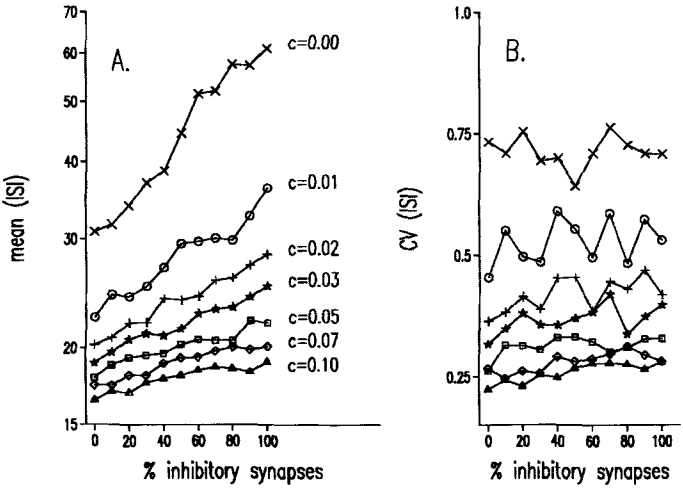

Fig. 2. Mean firing time(A) and CV(B) of the ISI output of the HH model subject to $p \approx 100$ excitatory EPSPs arriving according to Poisson processes, with EPSPs of amplitude 0.5 mV, and $q = rp$ inhibitory inputs also Poisson distributed, with correlation, c, between the 100 excitatory (and $100r$ inhibitory) processes lying between 0 and 0.1, as indicated in A. The model is simulated using the approximation to the Poisson input as described in the text.

also substantial reductions in mean(ISI) as c is increased over this range from between 30 msec when $r = 0$ to 60 msec when $r = 1$ for independence, to between 17 and 19 msec for $r = 0.1$. Again there is a great change for very slight positive correlation; increasing r from 0 to 0.01 involves a reduction in mean(ISI) of about one third over the whole range of r.

4 Discussion

Biological experiments indicate that the interspike interval CV of many neurons is frequently greater than 0.5, but how and when cells adjust their synaptic inputs so that their outputs take on an appearance broadly comparable to that of a Poisson process is unknown. Many analytical, numerical and simulation studies have attempted to predict when this would happen for the I&F models [8, 14, 15, 27, 23–26, 29]: one general conclusion is that firing becomes more regular as the frequency of inhibitory inputs falls to zero. In this paper, we

show that for the HH model, changes in the relative frequency of inhibitory input have little effect on CV when inputs are independent, but that very slight departures from independence involving positive correlation between input streams can change the CV very substantially. The reason why small correlations have such a profound effect is that there are $p(p-1)$ covariance terms in the expression for I_{syn} in equation Eq. (3), so that the covariance contribution increases much faster than p, even though the multiplier of $p(p-1)$, i.e. the correlation, is rather small. Furthermore, the effect of positive correlation on CV is qualitatively different in its effect for the HH and leaky I&F models. In [7], we demonstrate that positive correlation increases CV for the leaky I&F model, unlike as here for the HH model, decreasing it.

Considering the implications of these findings for neuronal functioning especially with reference to coding, changing the correlation between inputs slightly might well be easily accomplished by a neural network. Our present results suggest that such relatively minor changes - of the order of magnitude of those found in visual cortex [30] - can have major effects on the neuronal output, the effects depending strongly on the details of the neuronal mechanism, as demonstrated here and in [7]. This lends support to the hypothesis that population coding in networks composed of different neuronal types might well be an important communication mode in neuronal systems.

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