Asymmetry-induced nonclassical correlation

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Abstract In quantum resource theory (QRT), asymmetry recognized as a valid resource for the advantage of various quantum information processing. In this paper, we establish resource theory of asymmetry using quantum Fisher information (QFI). By defining the average Fisher information as a measure of asymmetry, it is shown that the discrepancy of bipartite global and local asymmetries naturally induces the nonclassical correlation between the subsystems. This measure satisfies all the necessary axioms of a faithful measure of bipartite quantum correlation. As an illustration, we have studied the proposed measure for an arbitrary pure state and Bell diagonal state.

1 Introduction

In the last two decades, quantum resource theory has attracted much attention owing to its potential applications in quantum information processing and considerable effort has been dedicated to the development and implications of quantum resource theory [1]. Any quantum system can exhibit many interesting properties such as entanglement [2,3], nonlocality and quantum coherence, which are consequence of superposition principle and representing fundamental departures from classical physics [4]. These peculiar properties are considered as an essential resources for quantum information processing tasks. Quantification of such features is a worthwhile task in the framework of resource theory. In recent years, many quantum resources in different context have been proposed such as entanglement, quantum correlation (beyond

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entanglement) [5,6,7,8,9,10,11,12,13,14,15], quantum coherence [16,17,18], asymmetry [19,20] etc.

Among the above, asymmetry plays an important role in the development of quantum information theory and deep understanding of quantum systems. The quantum state may or may not be invariant with respect to the action of the group and the asymmetry is defined as the degree of symmetry breaking. In general, the symmetry breaking, is a fundamental mechanism for the variety of matters, which is also useful in understanding of microscopic picture of fundamental interactions and emergence of macroscopic structure. Further, the significance of asymmetry can effectively identified in some significant condensed matter phenomena, such as accidental degeneracy and spontaneous symmetry breaking. With the development of qualitative characterizations of asymmetry, recently, the researchers paid wide attention on the quantification of asymmetry also. In the context various measures of asymmetry have been introduced and explored in detail, such as the entropic measure of asymmetry [20,21], robustness of asymmetry [22], asymmetry weight [23], measure of asymmetry based on commutators [24,25], Frobenius norm based measure of asymmetry [26].

The quantum correlation and its monogamy relation are the primary tools for the understanding of nature nonclassical properties of a quantum system. In general, asymmetry and correlations are the fundamental quantum resources and ubiquitous in quantum physics. In this work, we investigate the quantification of quantumness from the viewpoint of resource theory of asymmetry. By defining asymmetry of the bipartite quantum state, we propose the difference between the bipartite global and local asymmetries naturally induces the nonclassical correlation between the subsystems as a quantum correlation measure of the bipartite state. This measure satisfies basic requirements of bipartite quantum correlation measure.

The structure of the paper as follows. In Sec. 2, we present the review of quantum Fisher information. In Sec. 3, we establish the resource theory of asymmetry in terms of QFI. The bipartite quantum correlation measure defined and characterized in Sec. 4. Finally, the conclusions are given in Sec. 5.

2 Overview on QFI

Quantum Fisher information (QFI) is the most important and promising tool in quantum information theory to enhance the precision and efficiency of quantum metrology protocols. Further, QFI is also useful in quantification of resources such as quantum coherence and quantum correlation. The dynamics and behaviors of QFI in the physical systems are analyzed recently. In general, QFI quantifies the amount of information contained in state ρ with respect to any physical observable K [27] and is reduced to variance of the observable in the state ρ. For any arbitrary quantum state $\rho_\theta = U \rho U^\dagger$ that depends on the

parameter θ, we can define the QFI as

$$\mathcal{F}(\rho_\theta) = \frac{1}{4}\text{Tr}[\rho_\theta L_\theta^2],$$

where the state ρ_θ can be obtained from an initial probe state ρ subjected to a unitary transformation $U = e^{iK\theta}$. In order to extract the knowledge about θ from the parametric state ρ_θ, we set generalized quantum measurements, namely positive-operator-valued measures (POVMs), $M = \{M_k|M_k \geq 0, \sum_k M_k = \mathbb{1}\}$. Here, L_θ is symmetric logarithmic derivative defined as the solution of the equation

$$\frac{d\rho_\theta}{d\theta} = \frac{1}{2}(\rho_\theta L_\theta + L_\theta \rho_\theta).$$

Following the spectral decomposition $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, with $p_i \geq 0$ and $\sum_{i=1} p_i = 1$, the $\mathcal{F}(\rho, K)$ is computed as [28, 29]

$$\mathcal{F}(\rho, K) = \frac{1}{2}\sum_{i \neq j}\frac{(p_i - p_j)^2}{p_i + p_j}|\langle\psi_i|K|\psi_j\rangle|^2. \tag{1}$$

The quantum Fisher information has the following theoretic-information properties:

$\mathcal{F}$1. QFI is a non-negative i.e., $\mathcal{F}(\rho, K) \geq 0$.

$\mathcal{F}$2. Quantum Fisher information reduces to variance $V(\rho, K)$ for pure states, namely, $\mathcal{F}(\rho, K) = V(\rho, K)$. In general, if ρ is mixed, then

$$0 \leq \mathcal{F}(\rho, K) \leq V(\rho, K)$$

$\mathcal{F}$3. $\mathcal{F}(\rho, K)$ is unchanged under any unitary transformation U commuting with the observable K [30], i.e.,

$$\mathcal{F}(\rho, K) = \mathcal{F}(U\rho U^\dagger, K).$$

$\mathcal{F}$4. $\mathcal{F}(\rho, K)$ is convex, which means that if several different quantum systems are mixed, the information content of the resulting system is not larger than the average information content of the component systems [30]. Mathematically,

$$\mathcal{F}\left(\sum_n \lambda_n \rho_n, K\right) \leq \sum_n \lambda_n \mathcal{F}(\rho_n, K)$$

where $\sum_n \lambda_n = 1$, $\lambda_n \geq 0$ and ρ_n are quantum states.

$\mathcal{F}$5. $\mathcal{F}(\rho, K)$ is independent of choice of orthonormal basis $\rho^{ab} = \rho^a \otimes \rho^a$.

$\mathcal{F}$6. For bipartite state the Fisher information is superadditive [31]i.e.,

$$\mathcal{F}(\rho^{ab}, K^a \otimes \mathbb{1}^b + \mathbb{1}^a \otimes K^b) \leq \mathcal{F}(\rho^a, K^a) + \mathcal{F}(\rho^b, K^b).$$

The equality holds only for product state.

3 Resource theory of asymmetry

In quantum mechanics, any resource theory consists of a set of free states and a set of free operations. The amount of resource contained in a state can be quantified using resource monotones. The free states are not possessing the resource under consideration and can not be used for quantum information processing tasks. On the other hand, free operations are quantum operations that map a free state into another free state or unable to create resource from the free states. In order to develop the resource theory of asymmetry based on QFI, we recall the definition of free states and free operations with in the framework of resource theory.

Let $D(\mathcal{H})$ be the convex set of density operators acting on the Hilbert space $\mathcal{H}$ and G is a symmetry group with associated unitary representation $\{U_g\}$. Then we define,

$$\mathcal{U}_g(\tau) = U_g \tau U_g^\dagger \tag{2}$$

and the state $\sigma \in D(\mathcal{H})$ is defined as a symmetric state with respect to G if and only if

$$\mathcal{U}_g(\sigma) = \sigma \tag{3}$$

for all $g \in G$. Then we define a set which constitutes the set of free states for the resource theory of asymmetry as

$$\mathcal{S} := \{\sigma \in D(\mathcal{H}) : \mathcal{U}_g(\sigma) = \sigma\}. \tag{4}$$

Similarly, a set of covariant operations with respect to group G considered as free operation in the resource theory of asymmetry. Such an operation is defined by a superoperator $\mathcal{L} : D(\mathcal{H}) \to D(\mathcal{H})$ such that

$$\mathcal{L}(\mathcal{U}_g(\tau)) = \mathcal{U}_g(\mathcal{L}(\tau)), \forall \ \ g \in G. \tag{5}$$

With the above perspective, we define asymmetry of arbitrary quantum state based on the QFI on the system space $\mathcal{H}$ with respect to Lie group $G \subset S(U)$ with dimension d. Let $\{T_j : j = 1, 2, 3, \cdots, \dim\mathcal{L}_G\}$ be any orthonormal basis of $\mathcal{L}_G$, which is corresponding Lie algebra spanned by the generators of G and a subspace of the Hilbert space consisting of all observables (Hermitian operators) on K with the Hilbert-Schmidt inner product $\langle A|B \rangle = \mathrm{Tr}AB$. Here Tr denotes trace of an operator. The asymmetry of the state ρ with respect to the group G or the corresponding Lie algebra $\mathcal{L}_G$ is defined as

$$\mathcal{A}(\rho, \mathcal{L}_G) = \frac{1}{4} \sum_j \mathcal{F}(\rho, T_j), \tag{6}$$

where the numerical factor $\frac{1}{4}$ is a normalization factor inserted for our convenience. It is worth mentioning that $\mathcal{A}(\rho, \mathcal{L}_G)$ is independent of choice of the orthonormal basis $\{T_j\}$ of $\mathcal{L}_G$ and considered as a well-defined measure. The QFI based measure of asymmetry of the state ρ with respect to the group G has the following interesting properties:

i. $\mathcal{A}(\rho, \mathcal{L}_G) \geq 0$. The equality holds if and only if $[\rho, T_j] = 0, \forall\, j$.

ii. For any unitary operator U, $\mathcal{A}(\rho, \mathcal{L}_G)$ is unitary invariant in the sense that

$$\mathcal{A}(\rho, \mathcal{L}_G) = \mathcal{A}(U\rho U^\dagger, \mathcal{L}_G).$$

iii. $\mathcal{A}(\rho, \mathcal{L}_G)$ is convex with respect to ρ in the sense that

$$\mathcal{A}\left(\sum_i \lambda_i \rho_i, \mathcal{L}_G\right) \leq \sum_i \lambda_i \mathcal{A}\left(\rho_i, \mathcal{L}_G\right).$$

where $\sum_i \lambda_i = 1$, $\lambda_i \geq 0$ and ρ_i are quantum states on the system space H.

The above properties can be seen directly from the properties of Fisher information.

4 Asymmetry-induced nonclassical correlations

In order to quantify the quantum correlation contained in a bipartite quantum state, we first define the asymmetry for bipartite quantum state. Let us consider a bipartite quantum state ρ^{ab} on the composite separable Hilbert space $\mathcal{H}^{ab} = \mathcal{H}^a \otimes \mathcal{H}^b$ shared by a and b. Here ρ^a and ρ^b are the marginal states on the corresponding Hilbert spaces $\mathcal{H}^a$ and $\mathcal{H}^b$ respectively with dimension $\dim \mathcal{H}^{a(b)} = d_{a(b)}$. Then we define the orthonormal basis in the composite Hilbert space as

$$\{X_m \otimes \mathbb{1}^b, \mathbb{1}^a \otimes Y_n : m = 1, 2, 3, \cdots, d_a^2; \quad n = 1, 2, 3, \cdots d_b^2\},$$

where $\{X_m : m = 1, 2, \cdots, d_a^2\}$ and $\{Y_n : n = 1, 2, \cdots, d_b^2\}$ are any orthonormal bases of the Lie algebras $\mathcal{L}^a$ of $U(H^a)$ and $\mathcal{L}^b$ of $U(H^b)$ respectively.

With these settings, one can define asymmetry of a bipartite state as average Fisher information with respect to each generator of the basis. The measure of asymmetry of ρ^{ab} with respect to the group $U(H^a) \times U(H^b)$, or equivalently, the corresponding Lie algebra $\mathcal{L}^{ab} = \mathcal{L}^a \oplus \mathcal{L}^b$:

$$\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) = \frac{1}{4} \sum_{m=1}^{d_a^2} \mathcal{F}(\rho^{ab}, X_m \otimes \mathbb{1}^b) + \frac{1}{4} \sum_{n=1}^{d_b^2} \mathcal{F}(\rho^{ab}, \mathbb{1}^a \otimes Y_n).$$

We can rewrite the asymmetry of bipartite state as

$$\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) = \sum_{m=1}^{d_a^2} \mathcal{A}(\rho^{ab}, \mathcal{L}^a) + \sum_{n=1}^{d_b^2} \mathcal{A}(\rho^{ab}, \mathcal{L}^b). \tag{7}$$

Similarly, we can define asymmetry of local state $\rho^a = \mathrm{Tr}_b(\rho^{ab})$ with respect to the group $U(H^a)$, and the corresponding Lie algebra $\mathcal{L}^a$ as

$$\mathcal{A}(\rho^a, \mathcal{L}^a) = \frac{1}{4} \sum_{m=1}^{d_a^2} \mathcal{F}(\rho^a, X_m). \tag{8}$$

The asymmetry of the local state $\rho^b = \mathrm{Tr}_a(\rho^{ab})$ with the corresponding Lie algebra $\mathcal{L}^b$ is

$$\mathcal{A}(\rho^b, \mathcal{L}^b) = \frac{1}{4} \sum_{n=1}^{d_b^2} \mathcal{F}(\rho^b, Y_n). \tag{9}$$

The asymmetries of the local states $\mathcal{A}(\rho^a, \mathcal{L}^a)$ and $\mathcal{A}(\rho^b, \mathcal{L}^b)$ are also independent of the choice of the orthonormal bases of $\mathcal{L}^a$ and $\mathcal{L}^b$ respectively. The sum of local asymmetries quantifies the total asymmetry of local states.

The measure of asymmetry of the global state $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ with respect to the local unitary group has the following interesting properties:

$\mathcal{A}1.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) \geq 0$, equality holds if and only if ρ^{ab} is a maximally mixed state i.e., $\rho^{ab} = \mathbb{1}^{ab}/d_a d_b$, where $\mathbb{1}^{ab}$ is the identity operator on the bipartite system space $H^a \otimes H^b$.

$\mathcal{A}2.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ is locally unitary invariant i.e.,

$$\mathcal{A}\left(\rho^{ab}, \mathcal{L}^{ab}\right) = \mathcal{A}((U \otimes V)\rho^{ab}(U \otimes V)^\dagger, \mathcal{L}^{ab})$$

$\mathcal{A}3.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ is convex with respect to ρ^{ab} in the sense that

$$\mathcal{A}\left(\sum_j \lambda_j \rho_j^{ab}, \mathcal{L}^{ab}\right) \leq \sum_j \lambda_j \mathcal{A}(\rho_j^{ab}, \mathcal{L}^{ab})$$

$\mathcal{A}4.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ is superadditive in the sense that

$$\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) \geq \mathcal{A}(\rho^a, \mathcal{L}^a) + \mathcal{A}(\rho^b, \mathcal{L}^b).$$

The equality holds only for the product state.

$\mathcal{A}5.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ is not increased under the completely positive and trace preserving (CPTP) map.

$\mathcal{A}6.$ $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ is independent of choice of orthonormal basis.

Now we sketch the proof of the above properties.

It is quite easy to understand the positivity of $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab})$ from the positivity of Fisher information. From Eq. (7), we can understand the fact that $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) = 0$ leads $\mathcal{A}(\rho^{ab}, \mathcal{L}^a) = 0$ and $\mathcal{A}(\rho^{ab}, \mathcal{L}^b) = 0$. Using the result given in ref. [], we can observe that

$$\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) - \mathcal{A}(\rho^a, \mathcal{L}^a) \geq 0. \tag{10}$$

For product state $\rho^{ab} = \rho^a \otimes \rho^b$, the asymmetry of global state is equal to its marginal asymmetry i.e., $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) - \mathcal{A}(\rho^a, \mathcal{L}^a)$. Hence, the above equality holds for only product state. If $\mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) = 0$, from the above equation, we find that $\mathcal{A}(\rho^a, \mathcal{L}^a) = 0$ and this is the case only when $\rho^a = \mathbb{1}/d_a$ is the maximally mixed state. Similarly, we conclude that $\rho^b = \mathbb{1}^b/d_b$ is also the maximally mixed state. Combining these facts, it follows that is $\rho^{ab} = \mathbb{1}^{ab}/d_a d_b$ the maximally mixed state on $H^a \otimes H^b$.

Properties $\mathcal{A}2$–$\mathcal{A}4$ follows straightforwardly from the properties of Fisher information $\mathcal{F}3$–$\mathcal{F}5$. To show the property $\mathcal{A}5$, we have to show that the quantum Fisher information do not increase under CPTP map. To show this, First we define a CPTP map on subsystem b and it can be expressed as

$$\mathcal{I}^a \otimes \mathcal{E}^{ab}(\rho^{ab}) = \text{Tr}_c(\mathbb{1}^a \otimes U^{bc} \rho^{ab} \otimes \rho^c (\mathbb{1}^a \otimes U^{bc})^\dagger) \qquad (11)$$

where ρ^c is a density matrix of an ancillary system c, and U^{bc} is a unitary operation in state space of composite system bc.

$$\begin{aligned}
\mathcal{F}(\mathcal{I}^a \otimes \mathcal{E}^{ab}(\rho^{ab}), T^a \otimes \mathbb{1}^b) &= \mathcal{F}(\text{Tr}_c(\mathbb{1}^a \otimes U^{bc} \rho^{ab} \otimes \rho^c (\mathbb{1}^a \otimes U^{bc})^\dagger, T^a \otimes \mathbb{1}^b) \\
&\leq \mathcal{F}(\mathbb{1}^a \otimes U^{bc} \rho^{ab} \otimes \rho^c (\mathbb{1}^a \otimes U^{bc})^\dagger, T^a \otimes \mathbb{1}^b) \\
&= \mathcal{F}(\rho^{ab} \otimes \rho^c, \mathbb{1}^a \otimes U^{bc}(T^a \otimes \mathbb{1}^b)\mathbb{1}^a \otimes U^{bc})^\dagger) \\
&= \mathcal{F}(\rho^{ab} \otimes \rho^c, T^a \otimes \mathbb{1}^b) \\
&= \mathcal{F}(\rho^{ab}, T^a \otimes \mathbb{1}^b).
\end{aligned}$$

Here the inequality follows from Eq. (10).

Next, we define bipartite quantum correlation measure in terms of asymmetry of bipartite state. In general, the asymmetry of ρ^{ab} with respect to $\mathcal{L}^a$ is larger than the asymmetry of ρ^a with respect to $\mathcal{L}^a$. Similarly, the asymmetry of ρ^{ab} with respect to $\mathcal{L}^b$ is larger than the asymmetry of ρ^b with respect to $\mathcal{L}^b$. We define the quantum correlation as the discrepancy between the asymmetries of a bipartite state and its marginal states with respect to the same group that comes from the correlations contained in the bipartite state. Mathematically, it can be written

$$\mathcal{Q}(\rho^{ab}) = \mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) - \mathcal{A}(\rho^a, \mathcal{L}^a) - \mathcal{A}(\rho^b, \mathcal{L}^b) \qquad (12)$$

as a measure of correlations contained in the state ρ^{ab}. Using the property of QFI ($\mathcal{F}4$.), the correlation measure is redefined as

$$\mathcal{Q}(\rho^{ab}) = \mathcal{A}(\rho^{ab}, \mathcal{L}^{ab}) - \mathcal{A}(\rho^a \otimes \rho^b, \mathcal{L}^{ab}). \qquad (13)$$

Using Eq. (7), it can be rewritten as

$$\begin{aligned}
\mathcal{Q}(\rho^{ab}) &= \mathcal{A}(\rho^{ab}, \mathcal{L}^a) + \mathcal{A}(\rho^{ab}, \mathcal{L}^b) - \mathcal{A}(\rho^a, \mathcal{L}^a) - \mathcal{A}(\rho^b, \mathcal{L}^b) \\
&= \mathcal{A}(\rho^{ab}, \mathcal{L}^a) - \mathcal{A}(\rho^a, \mathcal{L}^a) + \mathcal{A}(\rho^{ab}, \mathcal{L}^b) - \mathcal{A}(\rho^b, \mathcal{L}^b) \\
&= \mathcal{Q}^a(\rho^{ab}) + \mathcal{Q}^b(\rho^{ab}) \qquad (14)
\end{aligned}$$

Here $\mathcal{Q}^{a(b)}(\rho^{ab}) = \mathcal{A}(\rho^{ab}, \mathcal{L}^a) - \mathcal{A}(\rho^{a(b)}, \mathcal{L}^{a(b)})$, which quantifies the discrepancy between the asymmetries between ρ^{ab} and $\rho^{a(b)}$ with respect to the local unitary group on system $a(b)$. In general, a faithful measure of bipartite correlation should satisfy three necessary axioms such as (i) should be zero for tensor product states, and a positive for entangled states, (ii) should be invariant under local unitary transformations on each one of the two subsystems and (iii) should coincide with quantum entanglement for pure states. Here we demonstrate the some important properties of asymmetry based quantum correlation measure $\mathcal{Q}(\rho^{ab})$:

$\mathcal{Q}1.$ $\mathcal{Q}(\rho^{ab}) \geq 0$. The equality hold for product state $\rho^{ab} = \rho^a \otimes \rho^b$.

$\mathcal{Q}2.$ $\mathcal{Q}(\rho^{ab})$ is locally unitary invariant.

$\mathcal{Q}3.$ $\mathcal{Q}(\rho^{ab})$ is convex with respect to ρ^{ab} in the sense that

$$\mathcal{Q}\left(\sum_j \lambda_j \rho_j^{ab}, \mathcal{L}^{ab}\right) \leq \sum_j \lambda_j \mathcal{Q}(\rho_j^{ab}, \mathcal{L}^{ab}).$$

$\mathcal{Q}4.$ For any pure state $|\Psi\rangle$ with $d = d_a \leq d_b$, $\mathcal{Q}(|\Psi\rangle\langle\Psi|) \leq (d-1)/d$. The maximum is achieved when the state is maximally entangled.

The properties $\mathcal{Q}1$–$\mathcal{Q}3$ are straightforward to prove from the properties of asymmetry of bipartite state. Hence, the asymmetry based quantum correlation is a valid quantifier of bipartite correlation. We shall provide proof of the property $\mathcal{Q}4$ in the next section.

Now, we generalize these formalisms to the resource theory of asymmetry for multipartite system and define a measure of multipartite quantum correlation measure using asymmetry. Let $\rho^{a_1,a_2,\cdots,a_n}$ be the multipartite states of the system space $\mathcal{H}^{a_1} \otimes \cdots \otimes \mathcal{H}^{a_n}$. The asymmetry of the state directly defined as

$$\mathcal{A}(\rho^{a_1,\cdots,a_n}, \mathcal{L}^{a_1,\cdots,a_n}) = \sum_{i=1}^{n} \mathcal{A}(\rho^{a_1,\cdots,a_n}, \mathcal{L}^{a_i}), \qquad (15)$$

where $\mathcal{L}^{a_1,\cdots,a_n} = \mathcal{L}^{a_1} \oplus \cdots \oplus \mathcal{L}^{a_1}$ is the Lie algebra of the local unitary group $U(H^{a_1}) \times \cdots \times U(H^{a_n})$ on the total space $H^{a_1} \cdots \otimes H^{a_n}$. The corresponding measure of correlations can be defined as

$$\mathcal{Q}(\rho^{a_1,\cdots,a_n}) = \mathcal{F}(\rho^{a_1,\cdots,a_n}, \mathcal{L}^{a_1,\cdots,a_n}) - \mathcal{F}(\rho^{a_1} \otimes \cdots \otimes \rho^{a_n}, \mathcal{L}^{a_1,\cdots,a_n}),$$

and in terms of asymmetry

$$\mathcal{Q}(\rho^{a_1,\cdots,a_n}) = \sum_{i=1}^{n} \mathcal{A}(\rho^{a_1,\cdots,a_n}, \mathcal{L}^{a_i}) - \mathcal{A}(\rho^{a_i}, \mathcal{L}^{a_i}). \qquad (16)$$

The above measure quantifies the amount of correlations in multipartite quantum state $\rho^{a_1,\cdots,a_n}$ in terms of the differences between asymmetries of the global and local states.

4.1 Examples

In this section, we compute the asymmetry induced correlation measure for an arbitrary pure and Bell diagonal state.

Pure state: The Schmidt decomposition of the pure state as

$$|\psi\rangle = \sum_{i=1}^{d} \sqrt{\lambda_i} |i\rangle_a |i\rangle_b, \qquad d = \min\{d_a, d_b\}, \qquad (17)$$

with reduced state $\rho^{a(b)} = Tr_{b(a)}(|\psi\rangle\langle\psi|)$ and $\boldsymbol{\lambda}_{|\psi\rangle} = (\lambda_1, \lambda_1, \cdots, \lambda_d)$ be the Schmidt coefficients, with $\lambda_i \geq 0$ and $\sum_i \lambda_i = 1$. To compute $\mathcal{Q}(|\psi\rangle\langle\psi|)$, first we define the set of generators as

$$T_{ij}^{(1)} = \frac{1}{2}(|i\rangle\langle j| + |j\rangle\langle i|), \qquad 1 \leq i < j \leq d,$$

$$T_{ij}^{(2)} = \frac{1}{2\mathrm{i}}(|i\rangle\langle j| - |j\rangle\langle i|), \qquad 1 \leq i < j \leq d,$$

$$T_{ij}^{(3)} = \frac{1}{2\sqrt{i(i-1)}}(|1\rangle\langle 1| + |2\rangle\langle 2| + \cdots + |i-1\rangle\langle i-1| + (1-i)|i\rangle\langle i|), \qquad 2 \leq i \leq d.$$

Then, we compute

$$\mathcal{F}(|\psi\rangle\langle\psi|) = \frac{1}{2}\left(d - \sum_i^d \lambda_i^2\right), \qquad \mathcal{F}(\rho^a) = \frac{1}{4}\left(2d - \sum_{i,j} \frac{4\lambda_i\lambda_j}{\lambda_i + \lambda_j}\right),$$

and

$$\mathcal{Q}^a(|\psi\rangle\langle\psi|) = \frac{1}{2}\sum_{i \neq j}\left(\frac{2\lambda_i\lambda_j}{\lambda_i + \lambda_j} + \lambda_i\lambda_j\right).$$

Considering the same set of generators for the subsystem b, we have

$$\mathcal{Q}^b(|\psi\rangle\langle\psi|) = \frac{1}{2}\sum_{i \neq j}\left(\frac{2\lambda_i\lambda_j}{\lambda_i + \lambda_j} + \lambda_i\lambda_j\right).$$

Then

$$\mathcal{Q}(|\psi\rangle\langle\psi|) = \sum_{i \neq j}\left(\frac{2\lambda_i\lambda_j}{\lambda_i + \lambda_j} + \lambda_i\lambda_j\right).$$

$\mathcal{Q}^a(|\psi\rangle\langle\psi|)$ is a Schur concave, implies that $\mathcal{Q}^a(|\psi\rangle\langle\psi|) \geq \mathcal{Q}^a(|\phi\rangle\langle\phi|)$ iff the Schmidt coefficient $\boldsymbol{\lambda}_{|\psi\rangle}$ is majorized by $\boldsymbol{\lambda}_{|\phi\rangle}$, which means that $\sum_{l=1}^k \lambda_{|\psi\rangle}^{\downarrow} \leq \sum_{l=1}^k \lambda_{|\psi\rangle}^{\downarrow}$ with $k = 1, 2, \cdots, d$ and equality is required for $k = d$, where $\downarrow$ indicates rearranging elements in descending order. From Schur concavity, the bound of $\mathcal{Q}(|\psi\rangle\langle\psi|)$ is given as

$$0 \leq \mathcal{Q}(|\psi\rangle\langle\psi|) \leq \frac{d-1}{d}. \tag{18}$$

Here the lower bound is reached by product states with $\boldsymbol{\lambda}_{|\psi\rangle}^{\downarrow} = (1, 0, 0, \cdots, 0)$ and the upper bound is reached by maximally entangled states with $\boldsymbol{\lambda}_{|\psi\rangle}^{\downarrow} = (1/d, 1/d, \cdots, 1/d)$. The concurrence of the pure state is $C(|\psi\rangle\langle\psi|) = 2\sqrt{\lambda_1\lambda_2}$. The $\mathcal{Q}^a(|\psi\rangle\langle\psi|)$ is computed as

$$\mathcal{Q}^a(|\psi\rangle\langle\psi|) = 3\lambda_1\lambda_2, \tag{19}$$

and the correlation is connected with the concurrence in the following way

$$\mathcal{Q}(|\psi\rangle\langle\psi|) = \frac{3}{2}C^2(|\psi\rangle\langle\psi|). \tag{20}$$

Bell diagonal state: Next we study the asymmetry of the Bell diagonal state whose marginal states are maximally mixed. In Bloch representation of the state can be expressed as

$$\rho = \frac{\mathbb{1}}{4} + \sum_i c_i \sigma_i \otimes \sigma_i. \tag{21}$$

The asymmetries of the local states are $\mathcal{A}(\rho^a, \mathcal{L}^a) = \mathcal{A}(\rho^b, \mathcal{L}^b) = 0$. The Fisher information of the global state is

$$\mathcal{F}(\rho, T) = 3 - 4 \sum_{i>j} \frac{\beta_i \beta_j}{\beta_i + \beta_j}, \tag{22}$$

where β_i are the eigenvalues of Bell diagonal state with

$$\beta_1 = 1/4 - c_1 + c_2 + c_3, \qquad \beta_2 = 1/4 + c_1 - c_2 + c_3,$$
$$\beta_3 = 1/4 + c_1 + c_2 - c_3, \qquad \beta_4 = 1/4 - c_1 - c_2 - c_3.$$

Then,

$$\mathcal{Q}^a(\rho) = \mathcal{Q}^b(\rho) = \frac{1}{4}\left(3 - 4\sum_{i>j} \frac{\beta_i \beta_j}{\beta_i + \beta_j}\right),$$

and the quantum correlation is

$$\mathcal{Q}(\rho) = \frac{1}{2}\left(3 - 4\sum_{i>j} \frac{\beta_i \beta_j}{\beta_i + \beta_j}\right). \tag{23}$$

5 Conclusions

In conclusion, we have defined the asymmetry of state as average of quantum Fisher information with respect to Lie groups and Lie algebras. As a tool on the way, we have shown that the discrepancy between the asymmetry of bi-partite global state and asymmetries of local states quantifies the quantum correlation contained in a bipartite quantum state. Further, this quantity satisfies the axioms of a valid measure of bipartite quantum correlation and is nonincreasing under the action of general CPTP quantum channels on one of the subsystems. Hence, the proposed quantity is a good measure of bipartite quantum correlation.

Since the asymmetry-based quantum correlation measure does not have an optimization procedure, is a computable measure irrespective of dimension of the Hilbert space of the subsystems. We have established a simple relation

between the concurrence and the correlation measure. Moreover, we have generalized the measure to multipartite settings. Finally, the significance of this measure was illustrated with some examples.

The quantum Fisher information is also closely connected with parameter estimation and useful in quantum metrology. The proposed correlation measure has more insight into metrology and quantum information theory.

References

1. Chitambar, E., Gour, G.: Quantum resource theories. Rev. Mod. Phys. **91**, 025001 (2019).
2. Einstein, A., Podolsky, B, Rose, R.: Can quantum–mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935).
3. Schrödinger, E.: Discussion of probability relations between separated systems. Proc. Cambridge Philos. Soc. **31**, 555 (1935).
4. Nielsen, M. A., Chuang, I. L.: Quantum Computation and Quantum Information. Cambridge University Press; Cambridge (2000).
5. Knill, E., Laflamme, R.: Power of one bit of quantum information. Phys. Rev. Lett. **81**, 5672. (1998).
6. Datta, A., Flammia, S.T., Caves, C.M.: Entanglement and the power of one qubit. Phys. Rev. A **72**, 042316 (2005).
7. Datta, A., Shaji, A., Caves, C.M.: Quantum discord and the power of one qubit. Phys. Rev. Lett. **100**, 050502 (2008).
8. Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A: Math. Theor. **34**, 6899 (2001).
9. Ollivier, H., Zurek, W. H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. **88**, 017901 (2001).
10. Dakic, B., Vedral, V., Brukner, C.: Necessary and sufficient condition for nonzero quantum discord. Phys. Rev. Lett. **105**, 190502 (2010).
11. Luo, S., Fu, S.: Measurement-induced nonlocality. Phys. Rev. Lett. **106**, 120401 (2011).
12. Luo, S.: Using measurement-induced disturbance to characterize correlations as classical and quantum. Phys. Rev. A **77**, 022301 (2008).
13. Xiong, S., Zhang, W.J., Yu, C.-S., Song, H.-S.: Uncertainty-induced nonlocality. Phys. Lett. A **378**, 344 (2014).
14. Muthuganesan, R., Chandrasekar, V. K.: Measurement induced nonlocality based on afinity" Commun. Theor. Phys. **72**, 075103 (2020).
15. Muthuganesan, R., Sankaranarayanan, R.: Fidelity based measurement induced nonlocality, Phys. Lett. A **381**, 3028 (2017).
16. Baumgratz, T., Cramer, M., Plenio, M. B.: Quantifying Coherence. Phys. Rev. Lett. **113**, 140401 (2014).
17. Girolami, D.: Observable Measure of Quantum Coherence in Finite Dimensional Systems. Phys. Rev. Lett. **113**, 170401 (2014).
18. Muthuganesan, R., Chandrasekar, V. K., Sankaranarayanan, R.: Quantum coherence and correlation measures based on affinity. arxiv: 2003.13077.
19. Dowling, M.R., Doherty, A.C., Wiseman, H.M.: Entanglement of indistinguishable particles in condensed matter physics. Phys. Rev. Lett. **91** 097902 (2003).
20. Vaccaro, J. A., Anselmi, F., Wiseman, H.M., Jacobs, K.: Trade-off between extractable mechanical work, accessible entanglement, and ability to act as a reference system, under arbitrary superselection rules. Phys. Rev. A **77**, 032114 (2008).
21. Wiseman, H. M., Vaccaro, J. A.: The entanglement of indistinguishable particles shared between two parties. Phys. Rev. Lett. **91**, 097902 (2003).

22. Piani, M., Cianciaruso, , M., Bromley, T.R., Napoli, C., Johnston, N, and Adesso, G.: Robustness of asymmetry and coherence of quantum states. Phys. Rev. A, **93**, 042107 .

23. Bu, K., Anand, N., Singh, U.: Asymmetry and coherence weight of quantum states. Phys. Rev. A **97**, 032342 (2018).

24. Fang, Y. N., Dong, G. H., Zhou, D. L. Sun, C. P. Quantification of Symmetry. Commun. Theor. Phys. **65**, 423–433 (2016).

25. Dong, G.-H., Fang, Y-N., Sun, C-P.: Quantifying Spontaneously Symmetry Breaking of Quantum Many-Body Systems. Commun. Theor. Phys. **68**, 405 (2017).

26. Yao, Y., Dong, G.-H., Xiao, X., Sun, C-P.: Frobenius-norm-based measures of quantum coherence and asymmetry. Sci. Rep. **6**, 32010 (2016).

27. Helstrom, C. W.: Quantum Detection and Estimation Theory, Mathematics in Science and Engineering, Elsevier, New York (1976)

28. Braunstein, S. L., Caves, C. M.: Statistical distance and the geometry of quantum states. Phys. Rev. Lett. **72**, 3439(1994).

29. Modi, K., Cable, H., Williamson, M., Vedral, V.: Quantum Correlations in Mixed-State Metrology. Phys. Rev. X **1**,021022 (2011)

30. Tóth, I. G., Apellaniz.: Quantum metrology from a quantum information science perspective. J. Phys. A: Math. Theor. **47**, 424006 (2014).

31. Li, N., Luo, S.: Entanglement detection via quantum Fisher information. Phys. Rev. A **88**, 014301 (2013).